

Bansho Example: Comparing Fractions

Lesson Plan

Topic: Comparing $\frac{4}{3}$ and $\frac{14}{15}$

Instructional Purposes:

1. Comparing two fractions which have different denominators
2. Representing fractions through diverse models (e.g., number line, area models (rectangles, circles, etc.), set models, algebraic forms, etc.)
3. Mapping different representations
4. Developing mathematical language to the parts of a fraction

Stage	Activity	Attention & Materials	Diagram #
		<ul style="list-style-type: none"> Before beginning a class, write today's topic. 	1
Introduction	Introduce today's topic.	<ul style="list-style-type: none"> After introducing this question, ask students what might make it hard to compare them. Encourage them to consider different representations when comparing. 	2
Development	<p>Student work independently (or in pairs) to compare $\frac{4}{3}$ and $\frac{14}{15}$.</p> <p>Talk in the whole group about the comparisons.</p> <ol style="list-style-type: none"> Ask students to present their solutions. Discuss about representations. 	<ul style="list-style-type: none"> Give students five to ten minutes to do activity. Look at student work, track on their answers, reasoning and representations. Students might have some difficulties such as symbols (<, > or =), remembering to use words like denominator and numerator, recognizing the whole and making equal partitioning etc. Help them individually. Have enough time to share their findings. Encourage students to present their representations to compare fractions on the board and explain their reasonings. Discuss findings based on the representations on the board. Connecting different representations and using mathematical language is important here. 	3 4
Extension	Compare $\frac{3}{4}$ and $\frac{14}{15}$.	<ul style="list-style-type: none"> This is more complicated problem because the least common multiple is much bigger than the prior one. 	5
Conclusion	Ask the students to talk about something they learned during the lesson.	<ul style="list-style-type: none"> Connect the representations on the board with what students mention. 	

Public Space Planning Diagram

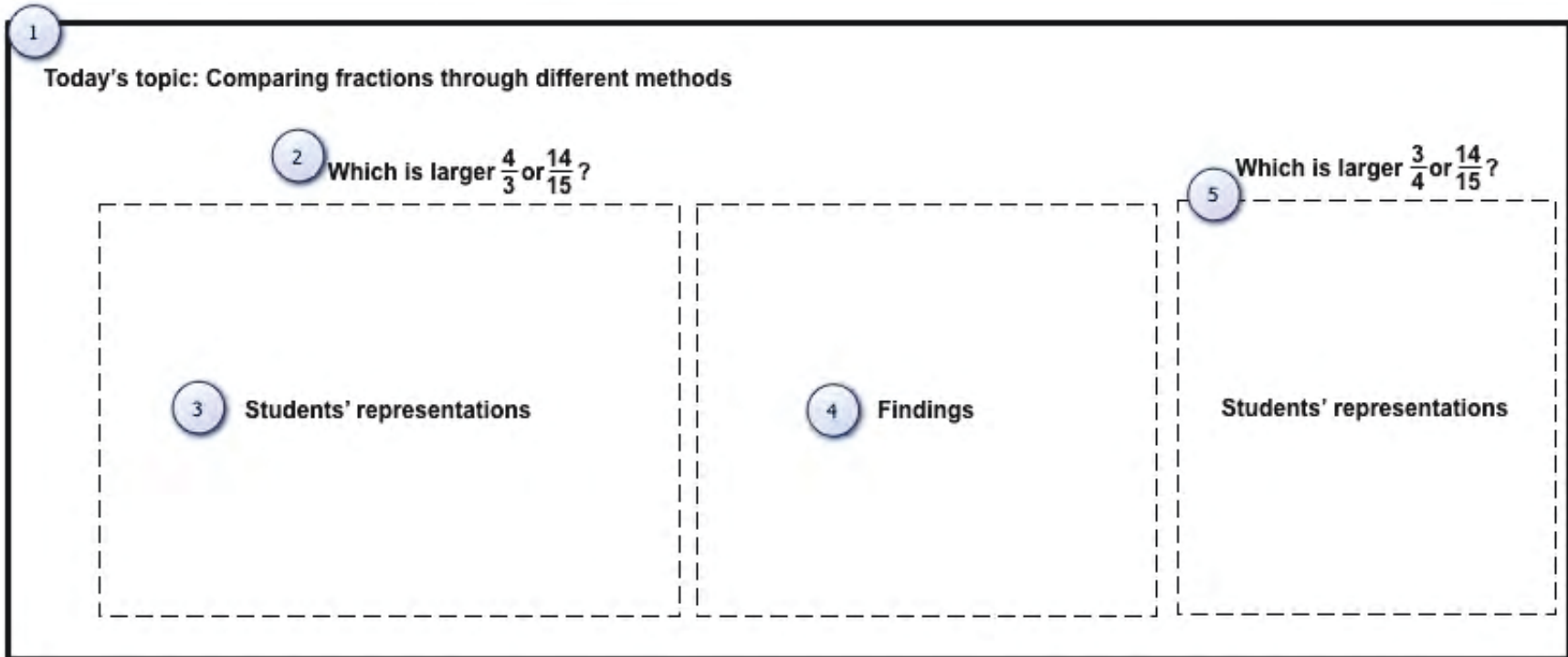


Image of the board

Today's Topic: Comparing fractions through different methods

which is larger $\frac{4}{3}$ or $\frac{14}{15}$

Representations used: number line
area (rectangles)

Representations not used: sets of objects
area (circles)

which is larger: $\frac{3}{4}$ or $\frac{14}{15}$

Findings:

- $\frac{4}{3}$ is bigger because it is more than 1 while $\frac{14}{15}$ is a little smaller.
- Both of these fractions are 1 part away from 1
- Even though the denominator is really small that does not mean that the fraction is going to be smallest
- You can change $\frac{4}{3}$ into a fraction with an equal value that is easier to compare with $\frac{4}{15}$.

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Reflections

When I made a plan for the board, I tried to make lots of space for students writing and drawing. For this lesson, I tried to have the first recordings written on the left side of the board and then have the next contributions written to the right of the ones that were there. I hoped that would make it easier for the students to see the progress of the discussion. Also, I made special spaces for to note the math topic, questions and "findings".

As was expected, the picture that I took of the board at the end of a class was different from what I planned. First of all, because we were short on time we didn't talk about comparing $\frac{3}{4}$ and $\frac{14}{15}$. Since that was an extension anyway I used that for homework and we could go over that the next day. Another difference is that I added some findings on the left of the students' representation, which was out of the left to right progression I was going to try. Overall, I think I used the board pretty well. It was great having the big space for students to write and writing the day's topic on the board was good for remembering the focus of the work.

The kids did not seem to have problems representing the fractions. There were a few issues that came up. It was not surprising that some children had trouble equally dividing the number line into 15 equal pieces from 0 to 1. Some of them used an area model rectangle. The problem was that some of them drew different sized rectangles. Since that showed a pretty important point I decided to have them show that one in class.

During the whole group discussion, I had four students presented their methods on the board. Jade drew one number line and pointed out two fractions. It was interesting because she divided into fifteen between 0 and 1, but showed thirds on the other side of one. She explained that $\frac{14}{15}$ stands on the left and $\frac{4}{3}$ is on the right from 1 so, $\frac{4}{3}$ is bigger than $\frac{14}{15}$. I think that she already knew an answer (probably using a benchmark), and she just used this representation to report her answer rather than to find the answer.

I asked Myles to show his representation of the different sized rectangles. First, he drew same two rectangles, shaded one of them, divided into three equal parts in the other rectangle, and shaded one of them. And then, he drew a bigger rectangle, divided into fifteen parts, and shaded fourteen parts of them. The problem was that $\frac{14}{15}$ looked bigger than $\frac{4}{3}$. Some students commented that he should have used same rectangles to compare fractions. I asked why we should use same rectangles. Desmond said different rectangle could make different fractions, and Tori said same rectangle meant using same one. I'm thinking now that I should have asked or showed how to do that correctly.

Next Marray showed the use of common denominators to make the comparison. He said a little about it and I saw several students nodding. But, I should have asked him how he knew that $\frac{4}{3}$ was equal to $\frac{20}{15}$ or how he knew to make the fraction into 15ths.

Matias used two number lines. One is for $\frac{4}{3}$ and the other is for $\frac{14}{15}$. It was not until looking at the picture later that I noticed that he wrote $\frac{14}{15}$ at the point of $\frac{13}{15}$. No one in class noticed that either.

I asked kids any other representations that they made and we talked about the different representations that were used. Eric said we used number lines and figures. I recorded his saying (I wrote area not figures) on the left side. I asked which representations were not used but could have been. Alyssa told objects, and Ryan (maybe Toni?) said other figures are possible such as circles or triangles. I also recorded them on the board.

I ended class by asking the student to say what we found out. Jade said $\frac{4}{3}$ is bigger because it is more than 1 while $\frac{14}{15}$ is a little smaller. I wrote his comment on the board. His comment was accorded with his representation (using one number line). When I pointed out Jade representation, Nia said "Both of these fractions are 1 part away from 1." I thought it was interesting, so I recorded that too. But, Carmen

commented that those parts are also different, one is one third and the other is one fifth. Elizabeth said a small denominator does not mean that a fraction is small.

I accomplished most of my purposes, but could have connected among the representations better. Taking a picture of the board after the class was helpful. It helped me to capture what happened so that I could think about it later when I had more time. It's funny how much one picture can help you remember what happened.