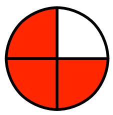
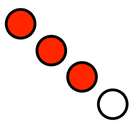
mod4 Materials Development Project (2009). Interpretations of representations of three fourths. In *mod4: Exploring Fractions (Session 1).* Ann Arbor, MI: University of Michigan. Available from http://sites.soe.umich.edu/mkt/materials. Adapted and used with permission.

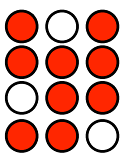
Math Notes: Analysis of Multiple Representations of ¾

Interpretations of representations of three fourths

Each of the given representations *can* be thought of as a representation of three fourths. This document provides sample interpretations for each.

a) 

b) 

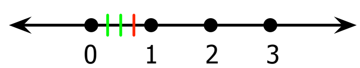
c) 

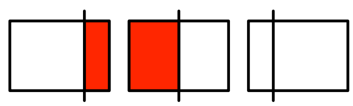
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

f) .75

g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

a) This is an area model, where the area of the circle represents the whole. In this case, the whole is divided into four equal (area) parts, and three of these parts are shaded.

b) This is a discrete (or set) model, where the group of four circles is taken as the whole. In this case, a subset consisting of three out of the four circles is shaded.

c) This is a discrete (or set) model, where 9 out of the 12 circles are shaded. And so it represents $\frac{9}{12}$. Here are two approaches for seeing that $\frac{9}{12}$ is equivalent to $\frac{3}{4}$ using this model. One approach is to notice that the circles in the representation are arranged in three rows and four columns. Each of the columns of the figure contains four circles and three of the circles in each column are shaded, hence $\frac{3}{4}$ of the set is shaded. Another approach is to rearrange the circles into four equal rows of three where three of the four rows contain all shaded circles and the remaining row contains the three circles that are not shaded. And so $\frac{3}{4}$ of the set is shaded. This observation provides some additional support for establishing the equivalence of $\frac{9}{12}$ and $\frac{3}{4}$.

d) Often, representations are thought of being diagrams or physical models, but language and mathematical notation can also be thought of as representations. In case d), the question, "How many 4s are in 3?" interprets the fraction $\frac{3}{4}$ as division using the measurement interpretation of division. If $\frac{3}{4}$ is interpreted as how many 4s are in 3, then it takes $\frac{3}{4}$ of a 4 to make a 3.

e) Like (c), this is a discrete (or set) model of fractions, where the whole is a collection of objects and the fraction represents a portion of the collection. In this case, the whole is the box of 24 crayons and 18 crayons is $\frac{3}{4}$ of the box of 24. There are multiple ways of establishing the equivalence of $\frac{18}{24}$ and $\frac{3}{4}$. One approach is to arrange the 24 crayons in four equal sets of six, of which three of those sets comprise the crayons that you have. Hence the fraction that you have is equivalent to $\frac{3}{4}$.

f) The equivalence of the decimal representation 0.75 and the fraction representation $\frac{3}{4}$ can by shown by reading ".75" as "75 hundredths". Then the question becomes "What part of 100 is 75?" Just as was the case in (e) and (c) splitting the whole 100 into 4 equal groups of 25, and showing that 75 is composed of 3 of the groups, facilitates the connection to $\frac{3}{4}$.

g) Similar to (d), this problem implies an interpretation of $\frac{3}{4}$ as division. However, instead of a measurement interpretation of division, this problem implies a fair-share, or partitive, interpretation of division. The idea here is that 3 bottles are being shared equally between four people. Here are two possible ways to see this representation as $\frac{3}{4}$. One involves dividing each of the bottles into fourths and giving the first person $\frac{3}{4}$ of the first bottle, the second person $\frac{3}{4}$ of the second bottle, the third person $\frac{3}{4}$ of the third bottle, and the fourth person the remaining $\frac{1}{4}$ from each of the bottles (which is the measurement approach to division). Another method involves dividing each of the bottles into four equal portions and giving each person $\frac{1}{4}$ of each bottle, for a total of $\frac{3}{4}$ of a bottle (which is the partitive approach to division).

h) The point on the number line representing $\frac{3}{4}$ is located at the point that is $\frac{3}{4}$ of the distance from 0 to 1. This can be thought of as taking the distance from 0 to 1 as the whole and dividing it into four equal parts and moving three of these to the right starting from 0.

i) This representation is similar to (g) in that it shows three divided by four (three divided into four equal parts). Here are two possible ways to see this representation as $\frac{3}{4}$. One involves taking the whole to be one of the rectangles. The first of the three rectangles shows $\frac{3}{4}$ of a whole not shaded with $\frac{1}{4}$ of the whole in red. When the red shading is attended to, $\frac{1}{4}$ of the first rectangle is red and $\frac{1}{2}$ of the second rectangle is red. Taken together this yields $\frac{3}{4}$ of a rectangle as red. Another approach involves considering the three rectangles as the whole. The red shaded area is $\frac{1}{4}$ of the three-rectangle whole meaning that the unshaded portion is $\frac{3}{4}$ of the three-rectangle whole.