Math Notes: Instances of Equivalence in Mathematics

Equivalence is a big idea in many areas of mathematics. Whether one is working with numbers and operations and place value, studying geometric ideas, or doing algebra, the idea of equivalence plays an important role. This document illustrates some instances of equivalence in number and operations, geometry, and algebra.

Equivalence in Number and Operations

We often consider different things to be essentially the same, because they share all of the features in which we are interested. We express this by writing "=" between them and calling them "equal." We sometimes use the term "equivalent" in place of "equal" in order to acknowledge that they are not *literally* the same.

In the early elementary grades, children experience the idea of equivalence as they learn number facts. For example, children learn that:

$$1+5=6 \qquad 4+2=6 \qquad 3+3=6$$

At first, students may think of these facts as distinct and unrelated. Later, students come to think of $1+5$, $4+2$, and $3+3$ as different ways of "making" 6. In other words, these three expressions are equal (equivalent) because they have the same (equal) value. The idea that numbers can be composed in different ways, like rewriting 43 to 30+13 to complete the problem 43 -17, is foundational to place value based computation algorithms, which depend on the ability to decompose and recompose numbers.

In the upper elementary grades, students learned that there are also equivalent names for fractional quantities. For example, students learn that $\frac{3}{4}$, $\frac{15}{20}$, 0.75, and 0.750 are all equivalent ways to name the same number. This equivalence can be illustrated in multiple ways such as showing that they are all names for the same point on the number line.

Equivalence in Algebra

Equivalence is also a key idea in algebra, notably in the notion of *equivalent algebraic equations*. We consider two algebraic equations to be equivalent if one can be obtained from the other by applying reversible transformations. We mainly use the following three basic transformations:

- Add (or subtract) the same quantity to both sides of the equation
- Multiply (or divide) both sides of the equation by the same non-zero constant
- Apply rules of arithmetic to one or both sides of the equation (usually to simplify expressions)

Because equivalent equations have the same solutions, skill in transforming one form of an equation into an equivalent form provides the possibility of finding a form that is easier to use for particular purposes, like solving for an unknown. This is illustrated with the following sequence of equivalent equations, leading to a solution for y:

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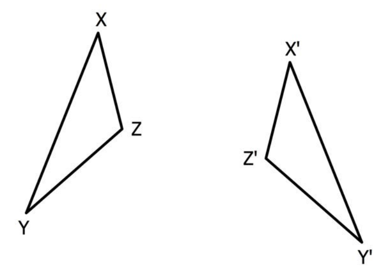
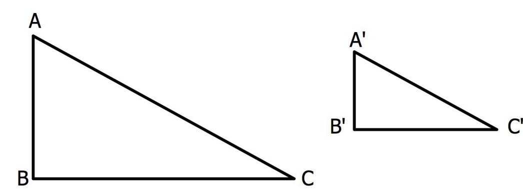
$3y + 10$	$=$	$y + 30$	
			Subtract 10 from both sides of the equation
$3y + 10 - 10$	$=$	$y + 30 - 10$	
			Apply rules of arithmetic to both sides of the equation
$3y$	$=$	$y + 20$	
			Subtract y from both sides of the equation
$3y - y$	$=$	$y + 20 - y$	
			Apply rules of arithmetic to both sides of the equation
$2y$	$=$	20	
			Divide both sides of the equation by 2
$2y \div 2$	$=$	$20 \div 2$	
			Apply rules of arithmetic to both sides of the equation
y	$=$	10	

Equivalence in Geometry

In geometry, we introduce new terminology for talking about equivalence. One way we consider two figures (such as triangles) to be equivalent (the "same") is if they have the same size and shape. When this is the case, we say that the two figures are *congruent*. The figures to the right illustrate two congruent triangles. To prove that Triangle *XYZ* is congruent to Triangle *X′Y′Z′*, we could superimpose one triangle on the other. Another approach would be to measure the sides and angles of each triangle and confirm that the corresponding sides and angles are equal.



A second way in geometry that we consider two figures to be the same is in the case of figures that have the same shape, but not necessarily the same size. We call two figures that have the same shape, *similar*. Triangle *ABC* and Triangle *A′B′C′* are similar because they have the same shape (i.e., Triangle *ABC* is congruent to a magnification of Triangle *A′B′C′*). For two triangles to be similar, it suffices to show that their corresponding angles have the same measure, but this not the case for all figures. Generally, two figures are similar if one is congruent to a magnification of the other. All circles are similar. All regular hexagons are similar. All squares are similar. But not all rectangles are similar, even though corresponding angles are always both right angles. For two rectangles to be similar, it is (necessary and) sufficient that the ratio of the long side to the short side be the same for each rectangle. For squares, this ratio is 1.



Concluding Remarks

Equivalence is an idea in mathematics that transcends particular strands and topics. As shown above, equivalence is a key idea that is relevant to topics that appear very early in elementary school mathematics and also to topics that appear much later. Further, equivalence is more than just an idea that should be recognized. Equivalence is useful when doing mathematics. Knowing how to use equivalence positions students to strategically manipulate ideas to make them more manageable, elegant, and situationally appropriate. Developing understanding of equivalence supports teaching and learning across grade levels. Work early on equivalence provides a firm foundation for students to use this key idea in later mathematics.

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