Description of the session

In this session, participants will be introduced to the goals of the module, which include (a) learning mathematics; (b) developing skill with high-leverage teaching practices; (c) developing usable knowledge of progressions that detail how students' ideas grow with respect to geometric measurement; and (d) developing ways of analyzing and learning from practice. Participants begin to explore the mathematics of length measurement by estimating the length and width of the room, measuring the room with a "personal ruler", and then discussing the issues that arise. Participants then explore how length measurement is addressed in the Common Core State Standards, and they examine a Learning Trajectory for length measurement. After that, participants discuss several examples of student thinking—including students' performance on the "Broken Ruler Task"—and consider the core concepts involved in length measurement. The session closes with an overview of the Classroom Connection Activities that will be completed prior to the next session.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
I. Module preview	10 minutes	Part 1	• Participants will be oriented to the work of the module and develop the sense that this module will be useful in helping to improve their mathematics teaching, knowledge of mathematics, understanding of student thinking, and ability to learn from teaching. • Participants will be oriented to the work of the session.
II. Studying the math of linear measure	50 minutes	Parts 2, 3, and 4	• Participants will begin to recognize and understand concepts and skills involved in measuring length through estimating the length/width of the room. • Participants will understand various ways of estimating length. • Participants will recognize the relationship between estimation of length and physical measurement. • Participants will begin to understand measurement error and how it results from different measurement techniques. • Participants will recognize and identify length measurement within the Common Core State Standards. • Participants will understand connections between the CCSS standards for length measurement across the grade levels.
III. Studying student thinking: Thinking about length measurement	30 minutes	Parts 5 and 6	• Participants will recognize foundational mathematical ideas of length measurement. • Participants will recognize the principles of measurement in student work.
IV. Wrap up	5 minutes	Part 7	• Participants will understand the function of the Classroom Connection Activities • Participants will understand ways of connecting the session content to their classroom.

Classroom Connection Activities

Required	Optional
Type of task: Practice and extension of in-class work Description: Complete the Broken Ruler Task with 3-4 students of different (hypothesized) achievement levels and ask students to record how they measured.	Type of task: Practice and extension of in-class work Description: Video record students as they work on the Broken Ruler Task and compare these videos with the videos shown in Session 1.

Preparing for the session

- ☐ Gather materials: rulers (with both inches and centimeters) and yard/meter sticks
- ☐ Make copies as needed: Handout: Learning Trajectory display of measurement standards (Part 4); Handout: Content cubes – Common Core State Standards Length (Part 4); Handout: Wally and the rug (Part 5)
- ☐ Customize and make copies of the Classroom Connection Activities
- ☐ Text technical setups: Internet connection, speakers, projector, document camera

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Developing an understanding of the principles of measurement

Foundational concepts of measurement include: understanding of the attribute, conservation, transitivity, equal partitioning, iteration of a standard unit, accumulation, origin, and relation between measurement and number.

- **Attribute –** understanding what is being measured
  - Key question: What is being measured?
- **Conservation –** understanding that an attribute being measured does not change when moved
  - Key question: Does the measurement change if I move what is being measured?
- **Transitivity –** understanding that a third object can be used to compare the measures of two other objects
  - Key question: How could I know how the measurements of these objects relate without directly comparing them?
- **Equal partitioning –** understanding that an attribute to be measured can be partitioned into the same-sized units
  - Key question: How can we partition this into equal sized parts?
- **Units and unit iteration –** understanding that an attribute can be measured with a smaller unit without gaps or overlaps
  - Key question: How can this small unit be used to measure something so large?
- **Accumulation –** understanding that as you iterate a unit the count represents the total of all units used
  - Key question: How many copies of this unit were used to measure this attribute?
- **Origin –** the notion that any point on a ratio scale can be used as the origin. Young children who lack this understanding often begin a measurement with "1" instead of zero.
  - Key question: Where could I start the process of measuring?
- **Relation between number and measurement –** Understanding that there is an inverse relation between the size of the unit and the number of those units in a given measure.
  - Key question: How does the number of units change when I use a larger unit of measure?

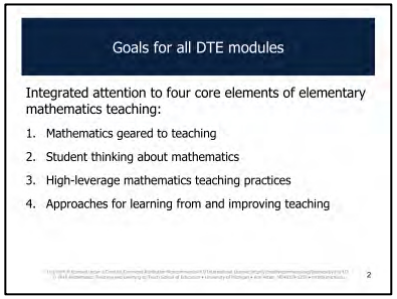
Scope of the module *(focal content of this session in* ***bold****)*

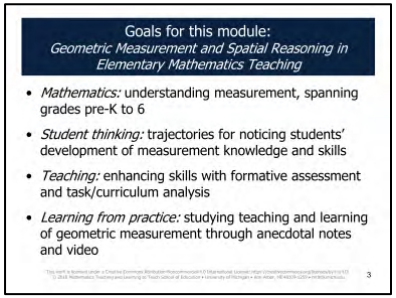
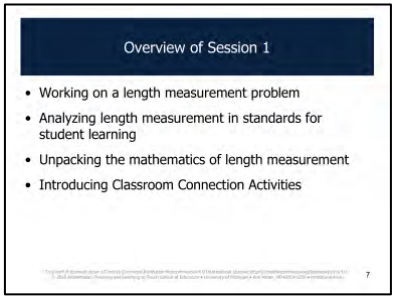
Mathematics	Student thinking	Teaching practice	Learning from practice
• **recognizing the mathematical goal as the first component of a complete Learning Trajectory** • **understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement)** • **understanding how measurement of length, area, and volume are represented and developed in the CCSS** • **understanding how measurement connects with the CCSS standards for mathematical practice** • **understanding concepts and skills involved in measuring length**, area, and volume • understanding connections between length, area, and volume measurement and between metric measurements for each	• recognizing student development as the second component of a complete Learning Trajectory • understanding children's development of measurement through Learning Trajectories for length, area, and volume • **recognizing principles of measurement in student work** • interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement • interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement • interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement	• recognizing instruction as the third component of a complete Learning Trajectory • using anecdotal notes to document what students say and do when working on measurement tasks • connecting measurement activities in curricula to measurement Learning Trajectory levels • modifying measurement tasks to target different and/or particular Learning Trajectory levels	• understanding the anecdotal notes workshop process • using the anecdotal notes workshop to improve the practice of note taking • using the anecdotal notes workshop to improve teaching

Part 1: Module preview and session overview (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the module and develop the sense that this module will be useful in helping to improve their mathematics teaching, knowledge of mathematics, understanding of student thinking, and ability to learn from teaching. • Participants will be oriented to the work of the session.	1. Have participants introduce themselves. 2. Introduce the module by having participants watch Video A.	• Video A (03:18): Module preview • Video B (01:39): Session overview

Detailed description of activity	Comments & other resources
1. Welcome participants to the session and have them introduce themselves.	*This could be done in many ways. One relatively quick way would be to ask participants to say their names, grade levels, years of experience teaching elementary grades, and the topic that they have recently been working on with students in math.* ***Be sure to measure the length and width of the room in advance of this session.*** *You will need to be aware of the length of the room during this session. Later, you will also need to know its area (Session 4) and volume (Session 7). Also note the objects in the room or characteristics of the room that might support estimation of the length of the room.*

Detailed description of activity	Comments & other resources
2. Introduce the module by having participants watch *Video A* in which Dr. Doug Clements and Dr. Julie Sarama, professors at University of Denver, provide an overview of the module. Each Dev-TE@M module focuses on four core elements of the work of elementary teaching: • Mathematics geared to the demands of teaching • Student thinking about mathematics • High-leverage mathematics teaching practices • Approaches for systematically learning from and improving teaching  Work on these elements is integrated across the ten sessions of a module, providing opportunities to practice, build on, and extend ideas over time. In addition, simultaneously working on the four core elements is important because the work of elementary mathematics teaching requires integrated attention to these elements in practice. The content of the module is applicable across grade levels and strands of mathematics. The module includes examples from elementary classrooms and professional development sessions. Many of the videos used in this module are taken from a professional development workshop for teachers led by Dr. Clements and Dr. Sarama. The teachers in the professional development workshop focused on the same content and many of the same activities as are used in this module. These videos are important resources that will support the work in the module. Video clips from the professional development session are often used to frame and summarize activities in the module. These clips are also used to provide opportunities to listen and respond to ideas raised by the teachers in the clips, as well as to analyze and discuss many of the issues with which they grappled during the course. Videos of students used in this module are taken from research done by Dr. Clements and Dr. Sarama. They will serve to provide a context for the work in the module. The module includes Classroom Connection Activities because learning about teaching is an extended process that needs to be connected with and supported by learning in and from one's own teaching. These activities provide significant opportunities to learn. Routine tasks in the activities encourage participants to use their teaching as a context for learning, connect professional development content with common classroom resources, support feedback on learning and teaching, as well as extend thinking about mathematics and teaching that arose in the session.	*In the module videos, Dr. Clements and Dr. Sarama often refer to one another as "Doug" and "Julie."* *To play video in the main viewer, simply click the "play" button towards the bottom-left of the video image, which will change to the "pause" button while the video is playing. To the right of the "play" button is volume control. To the right of the volume control is the time at which the video is currently out of the full length of the clip in minutes and seconds.* *There's a progress bar above the controls and just below the video image where the circular playhead shows where, in the video, you are located. You may notice yellow markings along the progress bar. This indicates a change in the slide associated with that part of the video. The slides are synchronized with the video and will change at the appropriate time.* *You can change whether you look at the video or the slide (if present) by clicking the "switch" button towards the bottom-right of the video image. To the left of the "switch" button is the "viewing options" button to toggle between showing only the video or slide and both at the same time, either one on top of the other or a picture-in-picture effect.* *To the left of the "viewing options" button is the full-screen/windowed view toggle. You can exit full-screen mode and return to windowed mode by either clicking the "fullscreen" button or pressing the "esc" key on your computer keyboard.* *The "CC" button will allow you to turn off and on the available closed captioning for the videos.*

Detailed description of activity	Comments & other resources
This module focuses on the following goals: • Mathematics: understanding measurement, spanning grades pre-K to 6 (Sessions 1, 4, and 7) • Student thinking: trajectories for noticing students' development of measurement knowledge and skills (Sessions 2, 5, and 8) • Teaching: enhancing skills with formative assessment and task/curriculum analysis (Sessions 3, 6, and 9) • Learning from practice: studying teaching and learning of geometric measurement through anecdotal notes and video (all sessions) 	
The module consists of 10 sessions: • Sessions 1-3 focus on length • Sessions 4-6 focus on area • Sessions 7-9 focus on volume • Session 10 provides a summary For each topic (i.e., length, area, and volume), the first session focuses on the mathematics, the second on student thinking, and the third on teaching practices that support children's learning of the topic.	
3. Introduce Session 1 by watching *Video B*. Session 1 launches consideration of length measurement by having participants work on a length measurement problem, explore how length measurement is represented in standards for student learning, and by discussing key concepts of length measurement. 	

Part 2: Estimating the length of the room (~20 minutes)

Goals

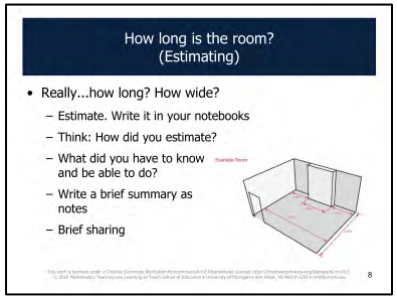
- Participants will begin to recognize and understand concepts and skills involved in measuring length through estimating the length/width of the room.
- Participants will understand various ways of estimating length.

Instructional sequence

1. Introduce the session.
2. Watch Video A to introduce Part 2 and have participants work on the problem independently.
3. Have a few participants share how they worked on the problem; watch and discuss Videos B-F as time and interest permit.

Resources

- Video A (00:38): Estimating the length of the room
- Video B (00:40): Method 1: Visualizing yards
- Video C (00:43): Method 2: Visualizing lying on the floor
- Video D (00:14): Method 3: Using the doorframe
- Video E (00:27): Method 4: Visualizing a football field
- Video F (00:18): Method 5: Sizing up the projector screen

Detailed description of activity	Comments & other resources
1. Introduce Part 2: In this part, participants will begin to consider the mathematics involved in length measurement by mentally estimating the length of the room.  Introduce the "How Long is the Room" problem by showing *Video A*, where Dr. Sarama talks about estimating the length and width of the room. In this part, participants will mentally estimate the length and width of the room and will discuss how they estimated. In Part 3, they will continue their work on this problem by using a "personal ruler" to measure the length and width of the room. Have participants first make an estimate of the length and width of the room independently. Then have them use the following prompts to reflect in their notebooks: • How did you estimate? • What did you have to know and be able to do? • Write a brief summary in your notebook.	*The focus of this activity should be on the mental unit. Push participants to use a mental unit by having them measure walls/lengths that are not visibly partitioned into units.* *Often people use their own body height as a unit when estimating. People also utilize objects/elements from their personal experience (e.g., a football field).*

Detailed description of activity	Comments & other resources
2. Allow a couple of participants to share what they wrote in their notebooks with the whole group. As time and interest permit, show *Videos B-F,* which provide several examples of the strategies teachers in the professional development series used to estimate the length of their room.	*Watching videos of other teacher learners may be useful for emphasizing important ideas during the discussion or helping participants consider and make sense of how others are reasoning (which is a part of the work teachers do every day). In this discussion, one or more of the following videos may be helpful to include.* *Video B: Method 1: Visualizing yards (using a mental unit)* *A teacher imagined a yardstick and used that mental image to estimate the length of half of the room. Then she doubled that estimate.* *Video C: Method 2: Visualizing laying on the floor (using one's own height)* *A teacher used his height as reference (estimating it to be "about 6 ft") and visualized lying on the floor. Using one's height as a visual estimate is a very common strategy.* *Video D: Method 3: Using the doorframe (using a visual reference in the room)* *A teacher estimated the size of the doorframe as 3 feet and used that as a unit. Dr. Sarama comments that it is interesting that the teacher thought of the unit as "one doorframe" rather than "one yard."* *Video E: Method 4: Visualizing a football field (partitioning a larger, visual unit)* *A teacher visualizes the length of a football field (larger than the room) and sees the room as some portion of that distance – an interesting and uncommon strategy that is good to talk about.* *Video F: Method 5: Sizing up the projector screen (using an intermediary unit)* *A teacher visualized herself lying along the screen and then visualized iterating the screen over the length of the room. This shows another example of using something in the room (i.e., the projector screen) as an intermediary unit.*

Part 3: Measuring the length of the room (~20 minutes)

Goals

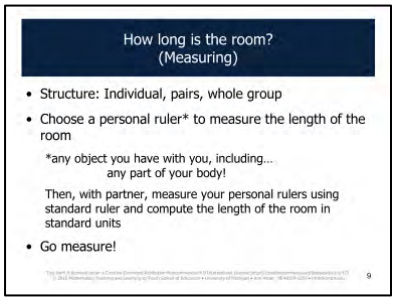
- Participants will recognize the relationship between estimation of length and physical measurement.
- Participants will begin to understand measurement error and how it results from different measurement techniques.

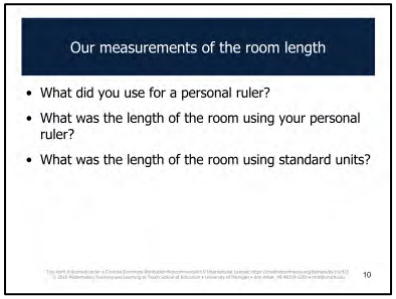
Instructional sequence

1. Watch Video A and have participants use a "personal ruler" to measure the room independently.
2. Have participants work in partners to calculate the dimensions of the room using standard units and then compare their answers with each other.
3. Introduce the whole-group discussion of the problem; watch Videos B-F as time and interest permit.
4. Discuss larger issues related to measurement, using Videos H-J as needed to support the discussion.
5. Have participants reflect on their work.

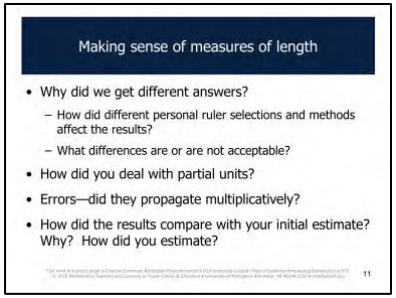
Resources

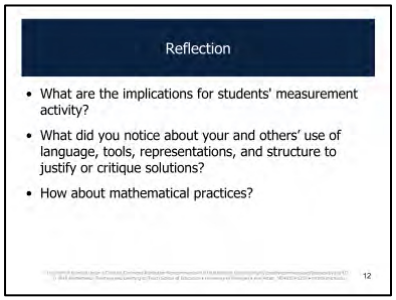
- Video A (1:33): Measuring the room
- Video B (00:20): Method 1: Pacing and using feet
- Video C (00:32): Method 2: Using forearms
- Video D (00:37): Method 3: Using clothing
- Video E (00:24): Method 4: Iterating an object of known length
- Video F (01:34): Method 5: Converting tables into other measurements
- Video G (0:23): Sharing measurements
- Video H (02:14): Why measurements of the rooms are so different
- Video I (00:34): How errors propagate
- Video J (01:09): Why estimates are off
- Video K (00:33): Reflection

Detailed description of activity	Comments & other resources
1. Introduce Part 3: In this part, participants will measure the room using a "personal ruler" and discuss their results. Have participants watch *Video A* in which Dr. Sarama talks about using personal rulers to measure one dimension of the room. Allow participants about 10 minutes to work individually to measure the length and width of the room with the personal rulers they have selected. 	*A "personal ruler" can be any object a person has in his/her possession at the time.*
2. Have participants work with a partner to measure their personal rulers using standard instruments and compute the length of the room in those standard units. Participants should discuss their results and justify why they think they have similar or (quite) different measures.	*Make sure to stress that rulers/meter sticks are to be used to measure "personal rulers"—not to measure the room.*

Detailed description of activity	Comments & other resources
3. Use the slide *Our measurements of the room length* to launch the discussion of their methods and results for the "How Long is the Room?" problem by responding to the following questions: • What did you use for a personal ruler? • What was the length of the room using your personal ruler? • What was the length of the room using standard units?  <u>Optional</u>: Use *Video G* to launch the discussion. Encourage both partners to contribute to the explanation. Show a selection of *Videos B-F* to support your discussion. These videos show examples of methods that people might use to measure with a personal ruler. • Video B: Method 1: Pacing and using feet • Video C: Method 2: Using forearms • Video D: Method 3: Using clothing • Video E: Method 4: Iterating an object of known length • Video F: Method 5: Converting tables into other measurements	*There will be errors in the measurements. The whole-group discussion should include discussion of these errors and why they happen.* *Consider showing one or more of the following videos to support this discussion (to diversify the methods or to see a new example of an approach teachers shared):* *Video B: Method 1: Pacing and using feet* *In this video, one teacher paces off the distance and another uses her feet. These are two common methods for measuring the length of a room.* *Video C: Method 2: Using forearms* *This teacher uses a different body part (her forearm) as her personal ruler. An interesting discussion point comes near the end as she encounters an obstacle (the cabinet) she has to deal with.* *Video D: Method 3: Using clothing* *These two use a sweater as their personal ruler. This brings up a good discussion point about measurement error and how measurement errors might arise, especially with a non-rigid object.* *Video E: Method 4: Iterating an object of known length* *In this video, both teachers are using a folder as a personal ruler. Note how one has the folder opened up and the other keeps it closed. Also, note how one teacher uses a pen to mark the endpoint each time, while the other simply visualizes the spot. Again, a good discussion point about measurement error.* *Video F: Method 5: Converting tables into other measurements* *This shows an example of using an object in the room as a personal ruler and then converting the personal ruler to other units (inches, feet). This provides a good opportunity to talk about precision. How does the precision change when using a ruler vs. using a body length?*

Detailed description of activity	Comments & other resources

4. Once participants have had a chance to share their methods for measuring the length of the room with a personal ruler, use the following questions to invite participants to discuss some of the larger issues related to measurement:
• Why did we get different answers?
  ○ How did different personal ruler selections and methods affect the results?
  ○ What differences are or are not acceptable?
• How did you deal with partial units?
• Errors—did they propagate multiplicatively?
• How did the results compare with your initial estimate? Why? How did you estimate?
 	
As needed, show a selection of *Videos H-J* to support discussion of these ideas:
• Video H: Why measurements of the rooms are so different
• Video I: How errors propagate
• Video J: Why estimates are off
 	
 
 	
	During the discussion, participants may bring up the following topics:
	• *estimation, measurement error, etc.*
	• *different strategies people used to estimate the length of the room*
	• *remainders, errors*
	• *features of measurement (continuous vs. discrete quantity; measurement errors)*
	
	Video H: Why measurements of the rooms are so different
	
	Dr. Sarama discusses three different personal rulers used to measuring the room (a ruler, a folder, and a jacket) and errors that might be associated with each. It may be a useful follow up to Videos D and E.
	
	Video I: How errors propagate
	
	In this video, Dr. Sarama discusses how small errors, done over and over, can result in a large difference.
	
	Video J: Why estimates are off
	
	This video shows a discussion of how actual measurements compared to original estimates and where errors occurred, setting the stage for why we need measurement.

5. Invite participants to think-pair-share about what they are thinking and learning about length, using the following questions: • *What are the implications for students' measurement activity?* • *What did you notice about your and others' use of language, tools, representations, and structure to justify or critique solutions?* • *How about mathematical practices?*  <u>Optional</u>: Launch this time of reflection by watching *Video K*.	*Think-pair-share is a format used throughout the module, which involves a sequence of independent work ("think"), partner work ("pair"), and then whole group discussion ("share").*

Part 4: Analyzing length in standards for student learning (~10 minutes)

Goals

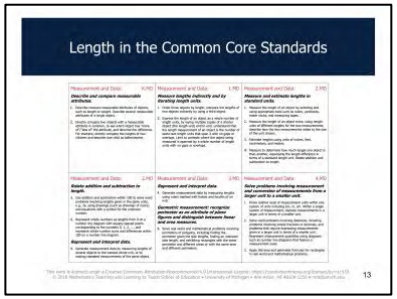
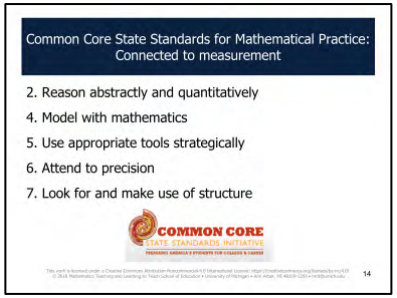
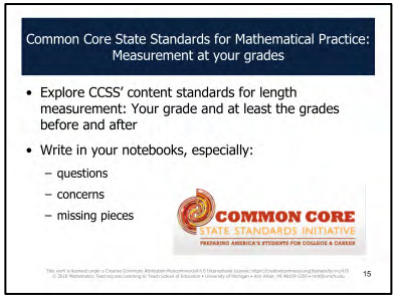
- Participants will recognize and identify length measurement within the Common Core State Standards.
- Participants will understand connections between the CCSS standards for length measurement across the grade levels.

Instructional sequence

1. Introduce the part by having participants review the first Handout; watch Videos A and B.
2. Have participants explore how Common Core State Standards address length measurement.
3. Discuss in whole group, using Videos C-D as time and interest permit.

Resources

- Video A (01:34): Measurement of length in the Common Core
- Video B (03:55): Mathematical practices and measurement standards
- Video C (01:31): Do standards on length "go away"
- Video D (00:42): Teaching measurement for exposure
- Handout: Learning trajectory display of measurement standards
- Handout: Content cubes – Common Core State Standards Length

Detailed description of activity	Comments & other resources
1. Introduce Part 4: In this part, participants explore the progression of length measurement standards in the Common Core State Standards. Distribute *Handout: Content cube – Common Core State Standards length standards* and give participants time to read this document. Then, watch *Video A,* in which Dr. Clements provides an overview of how the Common Core State Standards address length in the elementary grades. Then, watch *Video B,* in which Dr. Clements discusses how the following Common Core State Standards for mathematical practice relate to measurement: • Reason abstractly and quantitatively (MP #2) • Model with mathematics (MP #4) • Use appropriate tools strategically (MP #5) • Attend to precision (MP #6) • Look for and make use of structure (MP#7)	 
2. Distribute *Handout: Learning trajectory display of measurement standards*. Encourage participants to explore how measurement is addressed in the Common Core State Standards and to write down any questions, comments, observations, or concerns they have in their notebooks.	

Detailed description of activity	Comments & other resources
3. After participants have had a few minutes to review the handouts and record their ideas, encourage them to share what they noticed in the Common Core State Standards. If applicable, show *Video C and/or D* to support the discussion: • Video C: Do standards of length "go away"? • Video D: Teaching measurement for exposure	*The whole group discussion might include:* • *Discussion of units* • *Discussion about the value of experience in helping students develop measurement concepts* • *Opportunities to review the foundational concepts of measurement, including: understanding of the attribute, conservation, transitivity, equal partitioning, iteration of a standard unit, accumulation, origin, and relation between measurement and number* *Video C: Do standards of length "go away"?* *This video includes a short discussion of how the standards for length become more focused on application (e.g., conversion of units) and how laying the foundation at lower grades is important for meeting the standards at Grades 3 and 4.* *Video D: Teaching measurement for exposure* *Dr. Clements and Dr. Sarama talk about the decision many teachers make to only teach measurement for exposure rather than for understanding.*

Part 5: The mathematics of length measurement – Measurement stories (~15 minutes)

Goals

- Participants will recognize foundational mathematical ideas of length measurement.

Instructional sequence

1. Introduce the part by watching and discussing Video A.
2. Watch Video B, in which Dr. Clements tells a second story about children's thinking about measurement.
3. Have participants discuss concepts and skills for length measurement that are addressed in the Common Core State Standards.

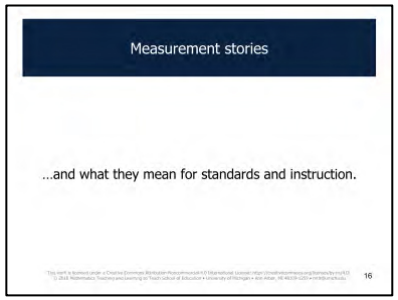
Resources

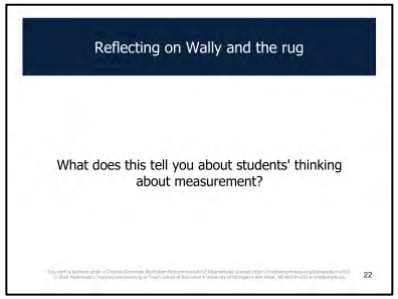
- Video A (03:47) Wally and the rug
- Video B (04:30): Doug, friends, and the room

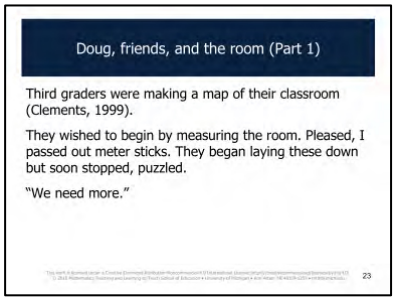
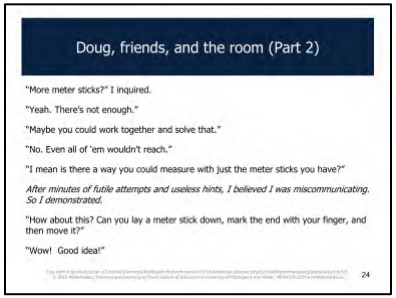
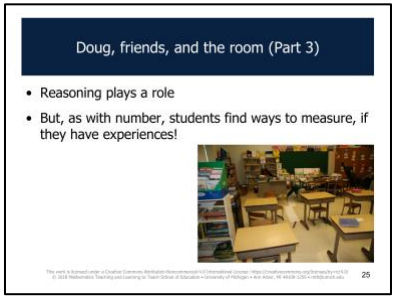
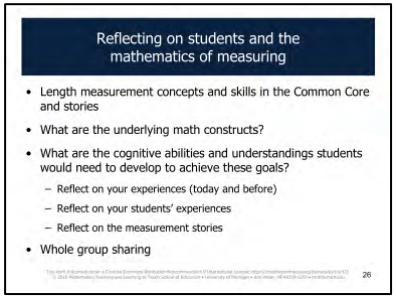
Supplements

- Video (01:19): Wally and the rug – The realness of non-standard measurement tools
- Video (00:38): Wally and the rug – Making measurement tools real
- Handout: Wally and the rug

Detailed description of activity	Comments & other resources
1. Introduce Part 5: In this part, participants will discuss classroom stories involving children's thinking about length measurement, and they will consider the mathematical ideas involved. Watch *Video A* in which Dr. Clements introduces classroom stories that include examples of student thinking about length measurement and tells a story from Vivian Paley's kindergarten class about "Wally and the Rug". Consider distributing *Handout: Wally and the rug* as a reference for participants. After watching the video, have participants think-pair-share about the question: "What does this tell you about students' thinking about measurement?"	*Make sure to guide participants to talk about children's thinking and cognition.* *As time and interest permit, show one or both of the videos included in the supplements.* *Video: Wally and the rug – The realness of non-standard measurement tools* *In this video, two teachers talk about how a physical unit (like Wally) means something, but a more abstract "ruler" doesn't really make sense or isn't "real" to young children.* *Video: Wally and the rug – Making measurement tools real* *In this video, a teacher discusses the importance of providing regular opportunities to use measurement in order to build familiarity and competence.*





Detailed description of activity	Comments & other resources
2. Play *Video B,* where Dr. Clements tells a story about a class of third graders and their attempt to measure their classroom. In this video, Dr. Clements uses this story to make the point that • Reasoning plays a role • But, as with number, students find ways to measure if they have experiences  **Doug, friends, and the room (Part 1)** Third graders were making a map of their classroom (Clements, 1999). They wished to begin by measuring the room. Pleased, I passed out meter sticks. They began laying these down but soon stopped, puzzled. "We need more." 23  **Doug, friends, and the room (Part 2)** "More meter sticks?" I inquired. "Yeah. There's not enough." "Maybe you could work together and solve that." "No. Even all of 'em wouldn't reach." "I mean is there a way you could measure with just the meter sticks you have?" *After minutes of futile attempts and useless hints, I believed I was miscommunicating. So I demonstrated.* "How about this? Can you lay a meter stick down, mark the end with your finger, and then move it?" "Wow! Good idea!" 24  **Doug, friends, and the room (Part 3)** • Reasoning plays a role • But, as with number, students find ways to measure, if they have experiences! 25	
3. In groups of four, have participants discuss the length measurement concepts and skills for length in the Common Core and in the measurement stories. Encourage participants to focus on • the underlying mathematical constructs, the cognitive abilities and understandings students would need to develop to achieve the goals • their own experiences (today and before) with length measurement, their students' experiences with measurement, and the measurement stories  **Reflecting on students and the mathematics of measuring** • Length measurement concepts and skills in the Common Core and stories • What are the underlying math constructs? • What are the cognitive abilities and understandings students would need to develop to achieve these goals? – Reflect on your experiences (today and before) – Reflect on your students' experiences – Reflect on the measurement stories • Whole group sharing 26	*As you circulate listening to discussions, you could also encourage participants to consider the mathematical progression and any "connective tissue" that may be appropriate for grade levels that do not have explicit standards for length measurement.*

Part 6: The mathematics of length measurement – Core concepts and the broken ruler (~15 minutes)

Goals

- Participants will recognize principles involved in length measurement.
- Participants will recognize the principles of measurement in student work.

Instructional sequence

1. Introduce Part 6 and play Video A, where Dr. Clements outlines the principles of length measurement.
2. Play Video B, where Dr. Clements and Dr. Sarama introduce the Broken Ruler Task.
3. Have participants watch, take notes about, and discuss Videos C and D (and other videos, as time and interest permit).
4. Conclude the part by showing Video E, which summarizes the mathematics of length measurement.

Resources

- Video A (01:50): Length concepts
- Video B (03:07): The broken ruler task
- Video C (00:36): Response to the Broken Ruler Task: Student 1
- Video D (00:44): Response to the Broken Ruler Task: Student 2
- Video E (02:01): Mathematics of length measurement
- Video F (00:57): Interpreting a student's response to the Broken Ruler: Student 1
- Video G (01:03): Interpreting a student's response to the Broken Ruler: Student 2

Supplements

- Video (00:39): Interpreting a student's response to the Broken Ruler: Student 3
- Video (00:59): Interpreting a student's response to the Broken Ruler: Student 4

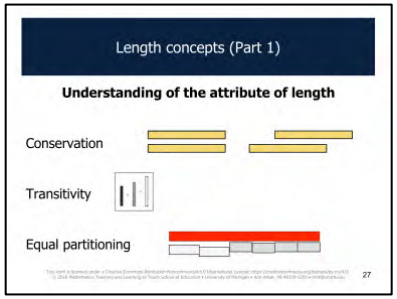
Detailed description of activity	Comments & other resources

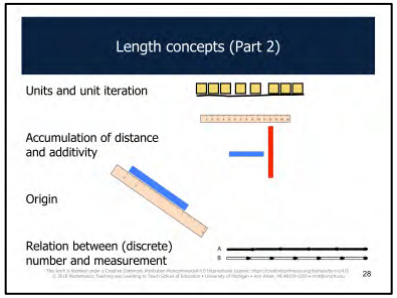
1. Introduce Part 6: In this part, Dr. Clements and Dr. Sarama outline concepts involved in length measurement and share an task that can be used to assess students' understandings of these concepts.

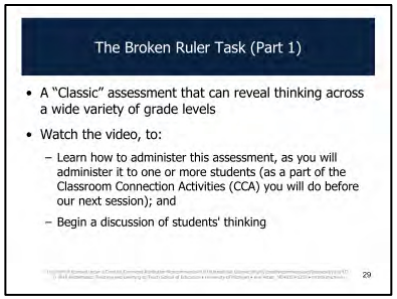
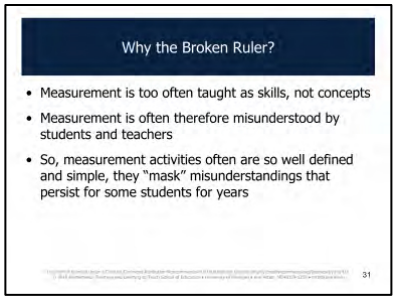
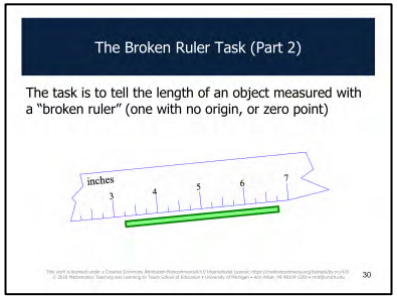
   Show *Video A*, where Dr. Clements talks through the various concepts involved in length measurement:

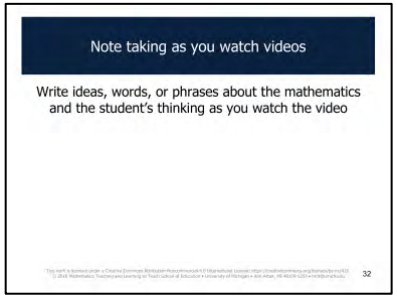
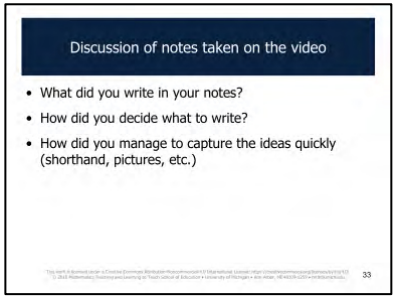
   - Conservation
   - Transitivity
   - Equal partitioning

   - Units and unit iteration
   - Accumulation of distance and additivity
   - Origin
   - Relation between (discrete) number and measurement

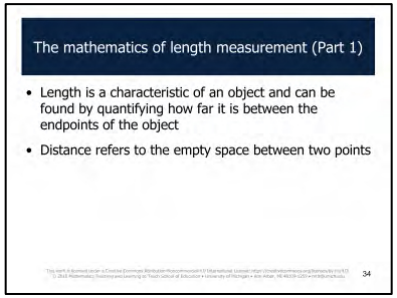


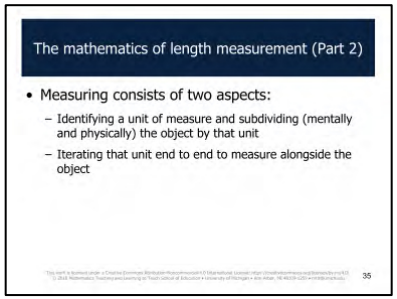


Detailed description of activity	Comments & other resources
2. Play *Video B,* where Dr. Clements and Dr. Sarama introduce the Broken Ruler Task. This is a "classic" assessment that can reveal thinking across a variety of grade levels.    Next, participants will watch videos of students' thinking about the task as a way to begin a discussion of how students think about length measurement. Foreshadow that those videos are important because they show participants how they can administer a task like the Broken Ruler. As part of the Classroom Connection Activities, participants will administer this assessment to one or more of their students before the next session. In the next session, participants will share about their experiences administering the assessment with their students.	*The Broken Ruler Task is useful because* • *Measurement is too often taught as skills, not concepts* • *Measurement is often therefore misunderstood by students and teachers* • *Due to these, measurement activities often are so well defined and simple, they "mask" misunderstandings that persist for some students for years* *Therefore, some of these misconceptions will be present in students throughout the early and elementary grades.*

Detailed description of activity	Comments & other resources
3. Explain that participants will watch some examples of the Broken Ruler task. As participants watch, encourage them to record ideas, words, or phrases about the mathematics and the students' thinking. Remind them that they will be administering this assessment with their own students before the next session. Play *Video C,* which shows a student working on the Broken Ruler task. Then lead participants in a discussion about their note-taking. 	*This video, like others in the module, is text tracked. To see the text tracking, move your cursor off of the video so that the tool bar disappears.* *Video C: Response to the Broken Ruler Task: Student 1* *In this video, the student says that the answer is 7 inches. When asked why, she points to each number that is marked on the ruler (i.e., 3, 4, 5, 6, 7) and shows that she ends at the 7.* *After Video C you might consider playing –* *Video F: Interpreting a student's response to the Broken Ruler: Student 1* *In this video, teachers point out that the student* • appears to measure by looking at the largest number on the ruler • does not appear to understand that measurement is like counting lengths • does appear to know how to look at the endpoint when measuring, but does not know how to take the starting point into consideration
Then play *Video D* which shows another student who is working on the Broken Ruler task. Again, have participants discuss the notes that they took while watching the video. If it would be useful during the discussion, show *Video: Interpreting a student's response to the Broken Ruler Task: Student 2* (in the Resources section), which shows teachers' discussing this student's thinking about the Broken Ruler task. 	*Video D: Response to the Broken Ruler Task: Student 2* *In this video, the student says that the answer is 9 inches. He starts at the 6 (marked on the ruler) and then counts up (i.e., 7, 8, 9) until he gets to the tick mark that is aligned with the end of the rod.* *After video D you might consider playing –* *Video G: Interpreting a student's response to the Broken Ruler: Student 2* *In this video, teachers consider whether the student* • was thinking of the ruler as a number line • lacked an understanding of fractions

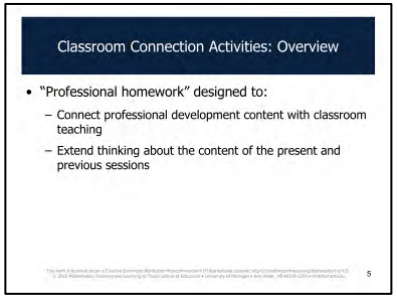
Detailed description of activity	Comments & other resources
4. Conclude this part by playing *Video E,* where Dr. Clements and Dr. Sarama summarize the mathematics of length that were discussed during this session. • Length is a characteristic of an object and can be found by quantifying how far it is between the endpoints of the object. • Distance refers to the empty space between two points. • Measuring consists of two aspects: ○ identifying a unit of measure and subdividing (mentally and physically) the object by that unit, and ○ placing that unit end to end (iterating) alongside the object being measured.	*If time permits, discuss the following videos of other students' responses to the Broken Ruler Task (included as Supplements):* • Response to the Broken Ruler Task: Student 3 • Response to the Broken Ruler Task: Student 4





Part 7: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand the function of the Classroom Connection Activities • Participants will understand ways of connecting the session content to their classroom.	1. Watch Video A. 2. Distribute the Classroom Connection Activities. 3. Summarize the work of the session.	• Video A (00:41): Classroom Connection Activities

Detailed description of activity	Comments & other resources
1. Introduce Classroom Connection Activities. These activities are professional homework designed to • Connect the professional development content with classroom teaching • Extend thinking about the content of the present and previous sessions Watch the *video* in which Dr. Sarama describes the use of Classroom Connection Activities in the module and introduces what is to come in the next session. 	*Learning about teaching is an extended process that can be supported by learning in and from one's own teaching, so each session includes "Classroom Connection Activities." These provide significant opportunities to learn. Routine tasks in the activities encourage participants to use their teaching as a context for learning, to connect professional development content with common classroom resources, support feedback on learning and teaching, as well as extend thinking about mathematics and teaching that arose in the session.* *Emphasize to participants that they do not need to be <u>teaching</u> length in order to complete these activities.*
2. Distribute the Classroom Connection Activities, reminding participants that they will be administering the Broken Ruler task to some of their students before the next session. Have participants think-pair-share about the question: "What do you think your students will do on this task?" Explain that, before the next session, participants should • Complete this task with 3-4 students of different (hypothesized) achievement levels • Ask the students to write down and/or draw how they measured • *<u>Optional</u>:* Video record students as they work on this task to compare with what the students did in the videos shown in Session 1.	*It may be helpful to ask participants to upload their responses or materials related to the Classroom Connection Activities so that you review what participants have been thinking and trying prior to the next sessions. In addition, uploading responses would allow participants to easily share their ideas with each other.* *The optional video task provides a good opportunity for participants to practice collecting video in their classrooms, which is something they would benefit from doing throughout the module. This work provides experiences that participants will be able to build upon in Session 2.*

Detailed description of activity	Comments & other resources
3. Summarize the work of the session for participants. Reiterate that their work in the Classroom Connection Activities will be taken up during the next session.