

***Description of the Session 5: Narrating representations and analyzing tasks***

Participants will begin this session by discussing the student work samples they analyzed as part of their session 4 CCA. In this session, participants examine a central practice of using representations in mathematics teaching: the work of “narrating” a representation. Narrating refers to the practice of describing the construction and use of a particular representation during instruction. It is a practice that explicitly aims to expand students’ understanding of when and how mathematical representations are used, as well as their key features. The focus of this session is on narration by the classroom teacher; supporting students’ narration will be the focus of later sessions. Through a series of activities on the number line, participants develop a process for narrating a representation, analyze the narrations of others, and practice narrating the construction and use of a number line to compare fractions. In addition, participants will analyze the instructional and learning possibilities of different pairs of fractions for working on the comparison of fractions.

***Activities and goals of the session***

<b>Activities</b>	<b>Times</b>	<b>Corresponding parts of the session</b>	<b>Goals</b>
Conversation about a CCA from the last session	10 minutes		<ul style="list-style-type: none"> <li>• Participants will be able to describe the steps that students likely took to place numbers on the number line.</li> </ul>
I. Preview	5 minutes	Part 1	<ul style="list-style-type: none"> <li>• Participants will be oriented to the work of the session.</li> </ul>
II. Narrating representations with a focus on the construction and use of a number line	80 minutes	Parts 2, 3, 4, & 5	<ul style="list-style-type: none"> <li>• Participants will understand the general steps involved in narrating the construction and use of a representation.</li> <li>• Participants will understand that there are multiple approaches to using the number line to compare fractions.</li> <li>• Participants will be able to:               <ul style="list-style-type: none"> <li>○ articulate features of the number line that may be important to make explicit when narrating the construction and use of a number line; and</li> <li>○ apply the general steps for narrating the construction and use of representation in a situation where a number line is being used to compare fractions</li> </ul> </li> </ul>
III. Analyzing fraction comparison tasks	20 minutes	Part 6	<ul style="list-style-type: none"> <li>• Participants will be able to analyze and strategically select fractions for comparison problems suited for particular instructional purposes.</li> </ul>
IV. Wrap up	5 minutes	Part 7	<ul style="list-style-type: none"> <li>• Participants will understand ways of connecting the session content to their classroom.</li> </ul>

***Classroom Connection Activities*****Required**

Type of task: Practice and extension of work

Description: Narrating the use of number line to compare fractions, creation and analysis of new fraction comparison sets

Type of task: Collecting records of practice

Description: Taking two photos of the medium used most often to record the work of the class during a mathematics lesson at two points: (1) the first photo, at the beginning of a mathematics lesson, and (2) the second photo, at the end of the mathematics lesson

***Preparing for the session***

- Colored pencils, rulers
- Make copies as needed:
  - *Resources:* Handout: Narrating the construction of a representation (Part 3); Handout: Blank number lines (Part 4); Handout: Fraction comparison problems (Part 5)
  - *Supplemental resources:* Handout: Key properties and conventions of the number line (Session 4) (Part 3); Math notes: Analysis of fraction comparison tasks (Part 6)
- Customize the Classroom Connection Activities and make copies as needed
- Test technical setups (Internet connection, speakers, projector)

***Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)***




1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
  - a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
  - b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
  - c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards for School Mathematics.

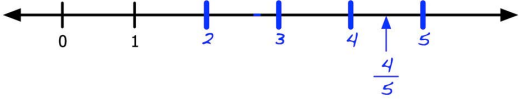
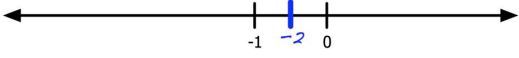
***Scope of the module (focal content of this session in bold)***

<b>Mathematics</b>	<b>Student thinking</b>	<b>Teaching practice</b>	<b>Learning from practice</b>
<ul style="list-style-type: none"> <li>• representing fractions</li> <li>• defining fractions</li> <li>• <b>using and explaining methods and representations for comparing fractions</b></li> <li>• understanding how equivalence (of fractions) can be represented and used</li> </ul>	<ul style="list-style-type: none"> <li>• identifying and analyzing student conceptions, explanations, and representations of fractions</li> <li>• identifying and analyzing student strategies for comparing fractions</li> </ul>	<ul style="list-style-type: none"> <li>• selecting and generating representations</li> <li>• connecting representations</li> <li>• <b>narrating the process of representing</b></li> <li>• supporting students in narrating the use of a representation</li> <li>• recording contributions and emerging mathematical ideas</li> </ul>	<ul style="list-style-type: none"> <li>• studying public recording space to learn from practice</li> <li>• using a conceptual framework to guide the planning, use, and analysis of public recording space</li> </ul>

**Conversation about a Classroom Connection Activity from last session (~10 minutes)**

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> <li>Participants will be able to describe the steps that students likely took to place numbers on the number line.</li> </ul>	1. Participants discuss samples of student work.	<ul style="list-style-type: none"> <li>Number lines in CCA #4</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. In this session, participants will focus on the CCA that asked them to examine samples of student work.</p> <p>For each sample of student work, ask participants to share:</p> <ul style="list-style-type: none"> <li>the reasoning that the student likely used to locate the number;</li> <li>whether the student's reasoning resulted in correctly locating the number.</li> </ul> <p>As participants share their responses, the following questions might be useful:</p> <ul style="list-style-type: none"> <li>What whole did the student appear to use?</li> <li>What understandings might the student still be developing?</li> </ul>	<p><u>Task a:</u></p> <p><i>The student correctly identified the location of 4 on the number line. It appears that the student used some measurement skill to find the location. For example, the student may have measured the unit interval (the interval from 0 to 1) and iterated this interval three times starting at 1 on the line to locate 4. This is a non-standard, correct solution because the student did not mark 2 and 3.</i></p>  <p><u>Task b:</u></p> <p><i>The student did not correctly identify the location of 3 on the number line. It appears that the student treated the interval from 0 to 2 as the unit interval. The student located 3 the same distance from 2 as 2 was from 0. The student appears to still be developing understanding of the "whole" on the number line.</i></p>  <p><u>Task c:</u></p> <p><i>The student did not correctly identify the location of 1/6 on the number line. It appears that the student correctly identified the whole, treated the denominator as the number of tick marks that need to be added between 0 and 1, and thought that the tick marks need to be equally spaced. Using this strategy, the student added six equally spaced tick marks, which partitioned the interval from 0 to 1 into seven parts with equal length rather than six equal parts. The student then labeled the point at the first of those tick marks 1/6. The student appears to still be developing understanding of the d in n/d.</i></p> 

Detailed description of activity	Comments & other resources
	<p><u>Task d:</u></p> <p>The student did not correctly identify the location of <math>\frac{4}{5}</math> on the number line. It appears that the student correctly identified the whole (the interval from 0 to 1), and that they iterated this whole four times starting at 1 to correctly label 2, 3, 4, and 5 on the number line. The student interpreted <math>\frac{4}{5}</math> as between 4 and 5. The student labeled <math>\frac{4}{5}</math> halfway between 4 and 5 on the number line. The student appears to still be developing understanding of the meaning of <math>n/d</math> notation.</p>  <p><u>Task e:</u></p> <p>The student did not correctly identify the location of -2 on the number line. The student placed -2 between -1 and 0 perhaps because they thought -2 has a larger magnitude than -1. This student appears to still be developing understanding of how the number line represents negative numbers.</p> 

**Part 1: Preview (~5 minutes)**

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> <li>Participants will be oriented to the work of the session.</li> </ul>	1. Introduce the session and watch the introductory video.	<ul style="list-style-type: none"> <li>Video (04:18): Overview of session</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. Introduce the session: Session 4 established key properties and conventions of the number line and explored students' thinking about the representation. Session 5 extends the work in Session 4 by examining two central practices of teaching mathematics: narrating the construction and use of a representation, and analyzing tasks. In particular, this session focuses on:</p> <ul style="list-style-type: none"> <li>Narrating the construction and use of a number line to compare fractions</li> <li>Analyzing the instructional possibilities of fraction comparison problems.</li> </ul> <p>In this module, "narrating" refers to the practice of "talking through" the construction and use of a representation during instruction. Narrating is a practice that explicitly aims to expand students' access to and understanding of mathematical representations and the ideas being conveyed. Through narrating, students have the chance to hear others describe the process of selecting and constructing a representation to address a particular task or to solve a problem. In this session, participants will practice narrating the use of a number line to compare pairs of fractions. As participants rehearse and observe others narrate, they will map out a process for doing the work and identify qualities that make a narration productive. Participants will also think about what knowledge narrating representations requires. These ideas will help participants narrate representations with their students. Participants will also be able to use these ideas as they support students in narrating their representations.</p> <p>Have participants watch the <i>video</i> in which Dr. Ball frames the work of the session.</p> <div data-bbox="758 516 1142 805" data-label="Complex-Block" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center;"><b>Overview of Session 5</b></p> <ul style="list-style-type: none"> <li>Narrating the construction and use of a representation</li> <li>Analyzing mathematics tasks</li> </ul> <p style="font-size: small; text-align: center;">This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License. <a href="https://creativecommons.org/licenses/by-nc/4.0/">https://creativecommons.org/licenses/by-nc/4.0/</a> © 2018 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • mttt@umich.edu 1</p> </div>	<p><i>In previous sessions participants worked on making connections and also considered mathematical aspects of the number line. This session provides opportunities to draw on these skills and insights as participants engage in narrating, a teaching practice through which teachers explicitly support student understanding of mathematical representations.</i></p>

**Part 2: Exploring the practice of narrating through the construction and use of a number line (~10 minutes)**

<b>Goals</b>	<b>Instructional sequence</b>	<b>Resources</b>
<ul style="list-style-type: none"> <li>Participants will begin to understand and be able to name what is involved in narrating the construction and use of a number line to compare fractions.</li> </ul>	<ol style="list-style-type: none"> <li>Introduce Part 2 and watch the video in which Dr. Ball introduces the narrating of the construction and use of a representation activity.</li> <li>Have participants work in pairs to complete the narrating the construction and use of a representation activity.</li> </ol>	<ul style="list-style-type: none"> <li>Video (01:21): Launch of task: Narrating the use of the number line</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 2: In this part of the session, participants will begin to explore what is involved in narrating the construction and use of a number line to compare fractions.</p> <p>Have participants watch the <i>video</i> in which Dr. Ball launches the “Narrating the construction and use of a representation” task.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center;"><b>Narrating the construction and use of a representation</b></p> <p>Which fraction is larger – <math>\frac{3}{4}</math> or <math>\frac{4}{3}</math> ?</p> <p>With a partner:</p> <ul style="list-style-type: none"> <li>One person talks through the use of a number line to solve this problem.</li> <li>The other person notes phrases or ideas that are shared during the “narration.”</li> <li>When the problem is complete, discuss the narration and think about which parts seem to be important when doing this kind of work.</li> </ul> <p style="font-size: small;">This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License. <a href="https://creativecommons.org/licenses/by-nc/4.0/">https://creativecommons.org/licenses/by-nc/4.0/</a> © 2018 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • mttt@umich.edu</p> </div>	<p><i>In the video, Dr. Ball explains that when constructing a number line, it is important to consider:</i></p> <ul style="list-style-type: none"> <li><i>How big is the unit on the number line?</i></li> <li><i>What marks do we need on the number line?</i></li> </ul> <p><i>Considering what are important things to narrate is part of the work that participants will be doing in this session.</i></p> <p><b>CCSSM Link:</b> <i>In working on this activity, participants are engaged in several of the CCSSM mathematical practices including: constructing viable arguments and critiquing the reasoning of others (#3), and precisely communicating their thinking to others (#6). Comparing two fractions with different numerators and denominators is a Grade 4 standard (4.NF.2).</i></p>

Detailed description of activity	Comments & other resources
<p>2. Have participants work with a partner to complete the task:</p> <ul style="list-style-type: none"> <li>• As indicated in the slide, one person should “narrate” or talk through the use of a number line to solve the fraction comparison task. Encourage participants to use the actual language that they would use with their students.</li> <li>• The other person should note both the process that is used for the narration as well as features of the number line or aspects of the problem that were made explicit during the narration.</li> <li>• When the problem is complete, pairs should discuss the narration to:               <ul style="list-style-type: none"> <li>○ Document the process that was used (what was done first, next, etc.)</li> <li>○ List features of the number line or aspects of the problem that were highlighted and/or made explicit during the narration.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• <i>A common tendency is to use language such as “I might say” or “I would say”. Encourage participants to try to talk as though they are talking to their students.</i></li> <li>• <i>A common tendency is to wait until the number line is constructed to begin narrating. Encourage participants to talk through the entire process.</i></li> <li>• <i>Participants may indicate that the mathematics content in the task is beyond the content that they teach. Try to help participants see that the act of narrating mathematical representations occurs across grades and mathematics content.</i></li> <li>• <i>Participants may bring up the idea that the two fractions can be compared using one number line or they can be compared using two number lines. In Part 4, there is more support for thinking about the affordances/constraints of each representation.</i></li> </ul> <p><i>In the next segment, participants will consider a general process for narrating the construction of a representation. This process includes:</i></p> <ul style="list-style-type: none"> <li>• <i>Make clear the mathematical problem or context</i></li> <li>• <i>Describe how a particular representation is useful for this problem</i></li> <li>• <i>Construct the representation and use it to solve the task, <u>while describing and giving meaning</u> to each step</i></li> <li>• <i>Summarize what the representation has helped to do.</i></li> </ul>



**Part 3: General steps for narrating the construction and use of a representation (~25 minutes)**

<b>Goals</b>	<b>Instructional sequence</b>	<b>Resources</b>
<ul style="list-style-type: none"> <li>Participants will understand the general steps involved in narrating the construction and use of a representation.</li> <li>Participants will be able to articulate features of the number line that may be important to make explicit when narrating the construction and use of a number line.</li> </ul>	<ol style="list-style-type: none"> <li>Introduce Part 3 and have participants read the slide listing the general steps involved in narrating the construction and use of a representation.</li> <li>Compare the general list with the list of processes constructed during Part 2.</li> <li>Watch the video clips of teachers discussing their ideas about what is important to do when narrating the construction and use of the number line.</li> <li>Develop a list of features of the number line that may be important to make explicit.</li> </ol>	<ul style="list-style-type: none"> <li>Video A (03:32): Marking 0 and 1</li> <li>Video B (01:20): Introducing any terms being used</li> <li>Video C (01:29): Using consistent language</li> <li>Video D (03:37): Making decisions about how to construct the number line based on instructional purpose</li> <li>Video E (00:28): A challenge of making a particular number of parts</li> <li>Handout: Narrating the construction and use of a representation</li> </ul> <p><b>Supplements</b></p> <ul style="list-style-type: none"> <li>Handout: Key properties and conventions of the number line</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 3: This part of the session articulates general steps for narrating the construction and use of a representation. Participants use these steps to think about the specific work of narrating the construction and use of a number line.</p> <p>Read aloud (or ask participants to read) the slide, which lists the general steps involved in narrating the construction and use of a representation.</p> <div data-bbox="527 894 911 1182" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center;"><b>Narrating the construction and use of a representation</b></p> <ul style="list-style-type: none"> <li>Make clear the mathematical problem or context.</li> <li>Describe how a particular representation is useful for this problem.</li> <li>Construct the representation and use it to solve the task while <u>describing and giving meaning</u> to each step.</li> <li>Summarize what the representation has helped to do.</li> </ul> <p style="font-size: small; text-align: center;">This work is licensed under a Creative Commons Attribution-NonCommercial-4.0 International License. <a href="https://creativecommons.org/licenses/by-nc/4.0/">https://creativecommons.org/licenses/by-nc/4.0/</a> © 2018 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • mttt@umich.edu 3</p> </div>	<p><i>Consider writing the "Narrating the construction and use of a representation" slide on chart paper so that it can be referred to later in the session and in subsequent sessions. This slide can also be downloaded and printed (see Handout: Narrating the construction and use of a representation).</i></p> <p><i>The practice of narrating the construction of a mathematical representation, in this case a number line, is something teachers do often in their work. One goal of this is to enable teachers to step back and study what it means to narrate a representation in a meaningful way. Being explicit about the practice of narrating the construction and use of a representation allows us to:</i></p> <ul style="list-style-type: none"> <li>identify it when it occurs in teaching</li> <li>talk about it</li> <li>work consciously on it</li> <li>give feedback on it</li> </ul>

Detailed description of activity	Comments & other resources
<p>2. In pairs, have participants compare the general list on the slide with the process that they documented during their discussion of the <math>\frac{3}{4}</math> and <math>\frac{4}{3}</math> activity (Part 2). Have pairs discuss the following question: <i>Is there anything you would add to the general list or revise about your process?</i></p> <p>Collect any additional steps in the process that the teachers identify.</p>	<p><i>Might consider having participants refer back to the "Key properties and conventions of the number line" introduced in Session 4. This handout can be found in the supplemental materials in the bottom right portion of the viewer.</i></p>
<p>3. As time permits, have participants watch and briefly discuss <i>Videos A – E</i> in which teachers discuss what may be important to do when narrating the construction and use of the number line. Some of the ideas include:</p> <ul style="list-style-type: none"> <li>• Marking 0 and 1 to show the unit (Video A)</li> <li>• Introducing any terms being used (Video B)</li> <li>• Using consistent language (Video C)</li> <li>• Making decisions about how to construct the number line based on instructional purpose; making equal parts (Video D)</li> <li>• A challenge of making a particular number of parts (Video E)</li> </ul>	<p><i>Video A: Marking 0 and 1 to show the unit</i></p> <p><i>The length of the interval from 0 and 1 is called the "unit interval"; it is a measure of the whole of the line, and determines the distance between consecutive integers on the line.</i></p> <p><i>Participants may have questions about the necessity of marking 0 and 1 on the number line. Based upon the mathematical task at hand, it may or may not be useful to mark 0 and 1 first. More generally, once any two numbers are located on a number line, the locations of all other numbers (including 0 and 1) are defined.</i></p> <p><i>Video B: Introducing any terms being used</i></p> <p><i>The teachers note the importance of attending to the terms being used by either defining them or asking students to define them. For example, an improper fraction can be described as greater than one whole. While not discussed in the video, participants might note:</i></p> <ul style="list-style-type: none"> <li>▪ <i>We call the numbers greater than 0 "positive" and the numbers less than 0 "negative"</i></li> <li>▪ <i>Zero is not positive or negative</i></li> <li>▪ <i>Every number is at a distance from 0 and has a direction, namely whether it is on the positive, negative, or equal to 0.</i></li> </ul> <p><i>Video C: Using consistent language</i></p> <p><i>The teachers discuss the importance of consistently using the same language. The teachers also note that when multiple terms can be used, teacher should explicitly link the related terms so that students know that they refer to the same thing.. Participants may have questions about the use of consistent language. The decision to hold terms consistent (example: hatch marks or tick marks) or to make them vary is based on the specific situation and your purpose at that particular point.</i></p>

Detailed description of activity	Comments & other resources
	<p><i>Video D: Making decisions about how to construct the number line based on instructional purpose</i></p> <p><i>The teachers discuss how a variety of instructional purposes might require the use of different number lines. The narration of the number line as a representation might be different depending on the purpose.</i></p> <p><i>Video E: A challenge of making a particular number of parts</i></p> <p><i>A teacher notes the importance of having students attend to the number of equal parts that they are making (i.e. adding three equally spaced tick marks between 0 and 1 creates four equal parts)</i></p>
<p>4. In small or whole group, list and discuss features of the number line that the participants see as important to make explicit when narrating its construction and use. Encourage participants to:</p> <ul style="list-style-type: none"> <li>• Give examples that are specific to number lines (like Videos B and C).</li> <li>• Draw upon the ideas in the video, their work on the number line in the last session, and the discussion of narrating the comparison of <math>\frac{3}{4}</math> and <math>\frac{4}{3}</math>.</li> <li>• Think about <i>when</i> it might be important to make a feature explicit.</li> </ul>	<p><i>Participants may discuss the general features of constructing number lines. Try to focus discussion on narrating the construction and use of a number line for comparing fractions.</i></p> <p><i>Encourage participants to think about the affordances of using a blank number line to compare fractions, instead of using pre-marked number line.</i></p>

**Part 4: Analyzing others' narrations of the number line to compare  $\frac{3}{4}$  and  $\frac{4}{3}$  (~20 minutes)**

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> <li>Participants will understand that there are multiple approaches to using the number line to compare fractions.</li> <li>Participants will be able to apply the general steps for narrating the construction and use of representation in a situation where a number line is being used to compare fractions.</li> </ul>	<ol style="list-style-type: none"> <li>Introduce Part 4 and watch the video launching the task: "Comparing <math>\frac{3}{4}</math> and <math>\frac{4}{3}</math>".</li> <li>Watch the four videos containing examples of narration and analyze each narration.</li> </ol>	<ul style="list-style-type: none"> <li>Video A (01:25): Launch of task</li> <li>Video B (02:02): Using two number lines</li> <li>Video C (03:09): Using one number line and different colors</li> <li>Video D (02:29): Using one number line with common denominator</li> <li>Video E (02:00): Using one number line and number sense</li> <li>Handout: Blank number lines</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 4: In this part of the session, participants will use the general steps in narrating a representation and their list of key features to include in the narration when talking about the number line to analyze four narrations of the comparison of <math>\frac{3}{4}</math> and <math>\frac{4}{3}</math>.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center;"><b>Narrating the construction and use of a representation</b></p> <ul style="list-style-type: none"> <li>Make clear the mathematical problem or context.</li> <li>Describe how a particular representation is useful for this problem.</li> <li>Construct the representation and use it to solve the task while <u>describing and giving meaning</u> to each step.</li> <li>Summarize what the representation has helped to do.</li> </ul> </div> <p>Have participants watch <i>Video A</i> which launches the task by introducing the idea that there are several different ways to compare fractions using the number line, such as: one number line using common denominators, one number line without using common denominators, and two number lines.</p>	<p><i>These examples are not meant to be exemplary; they are intended to spur thinking about the components of the work of narrating in mathematics teaching.</i></p>

Detailed description of activity	Comments & other resources
<p>2. Show <i>Videos B – E</i> which are four examples of using the number line to narrate the comparison of <math>\frac{3}{4}</math> and <math>\frac{4}{3}</math> on the number line.</p> <p>As they watch each video, ask participants to consider the following:</p> <ul style="list-style-type: none"> <li>• Notice the process of narration used and the key features of the number line that were made explicit. Are there any new ideas you would add to the list started in the last part of the session?</li> <li>• Is there anything that you would add to or change about the narration in the video clip to make it more clear and/or complete?</li> </ul> <p>After each video is shown, discuss briefly in pairs or in whole group.</p>	<p><i>Important things to narrate when constructing a number line to compare fractions might include:</i></p> <ul style="list-style-type: none"> <li>• <i>What numbers (whole numbers) need to be marked?</i></li> <li>• <i>What is the whole? Unit fraction? Denominator? Numerator?</i></li> <li>• <i>How do you decide (explain) which fraction is larger on the number line?</i></li> </ul> <p><i>Video B: Using two number lines</i></p> <p><i>Participants might note that the necessity of marking parts of equal size could be emphasized more strongly.</i></p> <p><i>Video C: Using one number line with different colors</i></p> <p><i>Participants might note the following as changes/improvements:</i></p> <ul style="list-style-type: none"> <li>• <i><math>\frac{4}{3}</math> was identified by iterating the unit fraction instead of using the whole.</i></li> <li>• <i>Reason why <math>\frac{4}{3}</math> is larger than <math>\frac{3}{4}</math> is not made explicit in the narration (Need to explain that <math>\frac{4}{3}</math> is further from 0 or <math>\frac{3}{4}</math> is closer to 0).</i></li> </ul> <p><i>Video D: Using one number line with common denominators</i></p> <p><i>Participants might note the following as changes/improvements:</i></p> <ul style="list-style-type: none"> <li>• <i>0 should have been marked on the line.</i></li> <li>• <i>The reason for a common denominator of 12 should have been mentioned.</i></li> </ul> <p><i>Video E: Using one number line with number sense</i></p> <p><i>Participants might note the following as changes/improvements:</i></p> <ul style="list-style-type: none"> <li>• <i>0, 1, and 2 are marked on the line without explicit attention to their marking.</i></li> <li>• <i>Reason for <math>\frac{3}{3}</math> and <math>\frac{4}{4}</math> being equivalents of 1 is not made explicit in the narration.</i></li> <li>• <i>Full reason for <math>\frac{3}{4}</math> being less than 1 and <math>\frac{4}{3}</math> being greater than 1 was not made explicit.</i></li> </ul>

**Part 5: Additional practice narrating the construction and use of a number line (~25 minutes)**

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> <li>Participants will be able to apply the general steps for narrating the construction and use of representation in a situation where a number line is being used to compare fractions.</li> </ul>	<ol style="list-style-type: none"> <li>1. Introduce Part 5 and watch the video of Dr. Ball's narration.</li> <li>2. Analyze Dr. Ball's narration in the video.</li> <li>3. Watch the video in which teachers discuss Dr. Ball's narration.</li> <li>4. With a partner, practice narrating the construction and use of the number line to compare another pair of fractions (see handout).</li> <li>5. Debrief the narrating task using the questions in the Debriefing slide.</li> </ol>	<ul style="list-style-type: none"> <li>Video A (04:24): Dr. Ball's narration</li> <li>Video B (01:23): Teachers' comments</li> <li>Handout: Fraction comparison problems</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 5: This part provides participants with additional opportunities to practice narrating the construction and use of a number line to solve fraction comparison problems.</p> <p>Have participants watch <i>Video A</i>, which shows Dr. Ball narrating the use of a number line to compare <math>\frac{3}{5}</math> and <math>\frac{5}{8}</math>. As participants view the video, they should use the general steps in narrating introduced in Part 4 (see the slide) and their list of key features of the number line to emphasize when narrating to consider:</p> <ul style="list-style-type: none"> <li>• What were some of the key features about the number line that were highlighted in this narration?</li> <li>• Is there anything that you would have done differently, and why?</li> </ul>	
<p>2. Have participants work with a partner to analyze the narration in the video using the general steps in narrating introduced in Part 4 (see the slide) and their list of key features of the number line to emphasize when narrating. Participants should consider:</p> <ul style="list-style-type: none"> <li>• What were some of the key features about the number line that were highlighted in this narration?</li> <li>• Is there anything that you would have done differently, and why?</li> </ul>	

**Narrating the construction and use of a representation**

- Make clear the mathematical problem or context.
- Describe how a particular representation is useful for this problem.
- Construct the representation and use it to solve the task while describing and giving meaning to each step.
- Summarize what the representation has helped to do.

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Detailed description of activity	Comments & other resources
<p>3. Have participants watch <i>Video B</i> in which the teachers discuss this narration in order to hear some additional reactions.</p> <p>Elicit a few comments/reactions in whole group.</p>	<p><i>Video B is useful to show because it both sets up the work for the rest of this part and sets up the work in Part 6.</i></p> <p><i>In the video, teachers comment on:</i></p> <ul style="list-style-type: none"> <li>• <i>How Dr. Ball marks fractions on the number line (using measurement increment).</i></li> <li>• <i>How Dr. Ball explains which fraction is bigger (the number to the right is the bigger number).</i></li> </ul>
<p>4. Distribute the <i>Handout: Fraction comparison problems</i>.</p> <p>Have participants work in pairs to practice narrating the construction and use of a number line to compare fractions: Partners should take turns narrating. Encourage participants to try to include all of the steps in the narrating process and to highlight key features of the number line in their talk.</p>	<p><i>The Fraction comparison problems are also available on a slide in the bottom right portion of the viewer.</i></p> <p><i>A common tendency is to use language such as "I might say" or "I would say". Encourage participants to try to talk as though they are talking to their students.</i></p> <p><i>A common tendency is to wait until the number line is constructed to begin narrating. Encourage participants to talk through the entire process.</i></p>
<p>5. In whole group, debrief the Fraction Comparison Activity using the focus questions contained in the Debriefing slide.</p>	<p><i>This slide can be found in the bottom right portion of the viewer.</i></p>

**Use number lines as you describe comparisons of the following fractions**

a.  $\frac{1}{5}$  or  $\frac{1}{8}$       b.  $\frac{6}{10}$  or  $\frac{7}{10}$       c.  $\frac{5}{6}$  or  $\frac{3}{4}$

d.  $\frac{5}{6}$  or  $\frac{16}{15}$       e.  $\frac{3}{3}$  or  $\frac{5}{5}$

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**Debriefing**

- Did anything arise that we didn't capture yet in our draft ideas about narration?
- What was easy or difficult about the work of narrating?
- When would it be useful to narrate the use of a representation? When might it be unproductive to narrate the use of a representation?

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**Part 6: Analyzing fraction comparison tasks (~20 minutes)**

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> <li>Participants will be able to analyze and strategically select fractions for comparison problems suited for particular instructional purposes.</li> </ul>	<ol style="list-style-type: none"> <li>Introduce Part 6 and watch the introductory video.</li> <li>Analyze the tasks in the slide using the focus questions.</li> </ol>	<ul style="list-style-type: none"> <li>Video (01:53): Introducing the work of analyzing problems</li> </ul> <p><u>Supplements</u></p> <ul style="list-style-type: none"> <li>Math notes: Analysis of fraction comparison tasks</li> </ul>

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 6: Different mathematics problems provide different opportunities for student learning and different challenges for the work of narrating. Analyzing the learning opportunities available in particular tasks allows teachers to use instructional time wisely and to maximize student opportunities to learn. In this part, participants will explore strategically choosing numbers and analyzing tasks for particular instructional purposes. In particular, participants will examine what mathematical ideas can be modeled and highlighted through various fraction comparison problems.</p> <p>Have participants watch <i>Video A</i> in which Dr. Ball introduces the work of analyzing problems in teaching.</p>	
<p>2. Introduce task: With a partner, use the focus questions on the slide to analyze the possibilities of each of the comparison problems that they narrated in the previous part.</p> <p>After participants have had time to analyze the comparison problems with a partner, discuss these tasks in whole group. Consider addressing the following points about each task:</p> <ul style="list-style-type: none"> <li>Task a: Students may think that <math>1/5</math> is greater than <math>1/8</math> because 8 is greater than 5 or because eighths have more pieces than fifths. This task could be used to support students in learning that when partitioning a whole into equal parts, the size of each part becomes smaller as the number of parts becomes greater. Mathematical terms that name components of fractions like numerator and denominator might be of use when considering this comparison. It may be more difficult to construct a representation of <math>1/5</math> since it has an odd denominator.</li> <li>Task b: Because the fractions in this task have the same denominator, students can solve it simply by comparing the numerators. Because the denominator is 10, this problem can also provide an opportunity to make connections with decimals.</li> </ul>	<p><i>Encourage participants to think about the following questions:</i></p> <ul style="list-style-type: none"> <li>What mathematics is highlighted when comparing each pair of fractions?</li> <li>Are there other representations for comparing fractions that seem more appropriate than using the number line?</li> </ul> <p><i>This activity is designed to help participants begin to think more explicitly about the work of strategically choosing numbers and analyzing problems for particular purposes. Participants will continue work on these ideas later in the module.</i></p>

Analyzing mathematics tasks

a.  $\frac{1}{5}$  or  $\frac{1}{8}$       b.  $\frac{6}{10}$  or  $\frac{7}{10}$       c.  $\frac{5}{6}$  or  $\frac{3}{4}$

d.  $\frac{5}{6}$  or  $\frac{16}{15}$       e.  $\frac{3}{3}$  or  $\frac{5}{5}$

- Mathematics:** What mathematical ideas or terms are used in comparing the pairs of fractions?
- Student thinking:** What strategies might students use to make these comparisons? What misconceptions might come up?
- Representations:** What is challenging or useful about a number line representation when comparing these fractions?

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Detailed description of activity	Comments & other resources
<ul style="list-style-type: none"> <li>• Task c: Because <math>\frac{5}{6}</math> and <math>\frac{3}{4}</math> are both “missing” only one part from the whole, students may compare them by reasoning about which fraction is “missing less”—in other words, which fraction is closer to 1. This task is difficult because, in this case, the fraction with the smaller number in the denominator is the smaller fraction.</li> <li>• Task d: <math>\frac{5}{6}</math> and <math>\frac{16}{15}</math> lie on opposite sides of a common benchmark, one whole. Using a benchmark such as <math>\frac{1}{2}</math> or 1 can be a helpful strategy in comparing fractions.</li> <li>• Task e: This task provides an opportunity to discuss the idea that points on the number line can be named in multiple ways. And it is fairly accessible for students to understand that there are (infinitely) many fractions that are equivalent to 1.</li> </ul> <p>Consider distributing the Math notes document to participants.</p>	

**Part 7: Wrap up (~5 minutes)**

<b><u>Goals</u></b>	<b><u>Instructional sequence</u></b>	<b><u>Resources</u></b>
<ul style="list-style-type: none"> <li>Participants will understand ways of connecting the session content to their classroom.</li> </ul>	<ol style="list-style-type: none"> <li>Summarize the session.</li> <li>Explain the Classroom Connection Activities.</li> </ol>	

Detailed description of activity	Comments & other resources
<p>1. Summarize the session: Session 5 provided an opportunity to examine central practices of teaching mathematics: narrating the construction of a representation and analyzing problems.</p> <p>In general, when narrating a representation, it is important for the narrator to describe and give meaning to each action he or she is taking to construct the representation as he or she is constructing it.</p> <p>When narrating the construction and using a number line to compare fractions, it is important to consider and describe: 1) how to mark the number line so that it can clearly demonstrate the fraction comparison, 2) how to represent each of the fractions that is being compared, and 3) how to decide which fraction is larger on the number line.</p> <p>Another important aspect of teaching practice is analyzing tasks. Particular tasks result in different opportunities for students to learn mathematics. When selecting mathematical tasks, it is important to consider the task in terms of mathematical, student thinking, representational affordances.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center;"><b>Summary</b></p> <p>In this session, you engaged in two central practices of teaching mathematics:</p> <ul style="list-style-type: none"> <li>Narrating the construction and use of a representation</li> <li>Analyzing mathematics tasks</li> </ul> <p style="font-size: 8px; margin-top: 5px;">This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License. <a href="https://creativecommons.org/licenses/by-nc/4.0/">https://creativecommons.org/licenses/by-nc/4.0/</a> © 2018 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • mttt@umich.edu</p> </div>	
<p>2. Distribute the handout you customized with selected Classroom Connection Activities and accompanying documents described below.</p> <p><u>Required:</u></p> <ul style="list-style-type: none"> <li>Narrating the use of number line to compare fractions and the creation and analysis of new fraction comparison sets</li> <li>Taking two photos of the medium used most often to record the work of the class during a mathematics lesson at two points: (1) the first photo, at the beginning of a mathematics lesson, and (2) the second photo, at the end of the mathematics lesson.</li> </ul>	<p><i>Stress to participants that photos can be done with whatever mathematics is currently being taught. In other words, it is not necessary to be teaching fractions.</i></p>