

Transcript: One-Eighth of Twenty-Four Circles

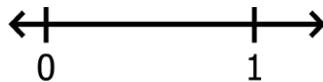
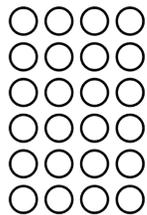
Elementary Mathematics Laboratory for incoming fifth graders
Park City Mathematics Institute
Tuesday, July 11, 2006

Seating Arrangement

Jessica								Maddie
Ally								Cozy
Sabrina								Holly
Paige								Luke
Tori								Arthur
Brianna								Britney
David	Vinnie	Rebecca	Sarah	Ben	Trevor	Michael	Sean	Autumn

Problem:

Teacher: ... With the number line and the group of circles, I want you to show what one-eighth means in each of those.



Teacher: So, now the other one is the twenty-four circles. Would somebody like to explain the way that they decided what one-eighth would look like in that drawing? Let's see, who's not had a chance to talk? Sean, you want to go up and do it?

Are you doing okay?

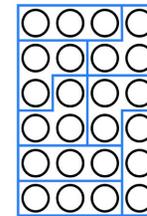
Ally: Yeah.

Teacher: Okay.

Sean: Okay, so the- There's twenty-four squares, and-

Teacher: Circles, I think. Circles.

Sean: Circles. Oh, circles, and eight goes into twenty-four three times, so you make it into three groups. Into groups of three. (*Draws:*)



And if you count them, you'll have eight. One, two, three, four, five, six, seven, eight.

Teacher: So what are those- What's the eight?

Sean: Three.

Teacher: Okay, but why- What- In each eight is- The eight- You made eight groups, and how much is in each one?

Sean: Three.

Teacher: Three. And why did you make it into eight groups?

Sean: Because- Eight- You need to make it into eight groups for an eighth.

Teacher: Eight- What kind of groups?

Sean: Into groups of three to make eight.

Teacher: Equal groups, right? Okay-

Sean: Equal groups.

Teacher: And so what are you calling one-eighth in this drawing?

Sean: Three circles.

Teacher: Three circles. Okay, do you understand what he did? Let's get some comments now. Rebecca?

Rebecca: Well, I'm wondering maybe if there's a different way to do it.

Teacher: Okay, but first let's comment on this and then we can see a different way. What- Can someone comment on what David did? He took the twenty-four and what did he do then?

Vinnie: David? Sean.

Teacher: I'm sorry, Sean. What did he do then? He took the twenty-four and did what? Yes?

Brianna: He divided it into three.

Teacher: He divided it into groups of three.

Brianna: Into groups of three.

Teacher: And how many groups of three did he get?

Brianna: Eight.

Teacher: Eight. So let's check it against- Can someone check it against the poster now? What's the whole in this drawing? Art?

Art: All the circles.

Teacher: How many?

Art: The twenty-four circles.

Teacher: Twenty-four circles. Did Sean divide it into equal parts?

Art: Yes.

Teacher: Yes. How many equal parts did he make? Maddie?

Maddie: Eight.

Teacher: Eight. Now, the name of the kind of part comes from the number of equal parts, which was eight equal parts, so it's one-eighth. And the answer was- How many was in each eighth? Three. Okay? Do- So you want to show a different thing? Okay, Rebecca, go ahead.

Autumn: After Rebecca if she doesn't use...

Teacher: Thank you, Sean. What?

Autumn: After Rebecca can I go show this way?

Teacher: Yes, in a minute.

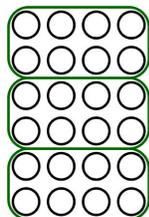
Autumn: Okay.

Teacher: Oh, let me give you a new drawing. I'm sorry.

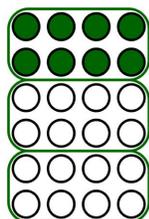
Rebecca: Okay.

Teacher: There you go. Can people see Rebecca?

Rebecca: Well, you have twenty-four as your whole, and then- Well, twenty- Eight goes into twenty-four three times, and so what I did is I just made eight groups- Well, three groups of eight. (*Draws:*)



And so- And then to get my one-eighth. (*Draws:*)



I just shaded in one of those groups.

Teacher: Okay, so that's very interesting. So, Rebecca knows that eight times three is twenty-four, and- How is her picture different from Sean's? Can someone explain? I should see more hands than that, because that's a pretty easy question. How is Sean's picture- How is Rebecca's picture different from Sean's? They're both on the board. You should be able to see them both. What's different? Ben?

Ben: On Rebecca's there are three groups, and on Sean's there are eight.

Teacher: Good. They have the same whole, they both have twenty-four circles and they both made equal groups, but their equal groups are different. So, Rebecca made three equal groups and Sean made eight equal groups. So, what does Rebecca's picture show? Can somebody use what we're developing about fractions to say what her picture shows? What kind of fraction did Rebecca show? Vinnie?

Vinnie: One-third.

Teacher: Okay, why one-third, Vinnie?

Vinnie: Because she's- she did three groups and she colored one of them.

Teacher: Okay, good. She made thirds because she divided into three equal groups and made one-third. And how many is in one- How many circles in one-third, Vinnie?

Vinnie: Eight.

Teacher: Eight. So that's very interesting, Rebecca. So, you made a different fraction, but you can see why you were thinking it. Why did you end up thinking about that one? You had a good reason. Why did you end up doing that? You made one-third, but why did you end up with thirds when you were thinking about eighths? Why did that happen?

Rebecca: Because I thought if you just make three groups of eight and shaded in one of them, it would equal to one-eight.

Teacher: It makes sense because eight times three is twenty-four, but in fact, what you ended up doing is showing us what it looks like when you make three equal groups. Very nice job. But that's not one-eighth, that's one-third. But you did a good job of thinking about that.

Okay, so now I think Autumn has one more to show. Do you want to show yours, Autumn, or do you not want to show it now?

Autumn: I don't care.

Teacher: What?

Autumn: I don't care. But I'll go.

Teacher: Okay. Watch carefully, because this is again a different one, and you need to decide: is it just a

different way of showing it? Or, is Autumn thinking about something else? Just like with what Rebecca just did. Here's a clean one. This is the last one we're going to look at, and then we're going to do a little bit of independent work with fractions. Here's a blank one for you.

Autumn: Okay.

Teacher: I'll move this up. Okay, there you go.

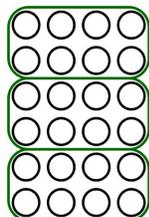
Autumn: Okay. I'm not sure if it's right, but I-

Teacher: Did you say you're not sure it's right?

Autumn: Yeah.

Teacher: Okay, so watch carefully. She's saying even while she starts to talk about it that she's not sure it's right. So, watch with her, and think about whether it's right or not.

Autumn: So, I did like what Rebecca and Sean did. I put them in groups of eight. (*Draws:*)



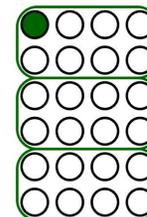
And then I- And then-

Teacher: So, first of all, wait one second. Let's ask a question: Whose picture is that more like, right now? Maddie?

Maddie: Rebecca's.

Teacher: It's more like Rebecca's. Now, what are you going to do next?

Autumn: And then, because each group is eight, and it's one-eighth, I just colored in one. (*Draws:*)



Teacher: Okay, so is that one-eighth of what?

Autumn: One-eighth of this (*gestures to the top group of eight circles*).

Teacher: Okay, so can someone say what question Autumn is answering? Lots of times when people get a different answer in math, they're actually answering a different question which is a useful thing for you to think about, because it helps you to learn more math, if you can figure out what question someone is actually answering. She doesn't get the same answer as Sean. Sean got the answer three for one-eighth of twenty-four, and we agreed that was right. But what is Autumn doing? Can someone explain what question she is answering? Yes.

Paige: She's answering: if you do all of them, it's actually- Well, since there's eight circles in it, she colored one in, which would be one of the eight, and that's what she was thinking, but if you- There's still the two parts left.

Teacher: Well, don't go to that, yet. So, what is that one-eighth of? It's one-eighth of something. What is it one-eighth of? Her drawing. Look at her drawing.

Paige: It's one-eighth of a third.

Teacher: One-eighth of what? Say it again.

Paige: One-eighth of a third.

Teacher: One-eighth of a third of the twenty-four. And why is Paige- Sorry. Why is Autumn calling that one-eighth? There's a good reason. Because there's something about that that's right, that she's doing. What is it? Sean?

Sean: Because there's eight circles in the rectangle

Teacher: Right.

Sean: that she shaded.

Teacher: So, in that group, it's one-eighth of those eight circles, right? Now go on to say something was missing, Paige?

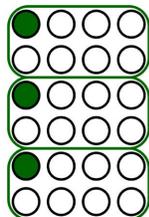
Paige: They're- The two ha- The two of the thirds are still left.

Teacher: Okay. So-

Paige: And, if you put them all together, it would actually be three-eighths instead.

Teacher: Can you go up and do that? So, there's a way of working with Autumn's drawing to see the- to see the one-eighth of twenty-four. Can you work with it?

Paige: Well, if you- Because these two are still left, and- So, you need to color in two more. (*Draws on Autumn's diagram.*)



But when you do that, then that would equal three-eighths instead of one-eighth, and it would be three-

Teacher: eighths of a third. She was right on the first part, but there's still those two left.

Teacher: Okay. So let me just clear this up a little and then we're going to do some work together. That's really very interesting what Autumn did and what Paige did with it, so see if you can follow this. Can everyone look up here for a minute? Everybody's eyes should be up here.

Okay, ready? So what Autumn did is she took the whole twenty-four circles and made three groups. That's a lot like what who did? Rebecca. And then she colored in one out of the eight. And we just said, well, one out of eight would be one-eighth, right? So, she was only looking- It's like this part didn't exist. (Folds the paper so only the top group of eight circles is showing). Okay. Is that one-eighth? Yeah. Because it's one out of eight parts. One equal part out of eight of the parts. So then, Paige did something else interesting. She said, well, there are two thirds left, and she took one-eighth of this one, and what did she do here?

Student: She took one-eighth.

Teacher: One-eighth of that one. So, she took one eighth of that one, one-eighth of this one, and one-eighth of this one. Now she's actually taken one-eighth of the whole thing, because she's just done it in steps. One-eighth of this group, then one-eighth of this group, then one-eighth of this group. How many circles are shaded when Paige is done?

Student: Three.

Teacher: Ben?

Ben: Three.

Teacher: Three, which is the same answer that who got? Who else got three for an answer? Sarah?

Sarah: Sean.
Teacher: Who?
Sarah: Sean.
Teacher: Sean. Now, where's Sean's drawing?
Sarah: Down there.
Teacher: Right there. So, they both arrived at the answer of three, but in a different way. And what this- Between Paige and Autumn together is a little bit harder to see. But, if you take one-eighth of one equal part, and then one-eighth of the next equal part, and one-eighth of the next equal part, you've taken one-eighth of the whole thing. This (*Sean's drawing*) is probably the easiest way to see it, because he took the twenty-four and made eight equal parts out of it. But that's very interesting, these different things that people tried to do.