Description of the Session 1: Studying mathematics teaching with a focus on fractions

In this session, participants will be introduced to the goals of the module, which include (a) learning mathematics; (b) developing skill with high-leverage teaching practices; (c) developing skill in anticipating, interpreting, managing, and using students' ideas and ways of thinking; and (d) developing ways of analyzing and learning from practice. Participants are oriented to records of practice (such as video) as a means to study mathematics teaching. Participants watch a video in which third grade students are solving a fraction comparison problem, and then analyze this video using three different lenses: mathematics, student thinking, and teaching practice. Participants then consider the instances of equivalence in mathematics, a topic that will be revisited throughout the module. The session closes with an overview of the classroom connection activities that will be completed prior to the next session.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
I. Module preview	15 minutes	Part 1	• Participants will be oriented to the work of the module. • Participants will be oriented to the work of the session.
II. Studying mathematics teaching	65 minutes	Parts 2, 3, 4, & 5	• Participants will understand challenges associated with studying teaching; how records of practice can be used to study mathematics teaching; and how this module uses records of practice to study mathematics teaching. • Participants will begin to be able observe and discuss mathematics teaching attending to mathematics, student thinking, and teaching practices (these are core elements of module content). • Participants will begin to develop norms for discussing records of mathematics teaching (a video). • Participants will be able to articulate the affordances and constraints of various representations for a particular fraction comparison problem.
III. Instances of equivalence in mathematics	20 minutes	Part 6	• Participants will begin to unpack the mathematical idea of equivalence. • Participants will see that the mathematical idea of equivalence emerges in many mathematics topics and is relevant across the K-5 spectrum.
IV. Wrap up	10 minutes	Part 7	• Participants will understand the function of the classroom connection activities and will understand the assignment.

Note: There are 10 minutes of this two-hour session that have been left open to address any local issues or logistics related to this professional development experience.

Classroom Connection Activities

Required
Type of task: Reading Description: Math notes: Instances of equivalence in mathematics
Type of task: Reading Description: Ball, Hill, and Bass (2005) on mathematical knowledge for teaching

Preparing for the session

☐ Make copies as needed:

- *Resources:* Transcript: Four-fourths and four-eighths (Part 3); Handout: Focus questions (Part 3); Handout: Possible instances of equivalence in mathematics
- *Supplemental resources:* Math notes: Instances of equivalence in mathematics (Part 6)

☐ Customize the Classroom Connection Activities and make copies as needed

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

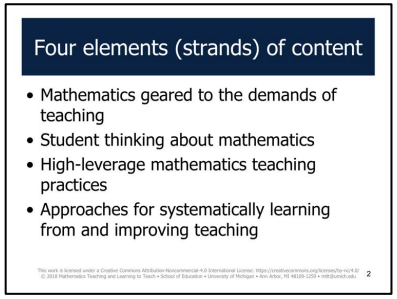
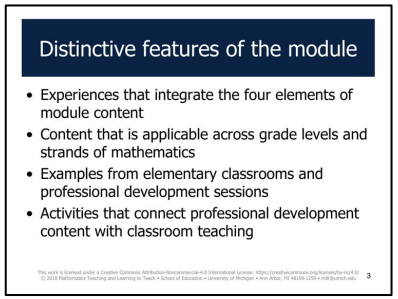
1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards for School Mathematics.

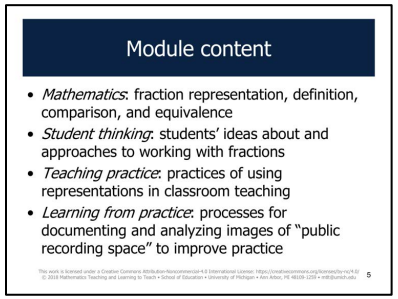
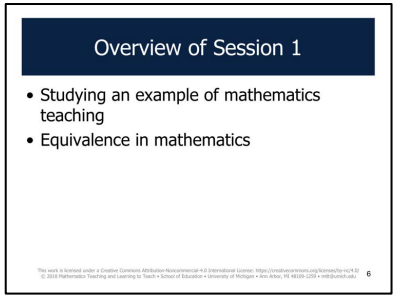
Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **representing fractions** • defining fractions • using and explaining methods and representations for comparing fractions • **understanding how equivalence (of fractions) can be represented and used**	• **identifying and analyzing student conceptions, explanations, and representations of fractions** • identifying and analyzing student strategies for comparing fractions	• selecting and generating representations • connecting representations • narrating the process of representing • supporting students in narrating the use of a representation • recording contributions and emerging mathematical ideas	• studying public recording space to learn from practice • using a conceptual framework to guide the planning, use, and analysis of public recording space

Part 1: Module preview (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the module. • Participants will be oriented to the work of the session.	1. Introduce the module by having participants watch *Video A.* 2. Introduce the session by having participants watch *Video B.*	• Video A (06:21): Module overview • Video B (02:03): Session 1 overview

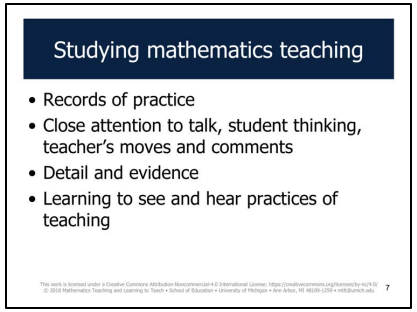
Detailed description of activity	Comments & other resources
1. Introduce the module by having participants watch *Video A* in which Dr. Deborah Loewenberg Ball, a professor at the University of Michigan, provides an overview of the module. Each Dev-TE@M module focuses on four core elements of the work of elementary teaching: • Mathematics geared to the demands of teaching • Student thinking about mathematics • High-leverage mathematics teaching practices • Approaches for systematically learning from and improving teaching  Work on these elements is integrated across the ten sessions of a module, providing opportunities to practice, build on, and extend ideas over time. In addition, simultaneously working on the four core elements is important because the work of elementary mathematics teaching requires integrated attention to these elements in practice. The content of the module is applicable across grade levels and strands of mathematics. The module includes examples from elementary classrooms and professional development sessions. Many of the videos used in this module are taken from a professional development workshop for teachers led by Dr. Ball. The teachers in the professional development workshop focused on the same content and many of the same activities as are used in this module. These videos will serve as a context (or text) for the work in the module. Video from the professional development session is often used to frame and summarize activities in the module. The video also is used to provide opportunities to listen and respond to ideas raised by the teachers in the video, as well as to analyze  	*Hold off on having participants introduce themselves. There will be an opportunity for introductions in Part 2.* *To play content in the main viewer, simply move your cursor over the image and click. When your cursor is over the image, you will also see controls for volume, resizing the image, and a player bar that allows you to rewind or move forward in the video. The slides are synchronized with the video and will change at a fitting moment.* *Participants may have questions about the use of video in the module.* *Emphasize that participants will take pictures to learn from their own teaching. These records of practice involve public recording spaces such as whiteboards and charts. In addition, the focus is on both the teacher's and students' use of public recording space.*

Detailed description of activity	Comments & other resources
and discuss many of the issues with which they grappled during the course. The module includes Classroom Connection Activities because learning about teaching is an extended process that can be supported by learning in and from one's own teaching. These activities provide significant opportunities to learn. Routine tasks in the activities encourage participants to use their teaching as a context for learning, to connect professional development content with common classroom resources, support feedback on learning and teaching, as well as extend thinking about mathematics and teaching that arose in the session.	
2. Introduce the content of this module. The content of this module—Representing and Comparing Fractions in Elementary Mathematics Teaching—focuses on: • Mathematics: fraction representation, definition and equivalence • Student thinking: students' ideas about and approaches to working with fractions • Teaching practice: using representations in classroom teaching • Learning from practice: protocols for learning from images of blackboards and other public recording space 	*This slide can be found in the resources section of the right side of the viewer.*
3. Introduce the session: Session 1 begins work on studying mathematics teaching and begins to consider instances of equivalence in mathematics. Have participants watch *Video B* in which Dr. Ball frames the work of the session. 	

Part 2: Using records of practice to study mathematics teaching (~15 minutes)

Goals	***Instructional sequence***	***Resources***

• Participants will understand:
  ○ challenges associated with studying teaching;
  ○ how records of practice can be used to study mathematics teaching; and
  ○ how this module uses records of practice to study mathematics teaching.
	1. Introduce Part 2 and watch Video A.
	2. Teachers share their experiences videotaping their own teaching.
	3. Watch the series of videos discussing the use of video in learning about teaching.
		• Video A (02:42): The challenges in learning about teaching
		• Video B (00:26): Experience with video (example 1)
		• Video C (00:39): Experience with video (example 2)
		• Video D (01:56): Studying mathematics teaching in records of practice

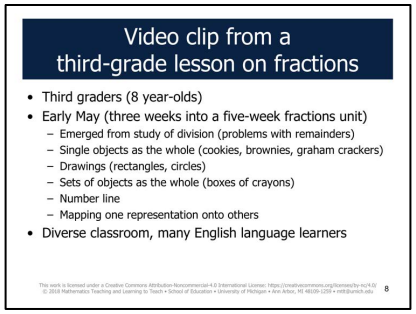
Detailed description of activity	Comments & other resources
1. Introduce Part 2: One challenge in studying teaching is that the work of teaching is often invisible. Much of what teachers do remains elusive and difficult to observe. Moreover, teachers often do things tacitly and therefore do not make visible what they are doing or easily explain it to others. In this part of the session, participants will explore challenges associated with studying teaching and the ways in which learning experiences in this module are designed to address those challenges.  Have participants watch *Video A* in which Dr. Ball describes both the challenges associated with learning about teaching and the ways in which learning experiences in this module are designed to address those challenges.	*In the video, Dr. Ball suggests:* • *Looking at records of teaching and learning practice such as videotapes or copies of student work closely to improve teaching.* • *There is a need to pay close attention to talk, student thinking, teacher moves' and comments when watching video.* • *There is a need to be detailed when talking about records of practice and to try to use evidence from the records to support assertions.* • *One goal is to learn to see, hear, and learn from practices of teaching.*
2. Have participants introduce themselves as they also share any experiences they have with using records of practice (i.e., video, audios, lesson plans, photos, work samples, etc.) to learn about their teaching. In cases where participants do not know each other already, have participants share information about their current teaching assignment and an interesting fact related to their teaching experience (grades, places, subjects they have taught, when they started teaching, etc.)	*This part serves two functions. (1) Having participants introduce themselves. (2) Having participants share and reflect on how they have used records of practice to learn about their own teaching.* *Encourage participants to explain how they used the records they collected to learn from their teaching. Participants may focus on the processes and activities that generate records of their own practice and not on the use of those records in learning from teaching.*

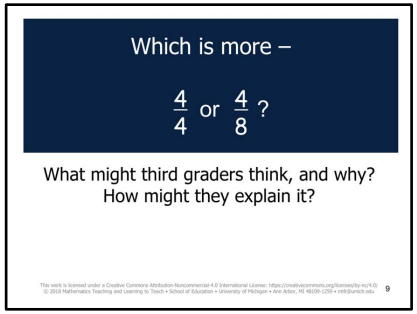
Detailed description of activity	Comments & other resources
3. (Optional) Have participants watch a sequence of videos in which the teachers and Dr. Ball discuss the use of video in learning about teaching: • *Video B*: A teacher describes looking at records of her own teaching as very hard, but she found it served as an opportunity to get feedback from others. • *Video C*: A teacher describes how videotaping allowed her to have a dialogue about her teaching with colleagues. • *Video D*: Dr. Ball describes the use of video records of teaching in this module. In particular, she notes that video can serve as a common text for discussion of teaching.	*Videos B and C might be used to spark conversation if these points have not been offered by participants. Video D can be used to orient participants to how video records are used in the module.*

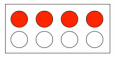
Part 3: Analyzing a video from a third-grade classroom (~20 minutes)

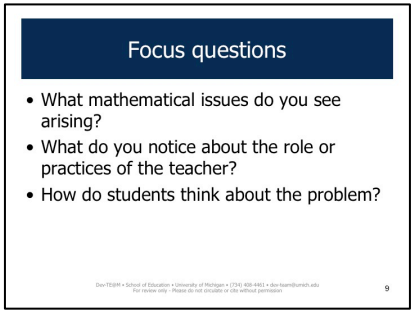
Goals	***Instructional sequence***	***Resources***
• Participants will begin to be able observe mathematics teaching attending to mathematics, student thinking, and teaching practices (these are core elements of module content).	1. Introduce Part 3 and watch Video A. 2. Participants anticipate solutions and explanations of third-grade students. 3. Watch Video B, introducing the focus questions that participants will use. 4. Watch the "4/4 and 4/8" classroom video.	• Video A (02:57): Setting the context for the video • Video B (01:39): Introducing the focus questions • Video C (06:05): Four-fourths and four-eighths • Transcript: Four-fourths and four-eighths • Handout: Focus questions

Detailed description of activity	Comments & other resources
1. Introduce Part 3: This part uses video from a third-grade mathematics class to begin to explore core elements of elementary teaching: mathematics, student thinking, and teaching practice. Have participants watch *Video A* in which Dr. Ball provides background context for watching the "4/4 and 4/8" video. She describes the classroom context (see slide) and the task posed to students (see slide).	CCSSM Link: Comparing fractions with the same numerator is a Grade 3 standard (3.NF.3).





2. Have participants anticipate how third graders might think about the problem: *Which is more: 4/4 or 4/8?* Encourage participants to think about how third grade students might explain their answers. Have participants briefly discuss their anticipations.	*Anticipating student thinking is a routine activity in the module. Practicing teachers are likely to know many ways in which students may think about math problems and may also hold strong opinions about whether or not problems are appropriate and the extent to which students are likely to succeed when presented with particular problems. You may need to persist or redirect participants to move past initial judgments and to engage in predicting what students might do with problems that were actually used with students.* *Participants may focus on the teaching of fractions and strategies that they tend to use. Encourage participants to focus on the mathematical aspects of those approaches. For example, when participants anticipate third graders' solutions and explanations they may talk about the practices they use to help students learn about fractions such as, "I teach my students to look at the bottom number of a fraction first." A facilitator could ask, "Why is it important to attend to the denominator at that point?"* *Predications about the case at hand may include third-grade students using area models, number line(s), or discrete models to compare the two fractions. Some children may use two representations to compare, while others might compare using only one. Children may think that the fraction with the greater denominator is larger because 8 is bigger than 4. Other children may believe that the fractions are equal because each fraction has 4 in the numerator.* *Students might use different wholes (see below).* *Example #1:* Using a discrete model (with different wholes) to compare. 4/4  4/8 

3. Have participants watch *Video B* in which Dr. Ball introduces focus questions for watching the video clip (see the slide). 	*The focus questions are also available in a handout format (Handout: Focus questions).*
4. Distribute *transcripts* and have participants watch *Video C* in which students compare 4/4 and 4/8. Tell participants to think about the focus questions as they watch the video and suggest using the transcript to keep track of things that stand out in the video.	*Watching a video of teaching in a professional development context may be a new experience for some participants. Some participants may think that the teaching in the videos is intended as an example of "good teaching" to be emulated. Try to make clear that the video is not intended to model "good teaching," rather the video is being used to study the practice of teaching mathematics.* *Responses to these focus questions will be discussed in the next part of the session.* *You may need to help participants navigate between noting ideas in the transcript and attending to the video. The video includes more information than the transcript, so it is important to watch the action. However, making some notations in the transcript will help participants in the next part when they are asked to refer to specific examples in the video to support their ideas.*

Part 4: Discussing the third-grade video (~15 minutes)

Goals

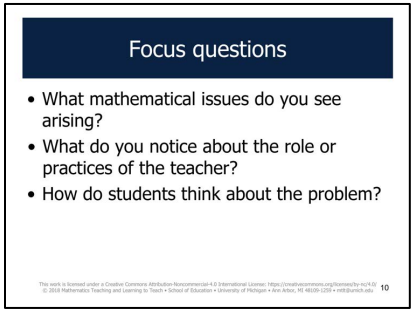
- Participants will begin to be able observe and discuss mathematics teaching attending to mathematics, student thinking, and teaching practices (these are core elements of module content).
- Participants will begin to develop norms for discussing records of mathematics teaching (a video).

Instructional sequence

1. Introduce Part 4 and discuss the video using the focus questions.
2. Watch some of the videos in which teachers raise issues about the video and then discuss.

Resources

- Video A (01:23): Mathematics: The use of mathematical language
- Video B (01:08): Mathematics: Attending to the whole when working with fractions
- Video C (02:00): Student thinking: "Taking" fraction parts

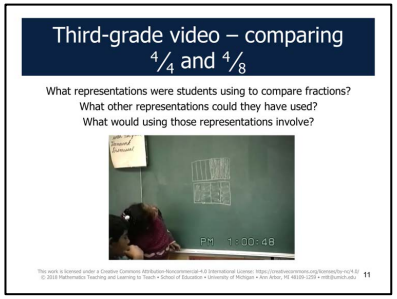
Detailed description of activity	Comments & other resources
1. Introduce Part 4: This part provides participants with an opportunity to discuss observations from the third-grade video. Remind participants that the classroom video is an example of teaching, not an instance of exemplary teaching. Participants should treat the video as a "text" with concrete instances of teaching that can be seen, heard, and used to support description and analysis. Have participants discuss the focus questions in either small or whole group.  	*Participants may make general comments about the teaching and learning in the video, which are focused on the causes/reasons for particular moves. Encourage participants to respond to the focus questions by listening for and if needed asking about facets related to mathematics, teaching, and student thinking. Support norms of attending to detail and supporting claims with evidence from the video/transcript.* *Ideas that participants may raise in response to the focus questions include:* *Mathematics: Discussion about how many "lines" to add without reference to the necessity of equal parts (see line 50).* *Teaching moves:* • *"Asking students to repeat their explanation" (see line 31).* • *"Agree with what" (see line 42). The teacher pushed a student to be explicit why (or what) he agreed with Mei's explanation.* *Student thinking:* • *To divide a whole into n parts, you add (n-1) lines (see line 51).* • *The use of area models to compare fractions.* *CCSSM Link: The classroom video contains examples of third grade students who are engaged in at least three of the mathematical practices (1, 3, and 6). In this segment, the students are making sense of the problem and persevering in solving the problem (#1), constructing viable arguments about their solution (#3) and communicating their reasoning to others (#6).*

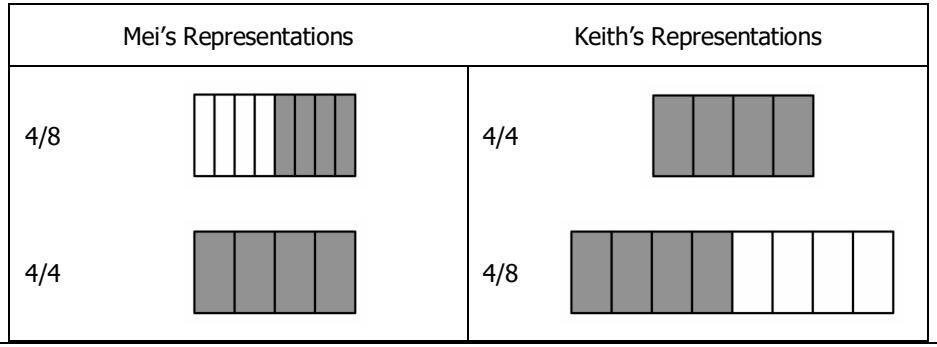
Detailed description of activity	Comments & other resources
2. Watch and discuss *Video A* and *Video B* in which teachers discuss their reactions to the third-grade video. For each clip selected, show the video and then have participants discuss their reactions to the issues raised. The issues raised in the clips include: • *Video A*: The need for students use mathematical language to give explanations. • *Video B:* The different ways in which students may have interpreted a whole. As time and interest permit, watch and discuss Video C. • *Video C:* The differing meanings of the use of the term "taking away".	*Very often sessions will have parts where collections of video contributions from teachers are available to show your participants. You may want to show clips that raise points you feel are important to discuss. You may also want to extend your consideration of a particular point by using clips where the teachers in the video consider the same issues that arose in your discussions.*

Part 5: Representations for comparing 4/4 and 4/8 (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be able to articulate the affordances and constraints of various representations for a particular fraction comparison problem.	1. Introduce Part 5 and have participants recall the representations used in the "4/4 and 4/8" video. 2. Discuss responses to focus questions. 3. Watch a video in which teachers discuss other representations that could have been used in the 4/4 and 4/8 comparison task. 4. Begin to develop a set of criteria for selecting a representation.	• Video (04:51): Teachers brainstorm representations that could have been used

Detailed description of activity	Comments & other resources
1. Introduce Part 5: Representation is a key aspect of doing, learning, and teaching mathematics. This part uses the third-grade video to discuss possible representations for comparing fractions and to begin to explore aspects of representation that are important in teaching. Have participants take a moment to recall what representations were used in the video to compare 4/4 and 4/8.	*Representation is a fundamental practice in both mathematics and mathematics teaching. Mathematical concepts and processes can be represented through diverse mediums – language, symbols, diagrams, images, physical materials, etc. – each with its own affordances and limitations. Some provide intuition, sense making, transparency, but may be cumbersome and inefficient for broad use. Others may be elegantly compact and efficient, at the cost of being abstract and conceptually opaque. The former are often appropriate for the introductory stages of instruction, while the latter may characterize the disciplinary end state toward which instruction is oriented. What is required is not only a wide and adaptable repertoire of representations, but also the making of explicit and precise correspondences establishing an equivalence of different representations of the same idea.*





Detailed description of activity	Comments & other resources
2. Have participants discuss the following questions with a partner or in whole group: • What other representations could they have used? • What would using those representations involve?	*As teachers suggest possible representations, you might probe their suggestions to unpack how the representations could be used to represent these numbers. Some possible questions include:* • *What would the whole be with that representation?* • *How would you represent 4/4? 4/8?* • *How does the representation show equal parts? How do use it to make fourths? eighths?* • *How would the representation show which fraction is greater?* • *Are there denominators that are more or less difficult to use this representation with?* • *What ideas about fractions does the representation highlight?*
3. Have participants watch the *video* in which teachers brainstorm ideas about representations that could have been used. These representations include: time, money, discrete (set) models, and a ruler. Encourage participants to listen for and make notes about the multiple criteria that the teachers are using to weigh those options. Consider pausing the video at 1:07 and ask participants to suggest possible representations and unpack the way in which they work with the comparison problem 4/4 vs. 4/8.	*In the context of the problem comparing 4/4 and 4/8, several of the potential representations mentioned by the teachers in the video may be problematic. Analyzing the limitations of representations can provide mathematical learning opportunities for participants, and also establish the importance of an open, but thoughtful stance toward teaching.*
4. Have participants review their list of criteria and discuss briefly with a partner. Ask whether there are additional criteria that participants use when selecting a representation.	

Part 6: Instances of equivalence in mathematics (~20 minutes)

Goals

- Participants will begin to unpack the mathematical idea of equivalence.
- Participants will see that the mathematical idea of equivalence emerges in many mathematics topics and is relevant across the K-5 spectrum.

Instructional sequence

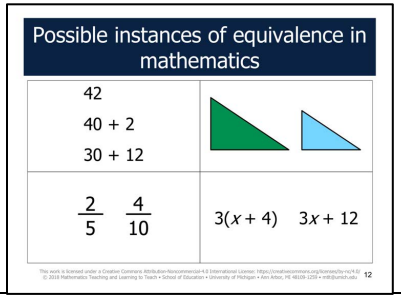
1. Introduce Part 6 and watch Video A.
2. Explore four possible instances of equivalence in mathematics.
3. Watch videos B-E.

Resources

- Video A (01:23): Overview of equivalence
- Video B (01:22): Place value example
- Video C (00:42): Geometry example
- Video D (01:48): Fraction example
- Video E (04:13): Algebra example

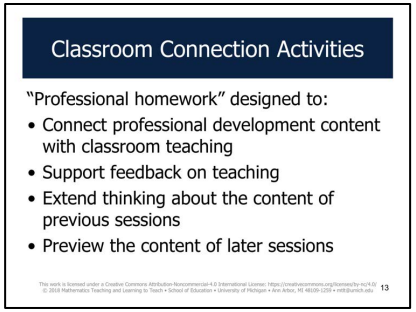
Supplemental

- Math notes: Instances of equivalence in mathematics

Detailed description of activity	Comments & other resources
1. Introduce Part 6: Equivalence is an important idea in mathematics and one that will be considered throughout the module. Watch *Video A* in which Dr. Ball provides an overview of equivalence.	*CCSSM Link: The notion of equivalence is implicitly and explicitly worked on throughout the elementary grades. Much of this work is based on the properties of equality.*
2. Have participants work with a partner to consider whether each of the four examples shown on the slide is an example of equivalence in mathematics. As participants work, encourage them to try to explain why an example is or is not an instance of equivalence. 	*A handout containing the slide is available in the resources section.* *The Math notes document includes explanations of each of these examples and can be found in the supplemental resources section in the bottom right portion of the viewer. Participants will read this document as part of their between-session work (the Classroom Connection Activities).*
3. Discuss each example in whole group and show *Videos B-E* as needed to support participants' understanding of how each example is an instance of equivalence. • *Video B*: Dr. Ball explains the place value example (upper left portion of the slide) • *Video C*: Dr. Ball explains the geometry example (upper right portion of the slide) • *Video D*: Dr. Ball explains the fraction example (bottom left portion of the slide) • *Video E*: Dr. Ball explains the algebra example (bottom right portion of the slide)	*If participants are struggling to articulate why each example is an instance of equivalence in mathematics, consider showing the corresponding video and then having participants comment.* *Support participants in noticing how the examples show that the idea of equivalence is relevant across grades and mathematical strands.* *It is important that participants leave this session knowing that all four examples are instances of equivalence in mathematics. The point of this part, however, is not to have participants be able to explain these examples to students.*

Part 7: Wrap up (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand the function of the classroom connection activities and will understand the assignment.	1. Explain and distribute the Classroom Connection Activities. 2. Summarize the work of the session.	• Video (02:35): Introduction to Classroom Connection Activities

Detailed description of activity	Comments & other resources
1. Introduce Classroom Connection Activities. These activities are professional homework designed to: • Connect the professional development content with classroom teaching • Supporting feedback on teaching • Extend thinking about the content of previous sessions • Preview the content of later sessions. Watch the *video* in which Dr. Ball describes the use of classroom connection activities in the module. Distribute the handout you customized with selected *Classroom Connection Activities* and accompanying documents. Required: • Read the Math Notes document on instances of equivalence in mathematics • Read the Ball, Hill, & Bass (2005) article on mathematical knowledge for teaching 	*Because learning about teaching is an extended process that can be supported by learning in and from one's own teaching, each session includes "Classroom Connection Activities." These activities provide significant opportunities to learn. Routine tasks in the activities encourage participants to use their teaching as a context for learning, to connect professional development content with common classroom resources, support feedback on learning and teaching, as well as extend thinking about mathematics and teaching that arose in the session.* *Emphasize to participants that they do not need to be teaching fractions in order to complete these activities.* *It may be helpful to ask participants to upload their responses or materials related to the Classroom Connection Activities to review what participants have been thinking and trying prior to the next sessions. In addition, uploading responses would allow participants to easily share their ideas with each other.*
2. Summarize the work of the session for participants using the slide found in the resources section (Slide: Summary): This session began work on studying mathematics teaching and equivalence in mathematics.