

Session 2: Length Learning Trajectory – Developmental Progression

**Building
Blocks**



DTE@
MATHEMATICS



Overview of Session 2

- Unpacking learning trajectories for length by watching students measure
- Taking notes to support learning and note taking in teaching
- Applying learning trajectories to your work with students on the “Broken Ruler”

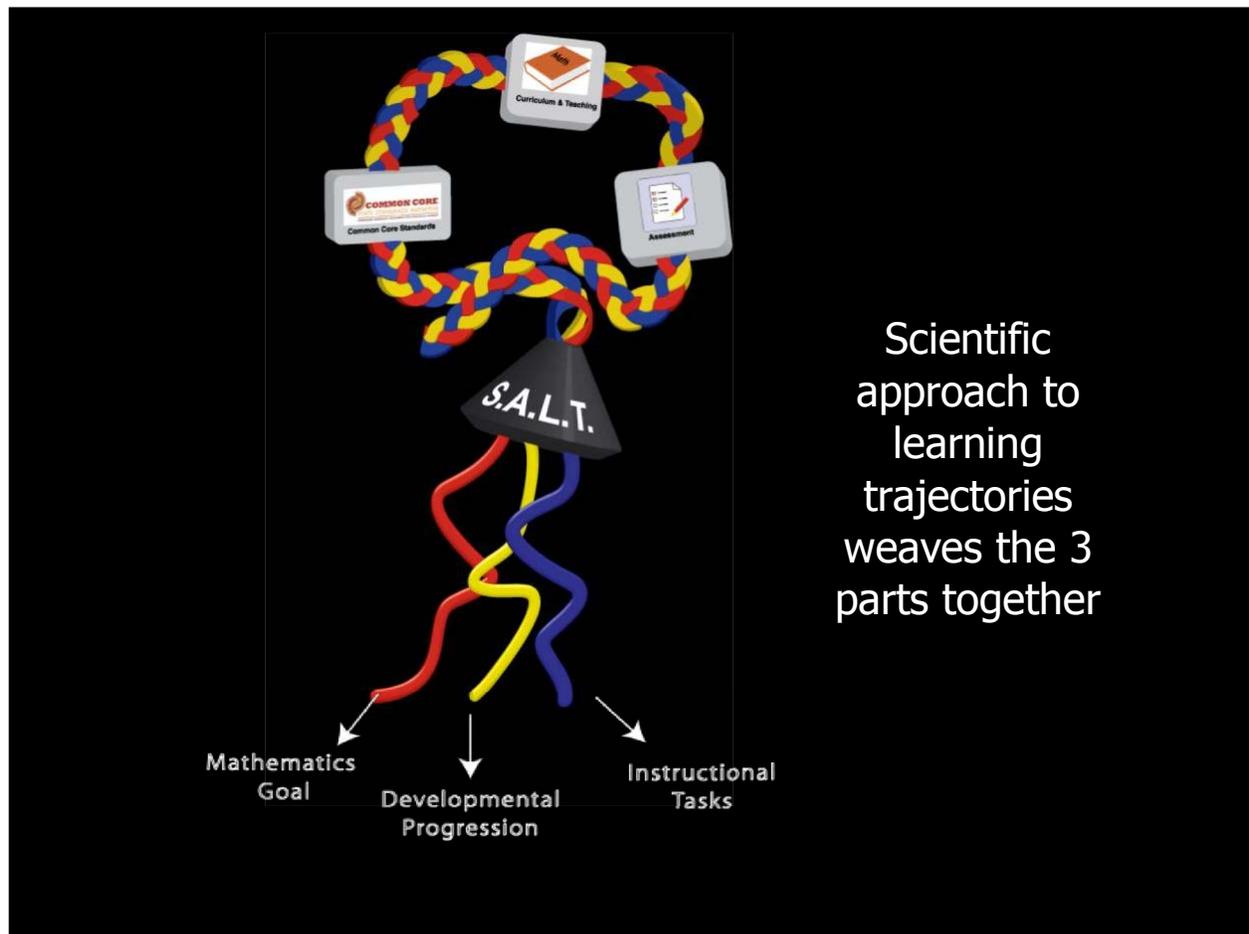
Learning trajectories approach

- Goal
- Developmental Progression
- Instruction



The diagram features three icons at the top: 'Common Core Standards' (with a logo), 'Assessment' (with a checklist icon), and 'Curriculum & Teaching' (with a book icon). Below these, three wavy lines in red, yellow, and blue represent paths. The red line starts at the 'Common Core Standards' icon and ends at the 'Mathematics Goal' label. The yellow line starts at the 'Assessment' icon and ends at the 'Developmental Progression' label. The blue line starts at the 'Curriculum & Teaching' icon and ends at the 'Instructional Tasks' label. The lines are disconnected from each other, illustrating a lack of integration.

Present day,
research on
mathematics goals
contributes to
standards (red line)
and so forth,
but...disconnected.



Learning trajectories in our sessions

- Goal (Session 1)
- Developmental Progression
- Instruction



Goals and the big ideas of mathematics

Number, Operations, and Algebraic Thinking

Numbers can be used to tell how many, describe order, and measure; they involve numerous relationships, and can be represented in various ways.

Operations can be used to model a variety of situations and to solve problems; they can be carried out in various ways.

Mathematics involves going beyond surface appearances to detect underlying structure or commonalities (patterns and relations).

Geometry

Geometry can be used to understand and represent the objects, locations, and directions in the world, as well as the relationships between them.

Geometric shapes can be described, analyzed, transformed, and composed and decomposed into other shapes.

Measurement

Comparing and measuring can specify how much of an attribute (e.g., length) objects possess.

Measures can be determined by repeating a unit or using a tool.

Patterns and Structure

Patterns and structures can be used to recognize relationships and can be extended to make generalizations.

Data

Data analysis can be used to represent, classify, and use information to ask and answer questions.



Learning trajectories in our sessions

- Goal (Session 1)
- Developmental Progression (Session 2)
- Instruction

Today!



Length developmental progression

- Let's study students at different levels
- View and analyze each video
- What characterizes the students' thinking?

Learning trajectory levels (Length) – Length Quantity Recognizer

- **Length Quantity Recognizer**
- Length Comparer
- End-to-End Length Measurer

Length Quantity Recognizer (LQR)

- Identifies length as attribute
- Another example:
 - “I’m tall, see?”

Learning trajectory levels (Length) – Length Comparer

- Length Quantity Recognizer
- **Length Comparer**
- End-to-End Length Measurer

Length Direct Comparer (LDC)

- Physically aligns two objects to determine which is longer or if they are the same length
- Another example:
 - Stands two sticks up next to each other on a table and says, “This one’s bigger”

Learning trajectory levels (Length) – Length Comparer

- Length Quantity Recognizer
- **Length Comparer**
- End-to-End Length Measurer

Indirect Length Comparer (ILC)

- Compares the lengths of two objects by representing them with a third object
- Another example:
 - Compares length of two objects with a piece of string

Learning trajectory levels (Length) – End-to-End Length Measurer

- Length Quantity Recognizer
- Length Comparer
- **End-to-End Length Measurer**

End-to-End Length Measurer (EE)

- Lays units end-to-end. May not see the need for equal-length units
- Another example:
 - Lays 9 inch cubes in a line beside a book to measure how long it is

End-to-End Length Measurer (EE)

- Lays units end-to-end. May not see the need for equal-length units
- Another example:
 - Lays 9 inch cubes in a line beside a book to measure how long it is

Learning trajectory levels (Length)

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Consistent Length Measurer
- Conceptual Ruler Measurer
- Integrated Conceptual Path Measurer
- Abstract Length Measurer

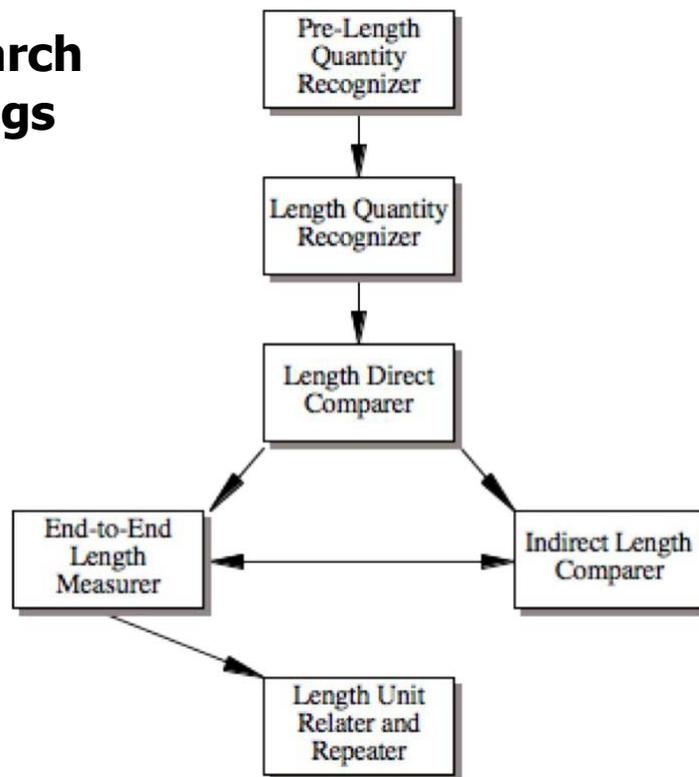
Learning trajectory levels (Length) – Length Unit Relater and Repeater

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- **Length Unit Relater and Repeater**
- Consistent Length Measurer
- Conceptual Ruler Measurer
- Integrated Conceptual Path Measurer
- Abstract Length Measurer

Length Unit Relater and Repeater (LURR)

- Relates size and number of units
 - “If you measure with centimeters instead of inches, you’ll need more of them, because each one is smaller”
- Repeats or iterates a single unit to measure. Sees need for identical units. Uses rulers with guidance
 - Measures a book’s length well with a ruler
- Uses rulers with guidance

**Research
findings**



Learning trajectory levels (Length) – Consistent Length Measurer and Conceptual Ruler Measurer

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- **Consistent Length Measurer**
- **Conceptual Ruler Measurer**
- Integrated Conceptual Path Measurer
- Abstract Length Measurer

Consistent Length Measurer (CLM)

- Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers
 - Begins to estimate
 - Considers the length of a bent path as the sum of its parts (not the distance between the endpoints)
 - “I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it's 3 meters, 14 centimeters in all”

Conceptual Ruler Measurer (CRM)

- Possesses an “internal” measurement tool. Mentally moves along an object, segmenting it, and counting the segments.
 - Estimates with accuracy
 - “I imagine one meter stick after another along the edge of the room. That’s how I estimated the room’s length is 9 meters”
 - Operates arithmetically on measures (“connected lengths”)

Learning trajectory levels (Length) – Integrated Conceptual Path Measurer

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Consistent Length Measurer
- Conceptual Ruler Measurer
- **Integrated Conceptual Path Measurer**
- Abstract Length Measurer

Integrated Conceptual Path Measurer (ICPM)

- Computes perimeter of a polygon
- Changes one part of a figure and adjusts other sides to compensate for length changes to maintain overall lengths
- In selection of units, shows well-developed ideas of precision and accuracy

Abstract Length Measurer (ALM)

- Constructs derived units with linear measures, such as miles per hour, and make appropriate unit conversions
- Measures to the degree of precision allowed by a tool by estimating to a fraction of the smallest calibration mark provided on the instrument

Reflecting on the length developmental progression

- Read and discuss handout
- How does each level make sense in view of your assessments and our work here?
- Questions or comments?

Length Unit Relater and Repeater (LURR)

- Relates size and number of units
 - “If you measure with centimeters instead of inches, you’ll need more of them, because each one is smaller”
- Repeats or iterates a single unit to measure. Sees need for identical units. Uses rulers with guidance
 - Measures a book’s length well with a ruler
- Uses rulers with guidance

Test ourselves

- Let's see if we agree on these levels!
- Take notes as you watch:
 What do they do and why?
- Think-pair-share about what level can be seen in each video.
- Why?...

Why take notes?

- For us, good aids to memory and discussion
- In school, several particular purposes
 - Compiling main solution strategies
 - Deciding who to call on for class discussions
 - Considering what to focus on next in formative assessment
 - Accumulating information over time to support summative assessment (report cards or rich information to share at conferences)

Test ourselves: Focus questions

Two focus questions:

- How are students reasoning about measuring?
- How are students making sense of the length?



Notes and interpretations for example 1

- Did you agree on the level of the learning trajectory?
- What in your notes helped you in the interpretation and discussion?
- What might you do differently?

End-to-End Length Measurer



Notes and interpretations for example 2

- Share the notes you took
- Any particularly helpful insights to share with the whole group?
- Trying pictures or diagrams may help

Length Direct Comparer

Revisiting the Broken Ruler

- Discuss your students responses to the broken ruler task
 - What was consistent or not with the learning trajectory (LT)?
 - How did or could the LT provide a framework for understanding their responses and strategies?
 - How could the LT help plan “next steps” (formative assessment)?

Using a form for taking anecdotal notes

Take notes on the performances of your students in ways that:

- Connect with the learning trajectory levels
- Support task selection and anticipation of student's thinking
- Structure space to record key information

<p>Pre-Length Quantity Recognizer (PLQR) Does not identify length as attribute. "This is long. Everything straight is long. If it's not straight, it isn't the long." Length Quantity Recognizer (LQR) • Identifies length/distance as attribute. • May understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative (e.g., one person is taller than another). "Both paths are the same length." </p>	<p>Length Comparer A. Length Direct Comparer (LDG) Physically aligns two objects to determine which is longer or if they are the same length. May use a ruler (as a stick rather than a measuring tool) to directly compare it and another object.  B. Indirect Length Comparer (ILC) Compares the length of two objects by representing them with a third object. Uses a piece of string to compare the lengths of two objects. </p>	<p>End-to-End Length Measurer (EE) Lays units end to end. May not recognize the need for equal-length units. The ability to apply resulting measures to comparison situations develops later in this level.  Length Unit Relater and Repeater (LURR) • Iterates a single unit to measure. Recognizes that different units will result in different measures and that identical units should be used, at least intuitively and/or in some situations. • Uses rulers with minimal guidance.</p>
<p>Consistent Length Measurer (CLM) • Considers the length of a bent path as the sum of its parts (not the distance between the endpoints). • Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers, and accumulation of distance. • Begins to estimate. </p>	<p>Conceptual Ruler Measurer (CRM) • Has an "internal" measurement tool. • Mentally moves along an object, segmenting it and counting the segments. • Operates arithmetically on measures. Projects or translates given lengths to determine missing lengths. • Estimates the length of an object that is not partitioned with accuracy and without any available image of the standard unit. • Employs explicit strategies to estimate lengths, including developing benchmarks for units and composite units and mentally iterating those units.</p>	<p>Integrated Conceptual Path Measurer (ICPM) • Computes length of complex bent path and perimeter of a polygon. • Can change one part of a figure and adjust other sides to compensate for length changes. • In selection of units, children show well-developed ideas of precision and accuracy. Abstract Length Measurer (ALM) • Organizes and synthesizes sets of objects based on perimeter or collections of complex bent paths. • Constructs derived units with linear measures and make appropriate unit conversions, including units and divisions of units. Can explain that this subdivision process is potentially unlimited. • Measures to the degree of precision allowed by a tool by estimating to a fraction of the smallest calibration mark provided on the instrument.</p>

Class Roster	LT code	Evidence (Notes/Images)

Session 2

Classroom Connection Activities (CCAs)

- Assessments of students
- Bring in a curriculum lesson or activity on length

Summary

In this session you:

- Analyzed examples of student engagement in measurement using the learning trajectory on length
- Considered the purposes and nature of note taking in teaching
- Connected the performance of your own students to the learning trajectories

Learning trajectories: Looking ahead

- Goal (Session 1)
- Developmental Progression (Session 2)
- Instruction (Session 3)

