Description of the Session 10: Supporting students' understanding and use of representations shared by classmates

In this session, participants will first have a conversation about a CCA from last session focused on a fraction-of-an-area task involving multiple wholes. In this session, participants revisit teaching practices that were focal in the module (using representations, narrating the use of representations, connecting representations, and using public recording space). In the context of videos showing a teacher and fifth-grade students working with a set model of fractions, participants describe and analyze the work that teachers do to help individual students record and talk about representations, as well as to support students in understanding how classmates use representations. Topics explored include supporting students in understanding one another's comments, using student errors to support understanding, supporting collective work on mathematics, and summarizing in order to support students' understanding. The session ends with participants reflecting on their learning in the module and considering the ways of capitalizing on what was learned in future teaching.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
Conversation about a CCA from the last session	10 minutes		• Participants will be able to explain solutions to a fraction-of-an-area task involving multiple wholes.
I. Preview	5 minutes	Part 1	• Participants will be oriented to the work of the session.
II. Analyzing and narrating a fraction-of-a-set task	15 minutes	Part 2	• Participants will be able to: ○ identify various methods for solving a fraction-of-a-set task; ○ anticipate student solution methods for a fraction-of-a-set task; and ○ narrate the use of a set model of fractions in the context of a fraction-of-a-set task.
III. Analyzing videos from a fifth-grade lesson to identify teaching moves that support the class in understanding and making use of representations shared by classmates	60 minutes	Parts 3, 4, 5, & 6	• Participants will be able to describe, and articulate rationales for, teacher moves that support students' understanding and use of representations shared by classmates.
IV. Teaching moves that support students' understanding and use of representations shared by classmates	15 minutes	Part 7	• Participants will be able to identify a set of teaching practices that support the class in understanding and making use of representations shared by classmates.
V. Wrap up	5 minutes	Part 8	• Participants will understand ways of connecting the session content to their classroom.

Note: There are 10 minutes of this two-hour session that have been left open to address any local issues or logistics related to concluding this professional development experience.

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Preparing for the session

☐ Make copies as needed:

- *Resources:* Handout: One-eighth of 24 circles (Part 2); Handout: Narrating and construction and use of a representation (Part 2); Transcript: One-eighth of 24 circles (Part 3); Handout: Connecting and summarizing (Part 6)

  Note: The *Transcript: One-eighth of 24 circles* is used in Parts 3, 4, 5, and 6. It is available in the resources section of each of these parts.

- *Supplements:* Math notes: Analysis of the fraction-of-a-set task (Part 2)

☐ Test technical setups (Internet connection, speakers, projector)

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards for School Mathematics.

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Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **representing fractions** • defining fractions • using and explaining methods and representations for comparing fractions • understanding how equivalence (of fractions) can be represented and used	• **identifying and analyzing student conceptions, explanations, and representations of fractions** • identifying and analyzing student strategies for comparing fractions	• selecting and generating representations • **connecting representations** • **narrating the process of representing** • **supporting students in narrating the use of a representation** • **recording contributions and emerging mathematical ideas**	• **studying public recording space to learn from practice** • using a conceptual framework to guide the planning, use, and analysis of public recording space

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Conversation about a Classroom Connection Activity from last session (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be able to explain solutions to a fraction-of-an-area task involving multiple wholes.	1. Discuss solutions to the Snack Cake Problem using the focus questions.	

Detailed description of activity	Comments & other resources
1. Have participants share different methods for solving the Snack Cake Problem from the CCA, which read: *Four friends find a box of snack cakes on the counter in the kitchen. Someone had eaten one of the snack cakes so there were seven left. The four friends want to share the seven snack cakes equally. How many snack cakes does each person get?* As participants share their solutions, pose questions for the group to consider such as: • How many steps [or key decision points] are shown in visual representation of the solution? • What are some of the key ideas about fractions that are shown in the solution? • Why do you think the numbers in the problem were chosen? If division is not shared as a method, suggest an analogous problem such as: There are eight donuts in the box, how many would each of the four friends get if the donuts were shared equally? Asking how they solved this problem should prompt *division* as an answer. Then return to the problem as written. A key question then becomes: Why is the answer given as a fraction and not with a remainder or a decimal?	*The problem as presented is a division problem. Since snack cakes can be cut into smaller pieces, the answer can contain fractions.* $7 \div 4 = \frac{7}{4} = 1\frac{3}{4}$ *A visual representation or concrete materials can be used to facilitate work on this problem.* *There are enough snack cakes for each person to take one, but not enough for each person to take two snack cakes. So each person takes one snack cake, leaving three snack cakes to share. There are several ways to share the other three snack cakes. One way to do this would be to cut them in half. This will yield six one-half snack cakes, allowing each person to take one-half of a snack cake. Left with two halves to share among four people, each half can be cut into two equal pieces yielding four equal pieces out of one snack cake. Each piece was one-half of a one-half or one-fourth of a snack cake. In the end each of the four friends got one snack cake, one-half of a snack cake, and one-fourth of a snack cake or* $1 + \frac{1}{2} + \frac{1}{4} = 1\frac{3}{4}$ *of a snack cake. Solving the problem using division results in the same answer. By acting out the problem or using a visual model it is possible to see how the answer is derived and why it makes sense.* *Connection to module content:* *This problem recaps key mathematics content found in the module. For example, early in the module participants engaged in the representations of ¾ activity where they encountered the idea of fractions as division and multiple ways to represent the same fraction, both of which are relevant to this problem. In the third session, they developed a working definition of fractions that can be easily connected with work in this problem (e.g., what is the whole, what do equal parts mean in this context).* *Numbers selected for the task:* *The numbers for this problem were selected for specific reasons. The number allows each person to have a whole snack cake and for there to be cakes left that will need to be divided into smaller and smaller parts to share the cakes equally.*

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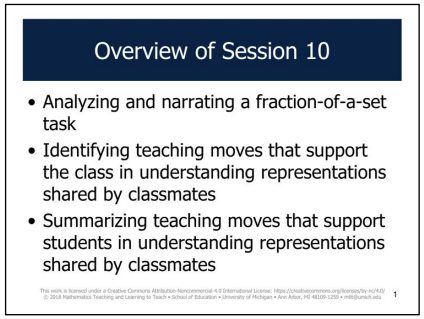
Part 1: Preview (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand the goals of the session.	1. Introduce the session and watch the video in which Dr. Ball frames the work of the session.	• Video (02:44): Overview of session

Detailed description of activity	Comments & other resources
1. Introduce the session: Session 10 brings together several of the teaching practices participants have developed throughout the module (connecting, narrating, and using public space) and extends their work on both fractions and student thinking. In Session 8, participants considered ways in which teachers can support students in narrating the use of a representation. This session builds on that work to consider how teachers can help the other students in the class understand and use representations shared by a classmate.  Have participants watch the *video* in which Dr. Ball frames the work of the session and introduces the work of supporting and using students' representations in teachings.	*The teaching practice of supporting and using student representation is important because it:* • *Scaffolds student opportunities to engage in a crucial mathematical practice* • *Provides the teacher with examples of student thinking* • *Models interest in the thinking of others* • *Helps the class to make sense of the ideas and their relationship to the mathematics of the lesson*

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Part 2: Analyzing and narrating a fraction-of-a-set task (~15 minutes)

Goals

- Participants will be able to:
  - identify various methods for solving a fraction-of-a-set task;
  - anticipate student solution methods for a fraction-of-a-set task; and
  - narrate the use of a set model of fractions in the context of a fraction-of-a-set task.

Instructional sequence

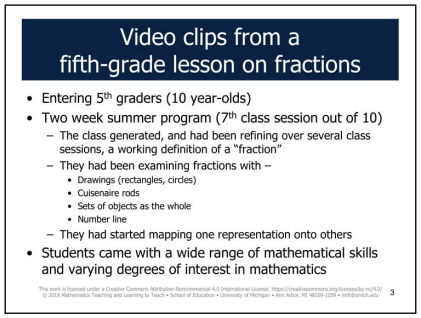
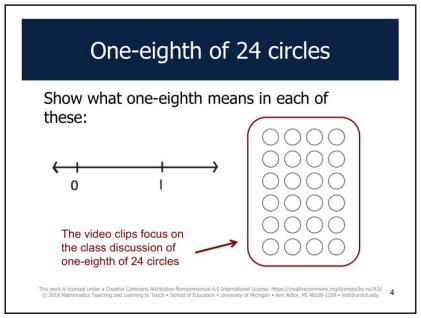
1. Introduce Part 2 and watch the video in which Dr. Ball describes the video context and the task in which students engage.
2. Have participants analyze the One-eighth of 24 circles task with a partner.

Resources

- Video (01:31): Context and the task
- Handout: One-eighth of 24 circles
- Handout: Narrating the construction and use of a representation

Supplements

- Math notes: Analysis of the fraction-of-a-set task

Detailed description of activity	Comments & other resources
1. Introduce Part 2: This part of the session brings together considerations of student thinking and the teaching practice of narration. It also provides an opportunity to think about the use of public space. The work in this session will be grounded in examples from a fifth-grade lesson on fractions. Have participants watch the *video* in which Dr. Ball describes the context of the lesson. Participants will watch the part of the lesson in which students discuss their strategies for finding one-eighth of the set of 24 circles.   	*This task concerns showing a fraction-of-a-set of objects. It illustrates a part/whole interpretation of fractions using a set representation. The whole consists of 24 circles and the task is to show 1/8 of the whole. In the context of a set representation, a fraction refers to the number of objects in the whole that have a given attribute (e.g., shaded gray) compared to the total number of objects in the whole. Objects in the whole can also be grouped into subsets of equal size.*

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Detailed description of activity	Comments & other resources
2. Distribute the *handout* one-eighth of 24 circles. Have participants work with a partner to analyze the task. Partners should: 1. Anticipate strategies students might use to find one-eighth of the set of 24 circles. 2. Narrate the use of the circle representation to solve the problem. As participants walk through the narration process (on *handout*), encourage them to highlight key ideas about fractions such as attention to the whole and equal parts. 3. Consider the kinds of support students might need when explaining their use of this representation to the class and the supports that the class might need to understand the solution being shared. When pairs finish, discuss in whole group if time permits. If it is useful, distribute the *Math notes document*, which introduces three different strategies for thinking about the task.	*When participants are narrating, encourage them to make use of the work on narration from previous sessions:* • *Make clear the mathematical problem or context* • *Describe how a particular representation is useful for this problem* • *Construct the representation and use it to solve the task while describing and giving meaning to each step.* • *Summarize what the representation has helped to do.*

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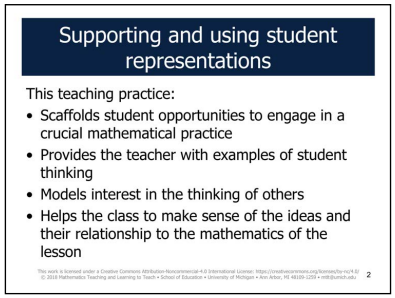
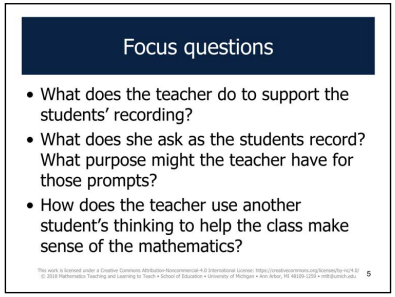
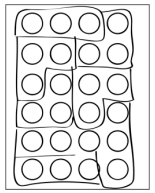
Part 3: Supporting students' understanding and use of representations shared by classmates (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be able to describe, and articulate rationales for, teacher moves that support students' understanding and use of representations shared by classmates.	1. Introduce Part 3 and watch Video A in which Dr. Ball frames the importance of supporting and using representations. 2. Watch Video B in which Sean explains his solution. 3. Watch Videos C–E to explore points made by other teachers.	• Video A (02:15): Supporting and using representations • Video B (02:53): Eight groups of three (Sean) • Video C (01:04): Examples of scaffolding • Video D (02:17): Filling in key points • Video E (00:20): Confirming • Transcript: One-eighth of 24 circles

Detailed description of activity	Comments & other resources
1. Introduce Part 3: This part introduces a focus on supporting students' understanding and use of representations shared by classmates. Have participants watch *Video A* in which Dr. Ball frames the importance of this work.  Supporting and using student representations This teaching practice: • Scaffolds student opportunities to engage in a crucial mathematical practice • Provides the teacher with examples of student thinking • Models interest in the thinking of others • Helps the class to make sense of the ideas and their relationship to the mathematics of the lesson 2	
2. Distribute copies of the transcript and have participants watch *Video A* in which a student, Sean narrates his use of the representation to solve the problem (transcript lines 1-58). Ask participants to read the focus questions (see the slide in the resources section) prior to watching the video.  Focus questions • What does the teacher do to support the students' recording? • What does she ask as the students record? What purpose might the teacher have for those prompts? • How does the teacher use another student's thinking to help the class make sense of the mathematics? 5 After viewing the video, ask the participants to try describing Sean's strategy to make sure they understand his mathematical thinking. (Note: An image of the Sean's solution is available as a handout in the resources section) Then discuss the focus questions to identify specific moves used by the teacher and their possible purposes.	*CCSSM Link: The classroom video contains examples of fifth grade students who are engaged in at least three of the mathematical practices (1, 3, and 6). In this segment, the students are making sense of the problem and persevering in solving the problem (#1), constructing viable arguments about their solution (#3), and communicating their reasoning to others (#6).* *Sean's narration includes:* • *There are 24 circles* • *Eight goes into 24 three times* • *Make groups of three* • *Count eight groups* • *He does not use language such as "whole" or "equal parts", and he does not explain what is 1/8 of 24.* 

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Detailed description of activity	Comments & other resources
	In his recording, Sean: • *Makes eight groups of three, but does not shade/record what is the eighth of 24* *After Sean's narration, the teacher asks him:* • *What's the eighth?* • *You made eight groups. How many are there in each one?* • *Why did you make eight groups?* • *What kinds of eight groups do you make?* • *What are you calling 1/8 in this drawing?* *To help students make sense of the mathematics of the task, the teacher asks other students:* • *Sean took 24, and then what did he do?* • *How many groups of three did he get?* • *What's the whole in this drawing (connecting to the definition of a fraction)?*
2. In order to extend participants' thinking, have them watch some of the videos (*Videos C-E)* in which teachers in the professional development course discuss the same focus questions. When watching *Video C*, participants should consider: • What specific questions were asked to scaffold students' understanding? When watching *Video D*, participants should consider: • How can "filling in information" support the engagement of the class? • Can you think of a time when you would not want to make those contributions? When watching *Video E,* participants should consider: • When might it be useful to confirm an answer as correct? • When might it less useful?	*Video C: Teachers in the professional development course noticed the following types of scaffolding:* • *In line 15, "What's the eight"* • *In line 17, "why?"* • *In line 18, "How much is in each one?"* • *In line 20, "Why did you make it into eight groups?"* • *In line 23, "What kind of groups?"* *Video D: Teachers in the professional development discuss how the teacher fills in key points and makes the explanation more accessible to other students.* • *Lines 40-41, following Brianna's explanation that "He divided it into three", the teacher uses very specific language, such as, "He divided it into groups of three".* • *Line 53, the teacher reinforces the importance of "equal parts".* • *Line 11, the teacher makes it more accessible to other students.* *Video E: Dr. Ball explains that sometimes it is useful to tell a student that their answer is correct so that they can focus on explaining the answer.* • *A move that should be used sparingly.*

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Part 4: Using student errors to support students' understanding and use of representations shared by classmates (~15 minutes)

Goals

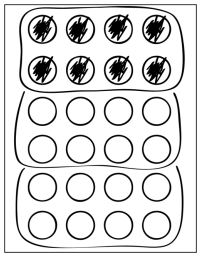
- Participants will be able to describe, and articulate rationales for, teacher moves that support students' understanding and use of representations shared by classmates.

Instructional sequence

1. Introduce Part 4 and watch Video A, in which Rebecca explains her solution. Have participants discuss Rebecca's error and identify specific moves used by the teacher.
2. Watch Video B in which teachers in the professional development discuss the same topics.

Resources

- Video A (02:56): Three groups of eight with one group shaded (Rebecca)
- Video B (03:06): Using student errors in making sense of representations
- Transcript: One-eighth of 24 circles

Detailed description of activity	Comments & other resources
1. Introduce Part 4: When students make errors during a whole-class discussion, there is often a tension between supporting an individual student through the error and worrying about the impact that dwelling on the error might have on the rest of the class. Have participants watch *Video A* (main viewer) in which the teacher helps the class understand and make use of an error that was made by a student, Rebecca (transcript lines 58-114). Participants should continue consideration of the focus questions (see the slide in the resources section) as they watch the video. **Focus questions** • What does the teacher do to support the students' recording? • What does she ask as the students record? What purpose might the teacher have for those prompts? • How does the teacher use another student's thinking to help the class make sense of the mathematics? After viewing the video, ask the participants to try to describe Rebecca's error to make sure they understand her mathematical thinking. (Note: An image of the Rebecca's solution is available in the resources section.) Then, discuss the focus questions to identify specific moves used by the teacher and their possible purposes.	*CCSSM Link: The classroom video contains examples of fifth grade students who are engaged in at least three of the mathematical practices (1, 3, and 6). In this segment, the students are making sense of the problem and persevering in solving the problem (#1), constructing viable arguments about their solution (#3), and communicating their reasoning to others (#6).* *Rebecca's narration includes:* • *Twenty-four is the whole* • *Eight goes into 24 three times* • *Make three groups of eight* • *To get 1/8, shade one of those groups*  *In her recording, Rebecca:* • *Makes three groups of eight* • *Shades one of three groups* *After Rebecca's narration, the teacher:* • *Explains that Rebecca knows that eight times three is 24* • *Asks Rebecca why she ended up with 1/3, when she was thinking about 1/8*

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	To help students make sense of the mathematics of the task, the teacher asks other students: • *How is Rebecca's picture different from Sean's?* • *What does Rebecca's picture show?* • *What fraction does Rebecca's picture show?* • *How many circles are in 1/3?*
2. To extend participants' thinking, have them watch *Video B* in which the teachers in the professional development course weigh different moves, purposes, and possible outcomes. Participants should note any new ideas or questions based on this conversation.	*Participants might note:* • *More judgmental language such as "good" and "interesting"* • *The mistake is one a lot of students make.* • *Great opportunity to teach* • *The teacher has to say that "this is not 1/8"* • *Need to confirm what Rebecca did (1/3), while also acknowledging that it isn't the answer to the question asked*

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Part 5: Encouraging collective work on errors (~15 minutes)

Goals

- Participants will be able to describe, and articulate rationales for, teacher moves that support students' understanding and use of representations shared by classmates.

Instructional sequence

1. Introduce Part 5 and watch the video in which Autumn made an error and Paige assists her in working through her error. Have participants discuss the error and identify specific moves made by the teacher.
2. Have participants discuss other moves a teacher could make to support the class.

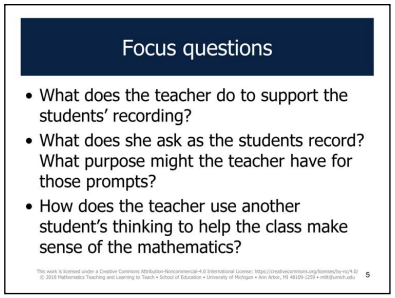
Resources

- Video (03:58): Three groups of eight with one shaded circle (Autumn and Paige)

  Transcript: One-eighth of 24 circles

Detailed description of activity	Comments & other resources

1. Have participants watch the *video* in which the teacher uses another student's thinking to help the class understand and make use of an error that was made by a student, Autumn (transcript lines 115-193). Participants should continue consideration of the focus questions on the slide as they watch the video.



   After viewing the video, ask the participants to try to do the following. (Note: Images of Paige's and of Autumn's representation are available in the resources section)

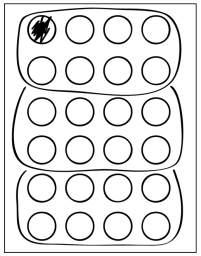
   Have participants discuss the video with colleagues:

   - Describe Autumn's error to make sure you understand her mathematical thinking.
   - Describe how Paige uses Autumn's drawing to try solve the problem. Is her explanation mathematically correct?

CCSSM Link: The classroom video contains examples of fifth grade students who are engaged in at least three of the mathematical practices (1, 3, and 6). In this segment, the students are making sense of the problem and persevering in solving the problem (#1), constructing viable arguments about their solution (#3) and communicating their reasoning to others (#6).

Autumn's narration includes:

- *Not sure whether the representation is right*
- *Make groups of eight*
- *Each group is eight, and then color one circle*
- *Colored circle is 1/8 of a group*



In her recording, Autumn:

- *Makes three groups of eight*
- *Shades one circle from only one group*

After Autumn's narration, the teacher asks:

- *Is that 1/8 of what? (While Autumn is narrating)*

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Detailed description of activity	Comments & other resources
Then discuss the focus questions to identify specific moves used by the teacher and their possible purposes.	*To help students make sense of the mathematics of the task, the teacher asks other students:* • *Whose picture is more like that now?* • *What question is Autumn answering?* • *What is that (Autumn's shaded circle) 1/8 of?* • *Why is Autumn calling that 1/8?* • *What is still missing?* *After the teacher's following-up question, another student, Paige, narrates that "Because these two [groups] are still left,… you need to color two more [circles]. But when you do that, then that would equal 3/8, instead of 1/8…."*
2. Have participants extend their thinking by considering other moves a teacher could make to support the class in making sense of the ideas of these two students. What would be the purpose of each move? Discuss in whole group.	

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Part 6: Summarizing to support students' understanding and use of representations shared by classmates (~15 minutes)

Goals

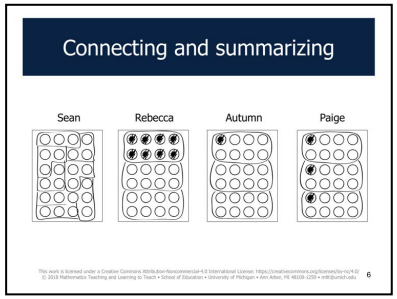
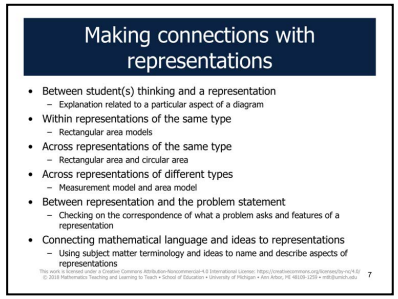
- Participants will be able to describe, and articulate rationales for, teacher moves that support students' understanding and use of representations shared by classmates.

Instructional sequence

1. Introduce Part 6 by explaining the work of summarizing to support the class' mathematical thinking and have participants discuss the connections that could be made in a summary of the class' work.
2. Watch and discuss the video in which the teacher summarizes the work on the task.

Resources

- Video (01:52): Summarizing the mathematical work
- Transcript: One-eighth of 24 circles
- Handout: Connecting and summarizing

Detailed description of activity	Comments & other resources
1. Introduce Part 6: Summarizing the mathematical work of the class is an important teaching skill. Providing summaries can help students make sense of and consolidate mathematical ideas. Skill in making connections, a teaching practice that you have worked on in the course of this module, is often essential in summarizing. The "Connecting and summarizing" slide shows the representations students shared in the discussion of the 1/8 of 24 circles problem. The "Making connections with representations" slide lists of the type of connections participants identified in earlier sessions:   Have participants discuss with a partner the connections that could be made in a summary of the work on the "One-eighth of 24 circles" problem. Participants should: • Identify different connections that could be made using the images on the slide, as well as the ideas raised in the video (see transcript). • Identify the mathematical ideas that each connection will help students make sense of.	*Support participants in making connections by trying to see particular elements of the solution in each of the diagrams. For instance, how are 24, 3, and 1/8 shown in each of the diagrams? How is the whole attended to in each diagram?* *Possible connections:* • *Between student(s) thinking and a representation: Rebecca's representation shows three groups of eight because she is thinking about how eight goes into 24 three times.* • *Within representations of the same type: Paige's representation has three circles shaded, which is similar to Sean's representation, where circles are divided into groups of three.* • *Between representation and the problem statement: Sean's representation shows 24 circles divided into eight equal groups, because the problem statement asks what one-eighth of 24 circles would look like.* • *Connecting mathematical language and ideas to representations: The teacher connects Sean's representation to the fraction definition by prompting Sean to explain that he made eight equal groups.*

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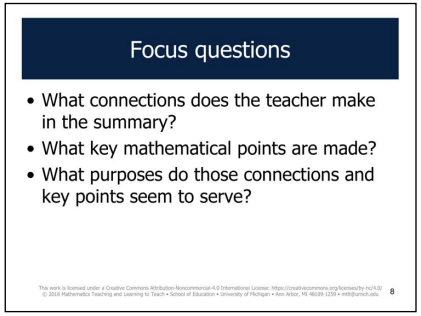
Detailed description of activity	Comments & other resources
Support participants in making explicit connections back to the slide that names the kinds of connections. They should link the name of the connection with the example of it from the discussion.	
2. Have participants watch the *video* in which the teacher summarizes the work on the problem (transcript lines 195-239). Participants should use the focus questions on the slide to guide their viewing. Have participants discuss the focus questions with colleagues.  	*In the video, the teacher summarizes:* *Autumn took the whole 24 circles and made three groups, similar to what Rebecca did. And then she colored one out of eight. Based on Autumn's work, Paige colored 1/8 of the second group and 1/8 of the third group. She ended up taking 1/8 of the whole thing—she just did it in steps. Paige shaded three circles just like Sean did. Paige and Sean got the same answer, but in different ways.* *In her summary, the teacher emphasizes that the students divided the whole into equal parts. She also emphasizes that Paige and Sean both arrived that the correct answer (3 circles) even though they used different methods and colored in different circles on the picture. She acknowledges that Sean's is the easiest representation to see because he divided the set of 24 circles into eight equal groups.*

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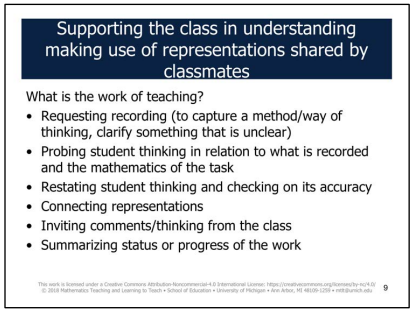
Part 7: Teaching moves that support students' understanding and use of representations shared by classmates (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be able to identify a set of teaching practices that support the class in understanding and making use of representations shared by classmates.	1. Introduce Part 7 and watch Video A in which Dr. Ball elaborates the task. 2. Watch Videos B-F in which teachers discuss ideas for moves, along with their potential purposes and outcomes.	• Video A (00:53): Distilling practices • Video B (01:09): Having representations available • Video C (01:40): Encouraging the class to work through a classmate's contribution • Video D (01:58): Asking for choral responses • Video E (00:23): Repeating key questions • Video F (00:24): Connection to previous learning

Detailed description of activity	Comments & other resources
1. Introduce Part 7: Throughout the session, participants studied videos from a fifth-grade lesson to identify specific examples of teaching moves that support the class in understanding and making use of representations shared by classmates. Participants also considered the ways in which the particular moves can impact the quality of the mathematics available to learn and students' engagement with that mathematics. Now participants should stop and think more generally about the types of moves that teachers can use to support the class in learning from each other's representations. Have participants watch *Video A* in which Dr. Ball sets up the task.	
2. Have participants make a list of their ideas about moves and practices teachers can use to support the class in understanding and making use of representations shared by classmates. Have participants compare their list with the moves found in the "Supporting the class in understanding and making use of representations shared by classmates" slide in the resources section. Ask participants to consider whether they have any new ideas or insights.  	*New insights might include the ideas listed on the slide in the resources section, titled "Supporting the class in understanding and making use of representations shared by classmates."*

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Detailed description of activity	Comments & other resources
3. Have participants watch selected videos (*Videos B – F)* of teachers in the professional development course discussing their ideas for moves, along with their potential purposes and outcomes.	*In Video B, teachers explain the need for having multiple copies of representations readily available so that the students can focus on their drawings and not have to redraw what is common. This saves time, which enables more students to share. This also ensures that there is enough consistency among different students' representations to make powerful connections among them.* *In Video C, teachers discuss benefits of asking students to restate a question that another student has asked. This technique can be used to keep students focused, check level of understanding, steer back to the questions at hand, help students save face, and help explain what is wrong.* *In Video D, teachers explain reasons for asking a closed question that students should answer with a choral response. This can increase participation, emphasize vocabulary, check understanding, reinforce that something is right, provide safe ways to answer, and encourage reluctant students to speak.* *In Video E, a teacher addresses the need for asking questions that draw attention to crucial points.* *In Video F, a teacher addresses the need for connecting to things that have already been studied to reinforce previous learning.*

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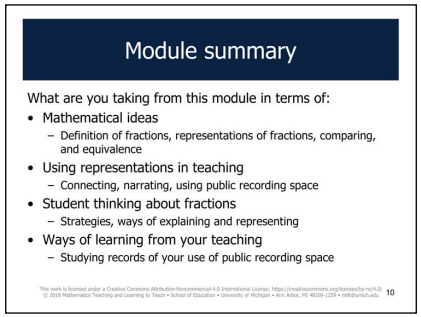
Part 8: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways in which they can continue to use ideas and practices from the module in subsequent teaching.	1. Watch the video in which Dr. Ball summarizes the work of the module.	• Video (04:12): Module summary

Detailed description of activity	Comments & other resources
1. Introduce Part 8: This marks the end of the module on representing and comparing fractions in elementary mathematics teaching. Have participants watch the *video* in which Dr. Ball summarizes the work of the module.	



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