DTE@ MATHEMATICS

Geometric Measurement and Spatial Reasoning in Elementary Mathematics Teaching

Session 10 Resource: Content Cube – Volume Learning Trajectories

Volume Quantity Recognizer (VQR)

Filling and Packing. Recognizes capacity as an attribute.

Building. Builds with blocks, associating more blocks with terms like "big" and fewer blocks with "small"

Comparing. Describes with words such as big, small, and tiny. Compares "volume" using only one dimension.

Volume Filler (VF)

Filling. Fills a container using smaller container, counts the number needed to completely fill the larger container.

Packing. Puts cubes into rectangular box to fill - may leave gaps; eventually, packs entire box leaving no gaps.

Building. Recognizes and counts cubes (the child may be counting "blocks" or even "squares") on multiple faces.

Comparing. Compares objects by physically or mentally aligning; refers to a least two dimensions of objects.

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Volume Quantifier (VQ)

Filling. Estimates number of scoops needed to fill. Attends to both portion filled and portion remaining unfilled. Recognizes when container is half full.

Packing. Packs box neatly and completely with cubes; counts one cube at a time, while packing.

Building. Builds without gaps. Counts on all faces of an object constructed of cubes, has a developing sense of the cube as a unit, begins to recognize that squares on adjacent faces of a rectangular prism sharing a side are faces of the same cube.

Comparing. Compares the volume of objects by counting the number of cubes or by physically or mentally aligning and explicitly recognizing three dimensions.

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Volume Unit Relater and Repeater (VURR)

Relates size & number of units explicitly; understands that fewer larger than smaller units will be needed to fill/pack a given container. Can accurately convert units in 1:2 ratio.

Filling. Uses simple units to fill containers with accurate counting, completely filling scoop each time. After one unit has been poured into the container can anticipate the volume of the container by iterating.

Packing. Packs completely using discrete units with accurate counting. Iterates unit throughout volume, maintaining equal unit size & spacing.

Building. Exhibits developing understanding of cubes as filling space. Counts cubes, not faces.

Comparing. When comparing two 3-D objects, describes correctly the relative volumes of objects by reasoning about unit size.

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Initial Composite 3-D Structurer (VICS)

Understands cubes as filling a space. Explicitly relates size and number of units to volume. Uses additive reasoning.

Filling. Relates number of cubes to cubic units as measured by capacity.

Packing. Begins to visualize and operate on composite units (rows or columns). Iterates accounting for "internal/hidden" cubes. Decomposes space with accurate use of units/subunits.

Building. Develops more accurate counting strategies. Counts systematically, accounting for internal/hidden cubes, and moves to operating on composites.

Comparing. Connects volume as number and volume as space. Develops sense of conservation for the cases in which transformation is involved.

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3-D Row and Column Structurer (VRCS)

Able to flexibly coordinate *filling*, *packing*, *building* aspects of volume. Uses additive comparisons (e.g., "this one has 12 more") but may show some multiplicative comparisons (e.g., "this one is four times as big").

Initially counts or computes the *number of cubes in one layer* then uses addition or skip counting by layers to determine total volume; eventually moves to multiplication.

Operates fluidly and flexibly on units (cubes), units of units (rows or columns), and units of units of units (layers). With perceptual support, can decompose 3-D arrays into other, complex 3-D arrays (not only layers, rows, or columns) and calculate the number of these smaller arrays in the larger array.

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3-D Array Structurer (3D AS)

Has an abstract understanding of the rectangular prism volume formula. Coordinates multiplicative and additive comparisons flexibly.

With linear measures or other similar indications of the three dimensions, multiplicatively iterates cubes in a row, column, and/or layer to determine volume. In multiple contexts, can compute the volume of rectangular prisms from their dimensions.

Visualizes and operates on both horizontal and vertical layers, even without perceptual support.

Decomposes 3-D arrays into other complex 3-D arrays (not only layers, rows, or columns) and calculates the number of smaller arrays in the larger array (by using repeated addition, multiplication, or division). Coordinates the spatial and symbolic decompositions.

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