

Overview of Session 4

- Analyzing a student's use of the number line
- Exploring properties and conventions of the number line
- Analyzing students' errors when using the number line

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Video clip from a third-grade lesson on fractions

- Third graders (8 year-olds)
- Diverse classroom, many English language learners
- Early May (three weeks into a five-week fractions unit)
 - Emerged from study of division (problems with remainders)
 - Single objects as the whole (cookies, brownies, graham crackers)
 - Drawings (rectangles, circles)
 - Sets of objects as the whole (boxes of crayons)
 - Number line
 - Mapping one representation onto others

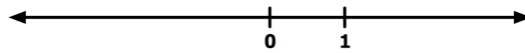
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Focus questions

- How does the student use the number line to solve the problem or explain her solution?
- What does the student appear to understand about the number line?

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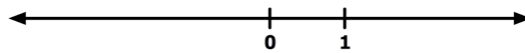
Key properties and conventions of the number line



The horizontally displayed number line has the following key properties and conventions:

- Locating Numbers:** The whole numbers are located on the number line by starting at 0 and repeatedly measuring the unit distance to the right. A fraction is similarly located on the number line by partitioning the unit interval into parts of equal length where the number of parts corresponds to the denominator. A fraction can be located by repeatedly measuring the length of one of these parts.
- Two infinite directions:** The number line has two directions and extends infinitely in both.
- Increase to the right:** Numbers are represented by points on the line. Numbers increase in value as you move to the right. In other words, $y > x$ if y is to the right of x .
- Positive and negative:** Once the number 0 is located, points to the right of 0 are “positive” numbers and points to the left of 0 are “negative” numbers.
- Unit interval:** The interval from 0 to 1 is defined to be the unit interval and its length is the unit distance.
- Labeling Points:** The number corresponding to a point is determined by its distance from 0 in relation to the unit distance and its direction from 0.
- Symmetry:** The number line can be reflected through 0. This reflection maps each number x to its additive inverse (opposite), $-x$.

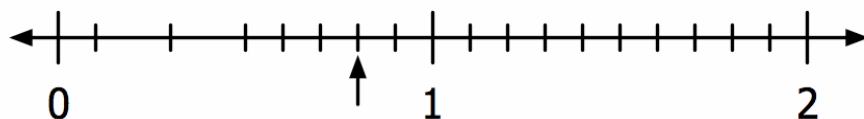
Other observations about the number line



- Every number can be written in infinitely many equivalent forms.
- All integer points can be written as fractions (e.g., the integer 2 can be written as $\frac{2}{1}$ or $\frac{4}{2}$), but not all fractions represent integers (e.g., $\frac{3}{2}$ cannot be written as an integer).
- Between any two points on the number line there is always a rational number (a number that can be written as a fraction). This is what we mean when we say that the rational numbers are dense in the line.
- Any number, x , and its additive inverse, $-x$, are the same distance from 0. For example, -2 and 2 are both at distance 2 from 0.
- The numbers represented by points on the number line are called the real numbers and so the number line is also called the “real line.” The real numbers include the rational numbers, but not all real numbers are rational; those that are not are called irrational numbers (e.g., square root of 2 and pi can be shown to be irrational).
- The number line is an important mathematical object. Some features of the number line are accessible to children in the early elementary grades.

Labeling locations on the number line

What value should be written where the arrow is pointing?



Shaughnessy, M. M. (2009). Students' Flexible Use of Multiple Representations for Rational Number: Decimals, Fractions, Parts of Area, and Number Lines. Unpublished doctoral dissertation, University of California, Berkeley.

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Summary

In this session, you:

- Analyzed a student's use of the number line
- Explored properties and conventions of the number line
- Analyzed students' errors when using the number line

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