Session 10: Connecting dimensions of measurement

Description of the session

In Session 10, participants will wrap up their work on measurement. First, they will engage in a final anecdotal notes workshop, based on their experiences teaching a measurement task in their classrooms. They will debrief the workshop, discussing the benefits of taking anecdotal notes and using the "Anecdotal Notes Workshop" learning from practice protocol to structure discussion with colleagues. Then, they will work on a task designed to highlight connections among different dimensions of measurement. Participants will then review the Learning Trajectories for length, area, and volume measurement by acting out scenarios of students' work on tasks and then identifying the level of the Learning Trajectory being represented in the scenario. Finally, participants will discuss the module as a whole and will talk about ways in which they can use what they have learned as they move forward.

*Activities and goals of the session**

Activities	Times	Corresponding parts of the session	Goals
I. Overview and learning from practice	40 minutes	Parts 1 & 2	• Participants will be oriented to the work of the session. • Participants will share the measurement instruction they tried in their classrooms. • Participants will distill key insights from the use of anecdotal notes as the basis for learning from practice and learning with colleagues.
II. Building metric units as a way of connecting measurements	20 minutes	Part 3	• Participants will explore connections between length, area, and volume measurements. • Participants will explore connections between metric measurements.
III. Reviewing student thinking and task selection	20 minutes	Part 4	• Participants will solidify their understanding of the Learning Trajectory levels for length, area, and volume. • Participants will solidify their understanding of how tasks make Learning Trajectory levels visible.
IV. Wrap up	10 minutes	Part 5	• Participants will be reminded of the work from all sessions in the module. • Participants will be encouraged to make changes in their teaching of measurement.

*Conversations about the CCAs from the last session are integrated into this session.

Preparing for the session

- Make copies as needed: Handout: Learning from Practice Protocol (Parts 1 & 2); Handout: Acting It Out cards (Part 4); Handout: Content cube – Length Learning Trajectory (Part 4); Handout: Content cube – Area Learning Trajectory (Part 4); Handout: Content cube – Volume Learning Trajectory (Part 4)
- Assemble and prepare materials needed for Part 3
  - Metric rulers
  - Scissors
  - Tape
  - Gallon-sized zip-top bags
  - Heavy tagboard or cardboard
  - Water (enough for each participant to fill a cube that has a volume of one cubic decimeter)
  - A one-liter container (such as a soda bottle or graduated cylinder)
  - Funnels
  - Metric scale
- Assemble and prepare materials needed for Part 4 "Acting it out"
  - 2x3x2 rectangular prism constructed from cubes
  - 24 cubes the same as used to construct the 2x3x2 prism
  - 30 square, inch tiles
  - Handouts with one 4" x 6" rectangle
  - Objects that are close to the same length (e.g., a marker and a pen)
  - An object that is close to an exact number of inches long
  - Handouts picturing a rectangular prism that is partially filled with cubes (see the picture on Handout: Acting it out cards)
  - Handouts picturing an 8 cm x 9 cm rectangle and a 3 cm x 2 cm rectangle with side lengths labeled (see the picture on Handout: Acting it out cards)
- Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Developing an understanding of the principles of measurement

Foundational concepts of measurement include: understanding of the attribute, conservation, transitivity, equal partitioning, iteration of a standard unit, accumulation, origin, and relation between measurement and number.

- **Attribute-** understanding what is being measured
  - *Key question: What is being measured?*
- **Conservation-** understanding that an attribute being measured does not change when moved
  - *Key question: Does the measurement change if I move what is being measured?*
- **Transitivity-** understanding that a third object can be used to compare the measures of two other objects
  - *Key question: How could I know how the measurements of these objects relate without directly comparing them?*
- **Equal partitioning-** understanding that an attribute to be measured can be partitioned into the same-sized units
  - *Key question: How can we partition this into equal sized parts?*
- **Units and unit iteration**- understanding that an attribute can be measured with a smaller unit without gaps or overlaps
  - *Key question: How can this small unit be used to measure something so large?*
- **Accumulation-** understanding that as you iterate a unit the count represents the total of all units used
  - *Key question: How many copies of this unit were used to measure this attribute?*
- **Origin** is the notion that any point on a ratio scale can be used as the origin. Young children who lack this understanding often begin a measurement with "1" instead of zero.
  - *Key question: Where could I start the process of measuring?*
- **Relation between number and measurement**- Understanding that there is an inverse relation between the size of the unit and the number of those units in a given measure.
  - *Key question: How does the number of units change when I use a larger unit of measure?*

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• recognizing the mathematical goal as the first component of a complete Learning Trajectory • understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement) • understanding how measurement of length, area, and volume are represented and developed in the CCSS • understanding how measurement connects with the CCSS standards for mathematical practice • understanding concepts and skills involved in measuring length, area, and/or volume • **understanding connections between length, area, and volume measurement and between metric measurements for each**	• recognizing student development as the second component of a complete Learning Trajectory • understanding children's development of measurement through Learning Trajectories for length, area, and volume • recognizing principles of measurement in student work • **interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement** • **interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement** • **interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement**	• recognizing instruction as the third component of a complete Learning Trajectory • using anecdotal notes to document what students say and do when working on measurement tasks • connecting measurement activities in curricula to measurement Learning Trajectory levels • modifying measurement tasks to target different and/or particular Learning Trajectory levels	• understanding the anecdotal notes workshop process • using the anecdotal notes workshop to improve the practice of note taking • **using the anecdotal notes workshop to improve teaching**

Part 1: Overview and sharing examples of teaching measurement (~30 minutes)

Goals

- Participants will be oriented to the work of the session.
- Participants will share the measurement instruction they tried in their classrooms.
- Participants will distill key insights from the use of anecdotal notes as the basis for learning from practice and learning with colleagues.

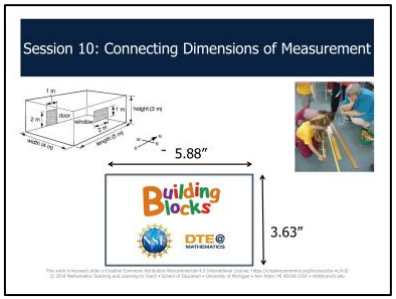
Instructional sequence

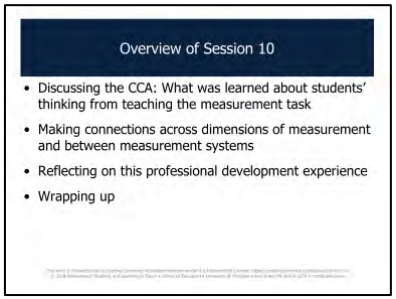
1. Introduce the session by watching Video A.
2. Watch Video B and have participants discuss their CCA tasks with small groups.

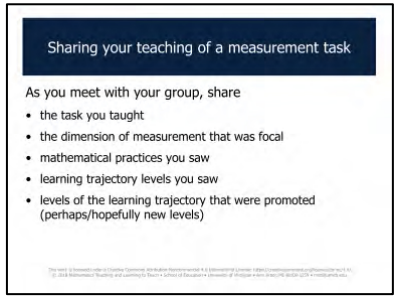
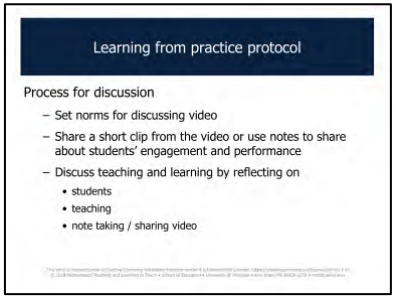
Resources

- Video A (01:08): Overview
- Video B (01:07): Sharing the teaching of a measurement task
- Handout: Anecdotal notes workshop protocol

Detailed description of activity	Comments & other resources
1. Introduce the session by watching *Video A*: In Sessions 9, participants finished their work on the Learning Trajectory for volume measurement. In Session 10, participants will focus on making mathematical connections and summarizing the work of the module. In this session, participants will • Discuss the CCA—what was learned about students' thinking from teaching the measurement task • Make connections across dimensions of measurement and between measurement systems • Reflect on this professional development experience • Wrap up the module	*In Video A, Dr. Sarama and Dr. Clements talk about the different connections that will be the focus of this session:* 1. *Connections between dimensions (e.g., how does length relate to area and how do length and area relate to volume?), and* 2. *Connections between different metric measures (units such as centimeters and meters).*

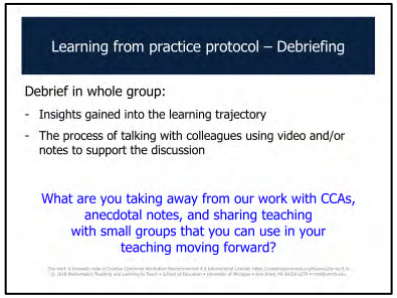




Detailed description of activity	Comments & other resources
2. Have participants watch *Video B*, where Dr. Sarama introduces the activity in this part: engaging in an anecdotal notes workshop in which they discuss their experiences teaching a measurement task as part of the Classroom Connection Activity. As participants meet with their groups, Dr. Sarama encourages them to discuss • The tasks they taught • The dimension of measurement that was focal in this task • Mathematical practices they saw • Learning Trajectory levels they saw • Learning Trajectory levels they promoted through their work with students on the task She also encourages participants to discuss whether or not they felt the need to take notes about what they were noticing when they also had a video. After watching the video, give participants time to share in small groups.   	*This module focuses on a specific method of learning from practice: using anecdotal notes to enhance and inform teaching. Even though participants may find it easier to use video in the discussions, point out that having long videos or collections of videos to go through from the classroom is not efficient for personal reflection or use for instructional improvement. When teachers develop the skill of using anecdotal notes, this practice can provide a wealth of information about student learning as well as the advantage of being easier to utilize on a day-to-day basis.*

Part 2: Debriefing anecdotal notes workshop (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will distill key insights from their use of anecdotal notes as the basis for learning from practice and learning with colleagues.	1. Debrief the anecdotal notes workshop with the whole group.	• Handout: Learning from practice protocol

Detailed description of activity	Comments & other resources
1. Introduce Part 2: In this part, participants will debrief their experiences with the anecdotal notes workshops during the module. During this debriefing discussion, focus on • Insights gained into the Learning Trajectory • The process of talking with colleagues using video and/or notes to support the discussion Ask, "What are you taking away from our work with CCAs, anecdotal notes, and sharing teaching with small groups that you can use in your teaching moving forward?" 	*Facilitate the discussion to bring out ideas about:* • *using the Learning Trajectories to talk about what students know and are able to do instead of what students are not able to do (strength-based vs. deficit-based)* • *how the Learning Trajectories help target understanding and guide instruction, especially in volume where the CCSS move so quickly to using the volume formula; how being more clear of the underlying understandings can help teachers better build a conceptual understanding of volume* • *the value of trying out the ideas from the PD in their own classrooms* • *what participants learned from colleagues when sharing their experiences (including expanding what they learned about the trajectories and approaches to note taking)* • *how participants' notetaking changed and/or still needs to improve in order to be useful in their teaching*

Part 3: Making connections between units and dimensions of measurement (~20 minutes)

Goals

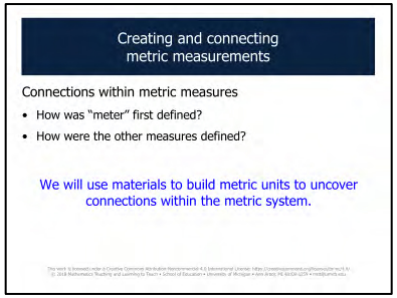
- Participants will explore connections between length, area, and volume measurements.
- Participants will explore connections between metric measurements.

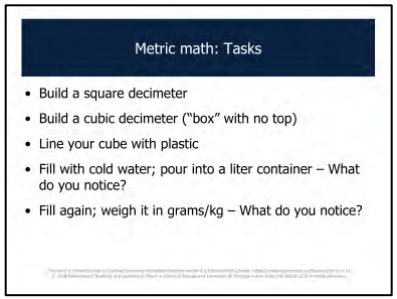
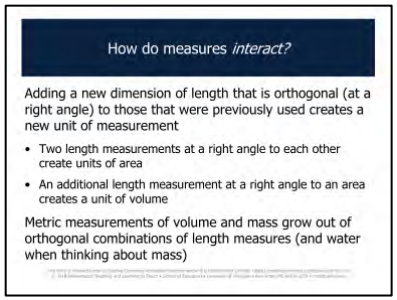
Instructional sequence

1. Introduce Part 3 and watch Video A.
2. Watch Video B and have participants work on the task.
3. Discuss participants' work on the task and watch Video C; discuss connections across the dimensions of length, area, and volume.
4. Continue the discussion, focusing on the connections between metric measurements; watch Video D.
5. Discuss how this task could be modified for use with students and watch Video E; discuss measurement error and precision.
6. Conclude the discussion by watching Video F.

Resources

- Video A (02:08): Connecting measurements: An early example
- Video B (02:30): Building metric units
- Video C (01:07): Connections among measurements
- Video D (02:22): Commentary on connections among measurements
- Video E (03:33): Teaching about connections among measurements
- Video F (01:46): Concepts central to measuring

Detailed description of activity	Comments & other resources
1. Introduce Part 3: In this part, participants will engage in a task that is designed to surface connections within metric measures and between different dimensions (i.e., between length, area, and volume). Have participants watch *Video A*, where Dr. Clements and Dr. Sarama discuss with teachers the origin of the "meter" as a unit of measurement. 	*Meters were first defined by the French in the 1700s. The French wanted to develop a standard unit and defined it as one ten-millionth of the distance between the North Pole and the equator in order to create a standardized unit. Later, scientists redefined it more precisely.*

Detailed description of activity	Comments & other resources
2. Watch *Video B*, where Dr. Clements sets up the task participants will work on. He explains that participants should do the following: • build a square decimeter • build a cubic decimeter (a "box" with no top) • line the cube with plastic (using a gallon-sized zip-top bag) • fill the cube with water and then pour it into a liter container • fill the cube again and weigh it in grams/kilograms  During this activity, participants should pay attention to the extent to which the water fills the liter container as well as to the mass of the cubic decimeter when filled with water.	*Materials needed for this activity:* • *Metric rulers* • *Scissors* • *Tape* • *Gallon-sized zip-top bags* • *Heavy tagboard or cardboard* • *Water (enough for each participant to fill a cube that has a volume of one cubic decimeter)* • *A one-liter container (such as a soda bottle or graduated cylinder)* • *Funnels* • *Metric scale*
3. After participants have completed the task, have them discuss as a whole group what they noticed. During the discussion, watch *Video C,* where Dr. Sarama and Dr. Clements explain how measurements of length relate to measurements of area and volume. Dr. Clements points out that • Two length measurements at a right angle to each other create units of area • An additional length measurement at a right angle to an area creates a unit of volume  Use this video to promote a discussion of the connections across dimensions. Ask participants about similarities and connections between the processes of measurement across the dimensions of length, area, and volume.	*Video C: Connections among measurements* *In this video, Dr. Clements talks explains that if two length measures are orthogonal, one of the dimensions can be moved without changing the other. When one length is swept against another length in an orthogonal dimension, the dimension of area is created. The dimension of volume is created when a base is swept up through an orthogonal direction to the plane. This is the nature of the space we live in.*

Detailed description of activity	Comments & other resources
4. Continue the discussion, turning the focus to connections between metric measures. Have participants consider how the task they completed helped them to make (or recall) the connections among metric units. Watch *Video D*, where Dr. Sarama and Dr. Clements talk about the connections between cubic centimeters/decimeters, milliliters/liters, and grams/kilograms and point out that this task was designed as a "hands on" way to highlight these connections.	*Video D: Commentary on connections among measurements* *In this video, Dr. Sarama and Dr. Clements talk about the relationships between metric measures. One cubic decimeter has a volume of 1 liter and a mass of 1 kilogram. One gram was originally defined as 1 cubic centimeter of cold water.*
5. Continue the discussion by focusing on measurement error, precision, and task modification. Ask participants to consider ways of modifying the task if they were going to use it with their students. Then, watch *Video E,* where teachers discuss how they might modify this task if they were using it with their students. In this video, participants talk about issues with precision and measurement error that influenced their results on this task.	*Video E: Teaching about connections among measurements* *In this video, Dr. Sarama asks how teachers might modify this task to use with students. This leads to a discussion of sources of measurement error participants experienced in their work on this task. As examples, there was extra space at the top of the one-liter soda bottle and the cardboard used to make the decimeter cube was so flexible that it bowed when filled with water. At the end of the video, Dr. Clements and Dr. Sarama point out that, even if this task is not used with students, it can be helpful for teachers to get "hands on" experience with the connections among the different units of measure.*
6. Conclude the discussion by showing *Video F,* where Dr. Clements and Dr. Sarama summarize concepts central to measuring.	*Video F: Concepts central to measuring* *In this video, Dr. Clements summarizes the similarities in measurement procedures and concepts against three dimensions. All measurement involves identifying a unit of measure, subdividing an object by that unit, and iterating that unit to quantify that attribute. Dr. Sarama and Dr. Clements then talk about how these procedures apply to other kinds of attributes, such as time, weight, wavelength, and even measures of abstract constructs like "happiness."*

Part 4: Reviewing Learning Trajectories by "acting it out" (~20 minutes)

Goals

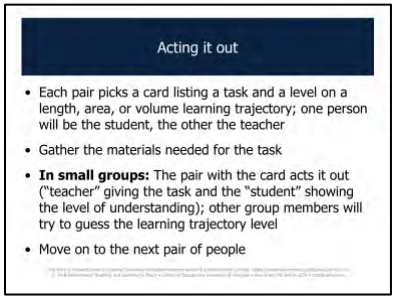
- Participants will solidify their understanding of the Learning Trajectory levels for length, area, and volume.
- Participants will solidify their understanding of how tasks make Learning Trajectory levels visible.

Instructional sequence

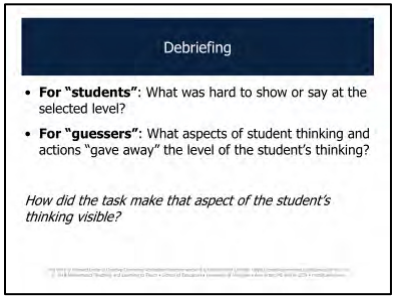
1. Introduce the part, watch Video A, and give participants time to do the activity in small groups (using the handouts provided).
2. Debrief the activity in whole group; watch Video B.

Resources

- Video A (02:15): Launching "Acting it out"
- Video B (02:39): Commentary on "Acting it out"
- Handout: Acting it out cards
- Handout: Acting it out resources
- Handout: Content cube – Length Learning Trajectory
- Handout: Content cube – Area Learning Trajectory
- Handout: Content cube – Volume Learning Trajectory

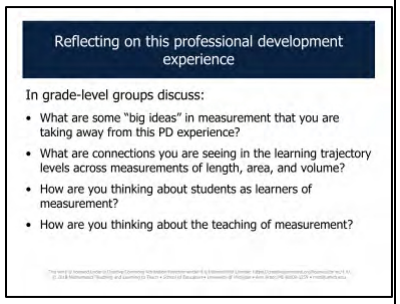
Detailed description of activity	Comments & other resources
1. Introduce Part 4: In this part, participants will review the Learning Trajectories for length, area, and volume by acting out scenarios involving students at various levels of each Learning Trajectory.  Watch *Video A*, where Dr. Sarama and Dr. Clements set up and model this activity. In this activity pairs of participants pick a card listing a task and a level on the length, area, or volume Learning Trajectory. One person plays the role of the student and the other plays the role of the teacher. The pair with the card acts out the scenario (i.e., the "teacher" gives the task and the "student" shows the level of understanding) and other group members try to guess the Learning Trajectory level being displayed. Then, another pair takes a turn selecting and acting out a different scenario. Give participants time to do this activity in small groups.	*The purpose of this activity is to help participants solidify their understanding of the Learning Trajectory levels for length, area, and volume.* *This activity involves the use of a variety of materials for length, area, and volume tasks, including* • *2x3x2 rectangular prism constructed from cubes* • *24 cubes the same as used to construct the 2x3x2 prism* • *30 Square inch tiles* • *Handouts with 4" x 6" rectangles* • *Objects that are close to the same length (e.g., a marker and a pen)* • *An object that is close to an exact number of inches long* • *Handouts picturing a rectangular prism that is partially filled with cubes (see the picture on Handout: Acting it out cards)* • *Handouts picturing an 8 cm x 9 cm rectangle and a 3 cm x 2 cm rectangle with side lengths labeled (see the picture on Handout: Acting it out cards)* *When participants act as students, they will work on the tasks with probing from the teacher (the partner of the "student"). If further questions are needed, the "guessers" can ask follow up questions which the "student" should answer as appropriate.*

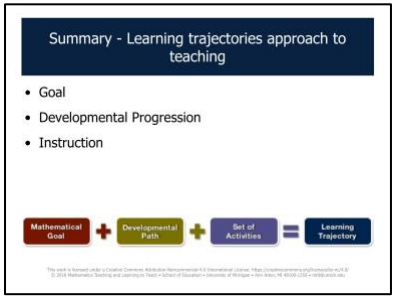
Detailed description of activity	Comments & other resources
2. Gather participants together in a whole group and debrief their experiences engaging in this activity using the following questions: • For "students": What was hard to show or say at the selected level? • For "guessers": What aspects of the student thinking and actions "give away" the level of the student's thinking? Ask all participants to consider: "How did the task make a particular aspect of the student's thinking visible?" During this discussion, show *Video B,* where Dr. Sarama describes the benefits of having a language to describe students' thinking (i.e., the levels of the Learning Trajectory).	*This slide can be found on the right-hand side of the viewer when Video B is displayed in the left-hand side. Display the slide and begin facilitating the debriefing discussion before showing Video B. Show the video as it becomes relevant to the discussion (or after participants have had time to discuss their initial thoughts).* *Video B: Commentary on "Acting it out"* *In this video, Dr. Sarama points out that, when teachers lack specific language to use to describe their students' thinking, it is easy to describe students' thinking using deficit language (e.g., "They don't get this" or "They can't do this"). Being able to describe students' behaviors using the levels of the Learning Trajectory, teachers are able to characterize what students <u>are</u> doing and understanding at a particular level – providing a more strength-based focus. Dr. Sarama acknowledges that it takes time to learn the names of the levels; however, she explains that these names can eventually become helpful tools for interpreting students' thinking.*



Part 5: Wrap up (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be reminded of the work from all sessions in the module. • Participants will be encouraged to make changes in their teaching of measurement.	1. Have participants reflect on the professional development module in grade-level groups. 2. Watch Videos A and B. 3. Summarize and discuss implications of the professional development experience for teaching.	• Video A (00:43): Wrapping up the module • Video B (01:44): Instruction aimed at supporting students' development

Detailed description of activity	Comments & other resources
1. Wrap up the module by having participants discuss the following questions in grade-level groups: • What are some "big ideas" in measurement that you are taking away from this professional development experience? • What connections are you seeing in the Learning Trajectory levels across measurement of length, area, and volume? • How are you thinking about students as learners of measurement? • How are you thinking about the teaching of measurement? After participants have had time to discuss these questions in grade-level groups, come back together and have participants share some insights with the whole group.	

Detailed description of activity	Comments & other resources
2. Have participants watch *Video A,* where Dr. Sarama and Dr. Clements summarize the Learning Trajectories approach to teaching and learning. Then, watch *Video B,* where Dr. Clements and Dr. Sarama talk about how instruction can support students' development through the Learning Trajectories. This might involve scaffolding students' work on measurement tasks depending on their developmental levels rather than "scaffolding" by encouraging to solve a task without actually understanding it (e.g., encouraging students to use the "length x width" formula when they do not yet have an understanding of area as the number of square units covering a surface). 	*In Video A, Dr. Sarama points out that understanding the progression of work on measurement in the Common Core can help teachers know what concepts are important to work on—even in grades in which they are not explicitly addressed. Dr. Clements points out that teachers can often "fold in" work on measurement into work on fractions and other mathematical topics.* *In Video B, Dr. Clements points out that teachers do not need to give students different tasks based on their developmental levels; rather, they can facilitate students' work on the same task by providing different kinds of support depending on their developmental levels.*
3. Talk with participants about implications of the professional development module. Encourage them to caitalize on what they have learned by • Using "big ideas" to make connections across the measurement of length, area, and volume • Learning about students' development of mathematical ideas in geometric measurement and in other topic areas • Using anecdotal notes to document and make sense of what students say and do as they learn mathematics • Designing instruction and choosing tasks that meet students where they are and help them take next steps • Engaging with colleagues on using notetaking to better understand what students are thinking and to further improve skill in taking notes