Description of the Session 7: Studying how tasks and public recording space support mathematics teaching

Participants will begin this session by discussing the Yoshida reading that they read as part of their Session 6 CCA. In this session, participants analyze a mathematics task in terms of the mathematics that can be worked on in the task, how students might solve the problem, and what representations could be used. That same task is then used to practice narrating the construction and use of a representation, namely a number line, to solve that task. After that, participants build on their work in Session 6 to begin developing criteria for planning, using, and reflecting on the use of public recording space by analyzing records of public recording space from their classroom.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
Conversation about a CCA from the last session	10 minutes		• Participants will understand the affordances and drawbacks of not erasing/removing written materials during a mathematics lesson.
I. Preview	5 minutes	Part 1	• Participants will be oriented to the work of the session.
II. Analyzing a task and narrating the construction of a representation to solve that task	55 minutes	Parts 2 & 3	• Participants will be able to analyze a task in terms of the mathematics, student thinking, and representations. • Participants will understand that different tasks create different mathematical learning opportunities for students. • Participants will understand how different representations support different types of mathematical work. • Participants will begin to identify strategies for comparing fractions. • Participants will begin to identify common student misconceptions and errors when comparing fractions. • Participants will continue to develop their skills at narrating the construction and use of a representation to solve a mathematics task.
III. Debriefing and improving the use of public recording space in a mathematics lesson	45 minutes	Parts 4, 5 & 6	• Participants will be able to analyze records of public recording space in terms of student thinking, mathematics, and teaching practice. • Participants will be able to identify ways of improving the use of public recording space during mathematics instruction.

			• Participants will be able to use ideas from the session to identify a goal for improving their recording in public space.
IV. Wrap up	5 minutes	Part 7	• Participants will understand ways of connecting the session content to their classroom.

Classroom Connection Activities

Required	Optional
Type of task: Collection and analysis of public recording space work Description: Engage in planning, enacting and analyzing public recording space work, providing more details about student participation	Type of task: Reading Description: Hiebert (1997) on the selection and use of worthwhile mathematics tasks and the role of tasks in mathematics instruction
	Type of task: Reading Description: NCTM (1991) on the selection and use of worthwhile mathematics tasks and the role of tasks in mathematics instruction. (*Note: This reading is not available in the module.)*

Preparing for the session

☐ Make copies as needed:

- *Resources:* Handout: A fractions-of-a-length task (Part 2)
- *Supplements:* Math notes: Analysis of the fractions-of-a-length task (Part 2)

☐ Customize the Classroom Connection Activities and make copies as needed

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:

a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.

b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.

c) Carefully using mathematical language.

5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards for School Mathematics.

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• representing fractions • defining fractions • **using and explaining methods and representations for comparing fractions** • understanding how equivalence (of fractions) can be represented and used	• identifying and analyzing student conceptions, explanations, and representations of fractions • **identifying and analyzing student strategies for comparing fractions**	• selecting and generating representations • connecting representations • **narrating the process of representing** • supporting students in narrating the use of a representation • recording contributions and emerging mathematical ideas	• **studying public recording space to learn from practice** • **using a conceptual framework to guide the planning, use, and analysis of public recording space**

Conversation about a Classroom Connection Activity from last session (~10 minutes)

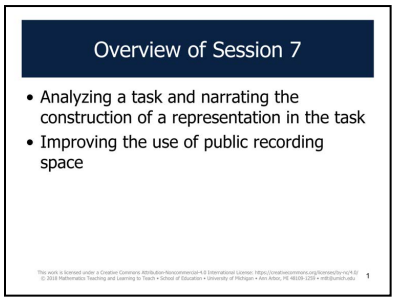
Goals	***Instructional sequence***	***Resources***
• Participants will understand the affordances and drawbacks of not erasing/removing written materials during a mathematics lesson.	1. Discuss the Yoshida (2005) chapter.	• Yoshida (2005) chapter from Session 6 Classroom Connection Activities

Detailed description of activity	Comments & other resources
1. Discuss the Yoshida chapter on using the blackboard effectively. In the article, Yoshida suggests that, during a class, one should not erase/remove something has been recorded in a public space. Ask participants: • What are the benefits of this "no-erase/remove" policy? • What drawbacks are there to not erasing/removing written material? • What are the implications of preserving what has been written when using different methods of public recording like overhead projectors, document projectors, or SMART boards?	*The article reports research that indicates that Japanese teachers rarely erase what they write on the blackboard during a lesson. Teachers in the United States, on the other hand, erase over half of what is written during the lesson, on average. Yoshida posits this difference in use of board space as a key difference in Japanese and U.S. teachers' work. Japanese teachers' board work provides students with a visual summary of "… what is learned in the lesson (p. 96)." The board work is a record of the lesson.* *Participants may note that there are many factors that limit U. S. teachers' use of a public recording space. For example, many classrooms do not have extensive board space. Also, many teachers are now focused on using newer technologies (SMART boards and overhead or document projectors) in their teaching. Having students share their solutions through these technologies does not leave a visual record of the set of representations that have been shared during the lesson. As a result, students are forced to remember the progression of the lesson rather than see it. They need to recall things like what originally prompted their work, what key questions were asked, how different students addressed the prompt, and what vocabulary was introduced. Often, at the end of the lesson, students see the conclusion that has been reached, but do not see how the class got to that conclusion.* *Though a key point in the Yoshida article is not to erase what is written on the board during a lesson, there are other questions that teachers can consider to enhance their use of public recording space, including:* • *Is the original question/problem/prompt written in a public space?* • *Do students show their work? Do they write on the board, chart paper, or overhead?* • *Does the teacher do most of the writing?* • *How is new vocabulary handled?* • *How are conclusions presented? By students? By teachers? Orally? Written?* • *How are connections made among parts of the lesson?* • *What remains of the progression of the lesson at the end of the lesson?*

Part 1: Preview (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the session.	1. Introduce the session and watch the video overview.	• Video (02:30): Overview of session

Detailed description of activity	Comments & other resources
1. Introduce the session: Tasks and problems are a foundation for mathematical work in the classroom. They influence students' opportunities to learn through the types of thinking they require and the types of tools that can be used. In this session, participants use tasks to explore core elements of practice: • Mathematics content: comparing and representing fractions • Student thinking: common strategies and conceptions • Teaching practice: narrating the construction and use of a representation to solve a problem The session also builds on the work in Session 6 to begin to develop criteria for planning, using, and reflecting on the use of public recording space. Have participants watch the *video* in which Dr. Ball frames the work of the session.	



Part 2: Analyzing a task: Which is longer: 3/8 of an inch or 7/16 of an inch? (~25 minutes)

Goals

- Participants will be able to analyze a task in terms of the mathematics, student thinking, and representations.
- Participants will understand that different tasks create different mathematical learning opportunities for students.
- Participants will understand how different representations support different types of mathematical work.
- Participants will begin to identify strategies for comparing fractions.
- Participants will begin to identify common student misconceptions and errors when comparing fractions.

Instructional sequence

1. Introduce Part 2 and watch Video A in which Dr. Ball sets up the task.
2. Analyze the fractions-of-a-length task using the focus questions on the slide.
3. Watch and discuss Videos B-D in which teachers comment on the task in terms of mathematics, student thinking, and representations.
4. (Optional) Watch and discuss additional video clips.

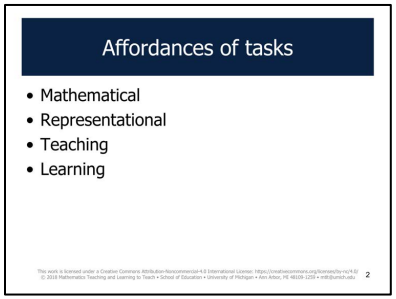
Resources

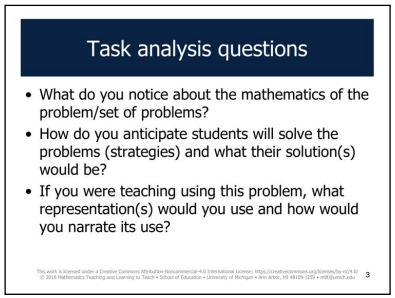
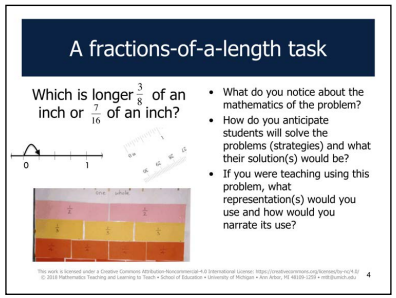
- Video A (01:57): Introduction to tasks
- Video B (01:19): Mathematics
- Video C (03:01): Student thinking
- Video D (01:00): Representations
- Handout: A fractions-of-a-length task

Supplements

- Math notes: Analysis of the fractions-of-a-length task
- Video E (01:32): Further analysis of the mathematics
- Video F (01:37): Further analysis of student thinking
- Video G (01:19): Further analysis of representations

Detailed description of activity	Comments & other resources
1. Introduce Part 2: Analyzing student tasks is a way to develop teaching knowledge and skill. Task analysis provides insights that can be useful when planning and implementing lessons, for example, identifying the key mathematical ideas and anticipating possible student solution methods. In this part, participants analyze a task for the mathematical, representation, teaching, and learning opportunities that are provided. Have participants watch *Video A* in which Dr. Ball describes the ways participants will be analyzing tasks in this session and sets up the task analysis questions.	*This task involves comparing two fractions of an inch. The task utilizes a measurement interpretation of fractions that can be represented with rulers, fraction strips, or number lines. Different strategies are possible for this comparison task.* • *Determining the distance of each fraction from a benchmark, such as 1/2. For example, 3/8 is 1/8 less than ½ and 7/16 is 1/16 less than ½. Since 1/16 is smaller than 1/8, 7/16 is closer to ½, making it the larger fraction.* • *Showing the two quantities using a fraction paper strip or a number line. Each fraction is located (in the case of the number line) or constructed (in the case of fraction strips), and their positions compared; the fraction the farthest from zero is larger.* • *Finding a common denominator and then comparing the numerators. For example, 3/8 can be rewritten as 6/16, and then 6/16 and 7/16 easily compared.*

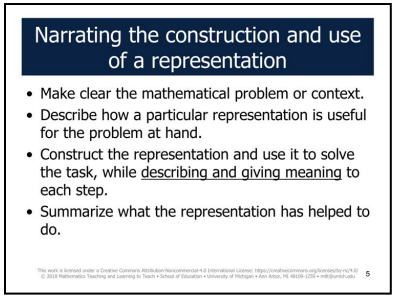


Detailed description of activity	Comments & other resources
 	*The Slide: Task analysis questions can be found in the resources section.* *The Slide: A fractions-of-a-length task can be found in the resources section. A handout with the slide is also available.*
2. Distribute the *Handout: Fractions-of-a-length task*. Have participants work with a partner to analyze the task using the task analysis questions on the Slide: Task analysis questions.	*Note: The task analysis questions slide can be found in the resource section.* *CCSSM Link: Comparing two fractions with different numerators and denominators is a Grade 4 standard (4.NF.2).* *Three ways that students might interpret this task include:* • *focusing on the number of parts (numerator) without considering the size of the parts (denominator);* • *focusing on the size of the parts (denominator) without considering the number of those parts (numerator); and* • *focusing on the numerators and denominators separately.*
3. Have participants watch *Videos B-D* in which teachers in the professional development course analyze the fractions-of-a-length task in terms of the mathematics, student thinking, and representations. As participants watch each video clip, have them consider the focal question for the clip, as well as whether the comments in the clip provide any new insights or questions about the task. • *Video B*: Mathematics - What mathematical connections do you see between the fractions that are being compared? • *Video C*: Student thinking - How are students likely to reason about the comparison? (Note: Students' reasoning may or may not be mathematically valid.) • *Video D*: Representations - What is being considered when suggesting a representation?	*You might choose to have participants make notes about their ideas or discuss them with a partner between each clip, and then, after watching all three clips, take comments in whole group.* *Video B: Mathematics* • *One denominator is a multiple of the second denominator; thus it is easy to find common denominator.* • *Each fraction is one unit fraction (i.e., 1/d, where d equals 8 or 16, respectively) less than one-half.*

Detailed description of activity	Comments & other resources
After watching the three video clips, have participants discuss any new insights or questions about the task. Consider distributing the *Math notes* document so that participants can read more about the task.	*Video C: Student thinking* • *Focusing on the denominators: Students might argue that 3/8 is greater than 7/16 because 1/8 is greater than 1/16.* • *Focusing on missing parts: Students might argue that 7/16 is greater by comparing pieces that are missing to make one half. In other words, 1/16 is less than 1/8, so 7/16 must be greater than 3/8.* • *Focusing on the numerators: Students might argue that 7/16 is greater than 3/8 because 7 is greater than 3.* • *Focusing on numerators and denominators separately: Student might argue that 7/16 is greater than 3/8 because 7 is greater than 3 and 16 is greater than 8.* *Video D: Representations* • *The purpose(s) of the lesson* • *Students' strategies*
4. (*Optional*) Three additional video clips are available for use depending upon time and the needs of the participants. These video clips include: Video E: Further analysis of the mathematics • Identifying equivalent fractions Video F: Further analysis of student thinking • Using number line, decimals, circles, rectangles • Depending on students' understanding about common denominator Video G: Further analysis of representations (in a small group) • Using a ruler rather than number line	

Part 3: Practice narrating (~20 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will continue to develop their skills at narrating the construction and use of a representation to solve a mathematics task.	1. Introduce Part 3 and watch the video in which Dr. Ball reviews the general steps of narrating. 2. Work with a partner to practice narrating the construction and use of a representation. 3. Discuss what was easy/difficult about narrating the comparison.	• Video (02:49): Narration process.

Detailed description of activity	Comments & other resources
1. Introduce Part 3: In this part, participants will use the steps in the narrating process to narrate the fractions-of-a-length task from Part 2: Which is longer: 3/8 of an inch or 7/16 of an inch? Have participants watch *Video A*, in which Dr. Ball frames the general steps of narrating. 	
2. Have participants work in pairs and talk through the construction and use of a number line to compare 3/8 and 7/16. While one partner narrates, the other partner should observe the performance, noting the presence of each of the narrating steps and the key information about fractions or the number line included in the narration. • What parts of the narration process did you see in the performance? • What would you add and/or do differently to improve the narration? After discussing the narration, partners should trade roles and repeat the activity.	*Participants may need a few minutes to think about how they will narrate the comparison before they talk through the comparison with their partner.* *CCSSM Link: In working on this activity, participants are engaged in several of the CCSSM mathematical practices including: making sense of problems and persevering in solving them (#1), reasoning abstractly and quantitatively (#2), constructing viable arguments and critiquing the reasoning of others (#3), and precisely communicating their thinking to others (#6).*

Detailed description of activity	Comments & other resources
3. Ask participants to reflect on what was easy/difficult about narrating the comparison in terms of: • Making clear the mathematical problem or context. • Describing how a particular representation (the number line) is useful for the task at hand. • Constructing the representation and using it to solve the task, while describing and giving meaning to each step. • Summarizing what the representation helped to do.	

Part 4: Analyzing records of public recording space (~25 minutes)

Goals

- Participants will be able to analyze records of public recording space in terms of student thinking, mathematics, and teaching practice.

Instructional sequence

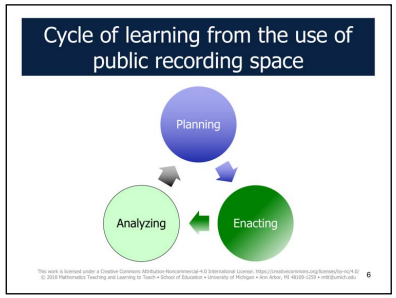
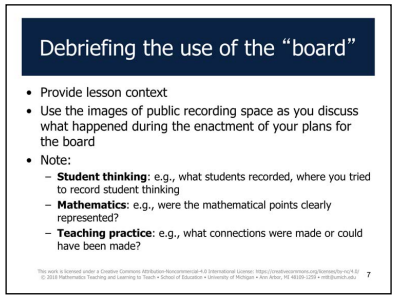
1. Introduce Part 4 and watch the video in which Dr. Ball introduces the task.
2. Analyze participants' use of the board in terms of the mathematics, student thinking, and teaching practice.
3. (Optional) Have participants share their records of public recording space in whole group.
4. (Optional) Watch Video B, in which the teachers discuss the use of the board.

Resources

- Video A (02:41): Overview of task

Supplements

- Video B (05:13): Debriefing process example & images

Detailed description of activity	Comments & other resources
1. Introduce Part 4: The rest of the session will focus on developing ways of learning from-and improving- recording in public space.   Have participants watch *Video A* in which Dr. Ball explains how participants will use the records they collected around their use of public recording space in mathematics lesson.	*The focus questions are designed to engage participants in considering student thinking, mathematics, and teaching practice when they are examining their images of public recording space. This has been a focus in many sessions, including the earlier parts of this session.*

Detailed description of activity	Comments & other resources
2. Have participants work with a colleague to analyze the records from their classroom. Participants should use the lenses of student thinking, the mathematics, and teaching practice to guide their attention, questions, and feedback.	*There are several types of things you can be listening and looking for as participants share their use of public recording space.* • *Make note of productive use of images and other records. Sharing an example of this with the whole group later could help in establishing norms of grounding discussions of public recording space in practice; it could also signal the idea that collecting and sharing those records is useful.* • *Look for instances when participants use the process to discuss mathematical ideas, student thinking, and teaching practices that relate to public recording space. Being able to share a few examples will provide images of how this process can be used to dig into these critical dimensions of mathematics teaching.* • *Listen for examples of ideas about public recording space use that will become explicit later in the module, including but not limited to: recording key points, organization of the space, labeling what is recorded, making space for and supporting students in recording their ideas, capturing recordings for later re-use.*
3. (*Optional)* Have a participant share his or her record of use of public recording space.	*You may want to select an example that highlights something that other participants might consider working on in their practice, not examples of things that show deficiencies.*

Detailed description of activity	Comments & other resources
4. *(Optional)* Have participants watch *Video B* in which two teachers discuss their use of public recording space.	*Sometime it helps to have an image of what the process of sharing images with a colleague could look like. This could help to build comfort with the process or to enhance attention to key content as images are shared. If you choose to watch this video you could encourage teachers to listen for ideas related to mathematics, teaching practices or student thinking. You could also have them think about what questions they might ask this teacher in order to learn more about his use of public recording space to support learning.* *In the video, the teachers discuss:* • *Mathematics: Naming fractional parts; comparing fractions; equivalent fractions* • *Student thinking: Some students made the connection that the two purple triangles represented 2/4 as well as ½, and that two purple triangles are equivalent to one green triangle; other students were hesitant to say that the two purple triangles and the one green triangle could be named the same way (as ½ of the rhombus).* • *Teaching practice: The teacher displayed the pattern blocks on the overhead and labeled each block or group of blocks with its fraction name once he had elicited the name from students. The teacher left each "scenario" on the overhead throughout the entire lesson so that students could make connections between different pattern blocks/groups of pattern blocks and the fractions they represented.*

Part 5: Improving the use of public recording space (~25 minutes)

Goals

- Participants will identify ways of improving the use of public recording space during mathematics instruction.

Instructional sequence

1. Introduce Part 5 and watch Video A, which sets up the task and provides examples of ways of improving the use of public recording space.
2. Have participants begin to develop criteria for planning, using, and reflecting on the use of public recording space.
3. Have participants share the criteria identified in (2).
4. (Optional) Watch Videos B-D in which the teachers discuss planning for using the board.

Resources

- Video A (04:18): Introduction of the task

Supplements

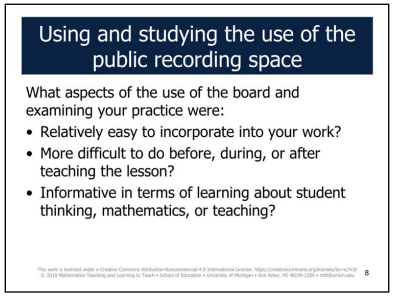
- Video B (01:10): Teaching practice: Making representations that can be saved
- Video C (01:14): Teaching practice: Pre-creating student workspaces
- Video D (01:26): Student thinking: List prior knowledge that is needed to solve the problem

Detailed description of activity	Comments & other resources
1. Introduce Part 5: In this part of the session, participants discuss ways of improving the use of public recording space in a mathematics class. These ideas can help them plan for and reflect on their recording in public spaces. The ideas can also be used during a lesson to inform instructional decisions while they or their students are recording. Have participants watch the *Video A* in which Dr. Ball and teachers discuss ways of improving the use of public recording space in a mathematics lesson.	*Considerations when using public recording space include:* • *Organizing where things get written and the amount of space available to write;* • *Summarizing what has been accomplished to the side of the centerpiece of the lesson;* • *Designating space for student use; and* • *Designing the board (physical layout of the board).*
2. In pairs, have participants discuss possible criteria for planning, using, and analyzing public recording space.	*Encourage participants to be explicit about what they think the different ideas will accomplish with respect to the mathematics, student learning, and/or teaching practice.* *Across the next few sessions, criteria will be elaborated and made more explicit. Criteria will available in different forms for use during the session and for the Classroom Connection Activities.*
3. Collect ideas of possible criteria for planning, using and analyzing public recording space.	

Detailed description of activity	Comments & other resources
4. (*Optional*) If it is useful and time permits, have participants watch videos in which the teachers in the professional development course discuss their use of public recording space. • Video B: Teaching practice: Making representations that can be saved • Video C: Teaching practice: Pre-creating student workspaces • Video D: Student thinking: List prior knowledge that is needed to solve the problem	*Video B: A teacher describes putting a general rule on a chart paper so that it can be used on multiple days.* *Video C: A teacher describes labeling workspace on the board with students' names before students record their work on the board.* *Video D: A teacher describing listing the things students already know that would be useful in solving the problem.*

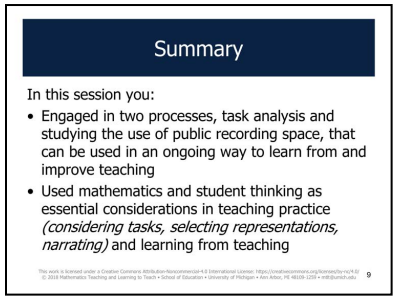
Part 6: Setting a goal for improving the use of public recording space (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be able to use ideas from the session to identify a goal for improving their recording in public space.	1. Discuss using and studying the use of the public recording space. 2. Participants write a goal for improving their use of public recording space in their mathematics lessons.	

Detailed description of activity	Comments & other resources
1. Have participants read and discuss with a partner the questions on the slide. 	*Earlier, participants were focusing on what to improve about their actual use of public recording space. The point here is to support participants in sharing their insights into the process of learning from one's own teaching. In other words, this part is about encouraging participants' engagement in studying their use of public recording space in order to learn. Encourage the sharing of "tips" that could help make the process more manageable or meaningful.*
2. Ask participants to write down a goal for improving their use of public recording space in mathematics lessons. Encourage participants to base their goal on the ideas discussed during this session. If time permits, you might ask a few volunteers to share their goals.	*Possible things for participants to work on:* • *selecting which ideas to record;* • *considering which pieces of what is shared to highlight or unpack, board organization;* • *labeling of different pieces that are included on the board, making clear representations; and* • *reserving space for students to record, and capturing public recording space for later use.* *Listen for the connection between the goals and the ways in which improvement in those areas is connected with student learning or mathematics.*

Part 7: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom.	1. Summarize the session. 2. Explain the Classroom Connection Activities.	

Detailed description of activity	Comments & other resources
1. Summarize the session by emphasizing that participants: • Analyzed a task • Studied the use of public recording space, a process that can be used to learn from and improve teaching • Used knowledge of mathematics and student thinking to inform teaching practice and the process of learning from teaching. 	
2. Distribute the *handout* you customized with selected Classroom Connection Activities and accompanying documents described below. <u>Required:</u> • Planning for, enacting, and analyzing public recording space <u>Optional:</u> • Hiebert (1997) reading on the selection and use of worthwhile mathematics tasks and the role of tasks in mathematics instruction • NCTM (1991). Standard 1: Worthwhile mathematical tasks. *Professional Standards for Teaching Mathematics*. (pp. 24-32). Reston, VA: NCTM.	*As was the case in this session, the records and responses participants generate in response to the required CCA are the basis for a substantial portion of Session 8. It is essential to establish a way for participants to have their responses and records available at the next session. If participants upload their responses and materials, you will need to be sure you have ways to access them during Session 8. If not, then you will need to be sure your participants physically bring their responses and all the materials they collected to Session 8.* *Reassure participants that CCAs related to teaching practices, like the collection and analysis of public recording space, can be done with whatever mathematics is currently being taught. In other words, it is not necessary to be teaching fractions to be engaging with the teaching practices in one's own classroom.*