Description of the session

Session 2 extends the work from Session 1 that focused on studying the mathematics of length measurement and introduced some examples of student thinking about length measurement. This session focuses on the second component of the Learning Trajectory for length measurement: the developmental progression of learning to measure length. First, participants will be introduced to the levels of the developmental progression. Then, they will practice observing students' performance on measurement tasks and identifying the level at which they are performing. This activity also launches work on a teaching practice – taking anecdotal notes – that will be the foundation later for learning from one's own teaching. Finally, participants will discuss their own students' responses to the Broken Ruler Task and will attempt to determine how these responses relate to the levels of the developmental progression.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
Conversation about a CCA from the last session	15 minutes		• Participants will share their students' responses to the Broken Ruler Task.
I. Overview	65 minutes	Part 1	• Participants will be oriented to the work of the session. • Participants will be introduced to the Learning Trajectory construct.
II. Studying student thinking: Introduction to the developmental progression	20 minutes	Parts 2, 3, & 4	• Participants will recognize and understand the early levels of the Learning Trajectory for length measurement. • Participants will recognize and understand the later levels of the Learning Trajectory for length measurement. • Participants will demonstrate understanding of the Learning Trajectory for length measurement. • Participants will be introduced to note taking and begin to use note taking in observation.
III. Connecting the Learning Trajectories with the Broken Rule Task	10 minutes	Part 5	• Participants will discuss their own students' responses to the Broken Ruler Task. • Participants will use the Learning Trajectory for length measurement to describe their students' understanding. • Participants will be introduced to particular note taking form.
IV. Wrap up	5 minutes	Part 6	• Participants will understand ways of connecting the session content to their classroom. • Participants will recognize instruction as the third component of a Learning Trajectory.

Classroom Connection Activities

Required
Type of task: Assessment task/Collecting records of practice Description: Complete tasks (the Broken Ruler Task and new tasks) with 3-4 students of different (hypothesized) achievement levels. Ask the students to write down or draw how they measured. Use the anecdotal notes form to record how students engage in the task. Respond to the reflection questions listed on the CCA handout.
Type of task: Analysis of curriculum materials Description: Select a lesson or a short sequence of instructional activities from your curriculum that focus on measuring length. Bring these materials with you to the next session.

Preparing for the session

☐ Make copies as needed:

- *Resources: Handout: Content cube – Length Learning Trajectory (Parts 2, 3, 4, 5); Handout: Research findings flowchart (Part 3)*

☐ Customize and make copies of the Classroom Connection Activities

☐ Test technical setups: Internet connection, speakers, projector, document camera

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards.

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• recognizing the mathematical goal as the first component of a complete Learning Trajectory • understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement) • understanding how measurement of length, area, and volume are represented and developed in the CCSS • understanding how measurement connects with the CCSS standards for mathematical practice • understanding concepts and skills involved in measuring length, area, and volume • understanding connections between length, area, and volume measurement and between metric measurements for each	• **recognizing student development as the second component of a complete Learning Trajectory** • **understanding children's development of measurement through Learning Trajectories for length, area, and volume** • recognizing principles of measurement in student work • **interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement** • interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement • interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement	• recognizing instruction as the third component of a complete Learning Trajectory • **using anecdotal notes to document what students say and do when working on measurement tasks** • connecting measurement activities in curricula to measurement Learning Trajectory levels • modifying measurement tasks to target different and/or particular Learning Trajectory levels	• understanding the anecdotal notes workshop process • using the anecdotal notes workshop to improve the practice of note taking • using the anecdotal notes workshop to improve teaching

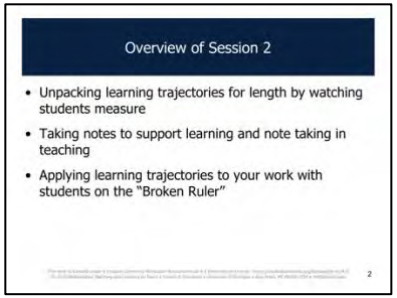
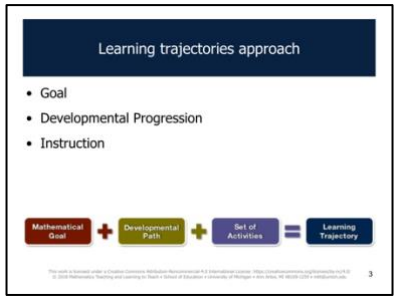
Conversation about a Classroom Connection Activity from last session (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will share their students' responses on the Broken Ruler Task	1. Participants share their students' responses on the Broken Ruler Task 2. *Optional:* Participants share about the logistics of video recording	

Detailed description of activity	Comments & other resources
1. Ask a few participants to share their students' responses to the Broken Ruler Task.	*Discussing CCA work from previous sessions establishes its importance in the module. Participants will revisit their students' responses to the Broken Ruler Task in Part 5.*
2. *Optional:* Ask participants about the logistics of video recording. Explain that, though video recording can be challenging (e.g., helping kids understand why the class is being video recorded, setting up the camera, and saving and sharing the video), it is worth the effort because it will allow participants to study student thinking in action with their own students. You might ask questions such as: • How did video recording the student interviews work out? • Where did you position your camera? • What were you able see in your video? • How well were you able to hear what was being said?	*This conversation will help support participants as they continue to experiment with video recording in their classrooms throughout the module. They will be recording after sessions 1, 4, and 7 to support their discussions of student thinking.*

Part 1: Overview (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the session. • Participants will be introduced to the Learning Trajectory construct.	1. Introduce the session by watching Video A. 2. Watch *Video B.*	• Video A (01:22): Session overview • Video B (02:46): Learning trajectories in our sessions

Detailed description of activity	Comments & other resources
1. Introduce the session by watching Video A: This session continues work on length measurement by focusing on developmental progression piece of the Learning Trajectory for length measurement. It also launches work on the learning from practice piece of the module, by providing participants with an opportunity to try out and discuss the practice of notetaking. In this session, participants will work on • Unpacking Learning Trajectories for length by watching students measure • Taking notes to support learning and note taking in teaching • Applying Learning Trajectories to your work with students on the "Broken Ruler" 	*The slides for this introduction are available in the slide player. By moving your curser over the slide, you will see that there are two slides that can be viewed. You can advance to the second slide by clicking on #2 or by clicking on the right-hand arrow.*
2. Have participants watch Video B, where Dr. Clements introduces the three parts of a Learning Trajectory: • the goal (i.e., the key concepts involved with measuring length) • the developmental progression (i.e., levels of thinking through which the typical child passes when learning these ideas), and • instruction (i.e., the ways teachers can support students' learning of these ideas). The scientific approach to Learning Trajectories weaves together the ideas of standards, curriculum, teaching, and assessment.  In the last session, the focus was on the "goal", or the mathematical ideas involved with length measurement. In this session, the focus is on the developmental progression for learning to measure length.	*Emphasize that the "mathematical goal" was discussed in the previous session. Those are the "big ideas" of math. In this session we will focus on students' thinking, which leads us to the developmental progression that will be at the heart of the professional development sessions.* *In the video, Dr. Clements mentions "big ideas" of mathematics. The mathematical domain of measurement includes many "big ideas" including:* • *Comparing and measuring can specify how much of an attribute (e.g., length) objects possess* • *Measures can be determined by repeating a unit or using a tool*

Part 2: Learning Trajectories – Early levels of the developmental progression (25 minutes)

Goals

- Participants will recognize and understand the early levels of the Learning Trajectory for length measurement.

Instructional sequence

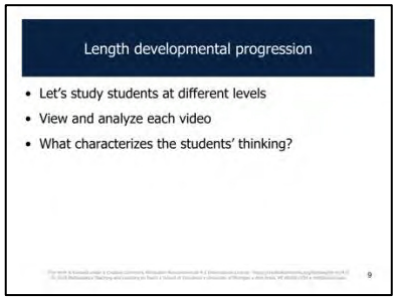
1. Introduce Part 2 and watch *Video A*.
2. Introduce the Length Quantity Recognizer level by watching and discussing *Videos B-D*. Introduce the Length Direct Comparer level by watching and discussing *Videos E and F*. Introduce the Indirect Length
3. Comparer by watching and discussing *Videos G and H*.
4. Introduce the End-to-End Length Measurer by watching and discussing *Videos I and J*.

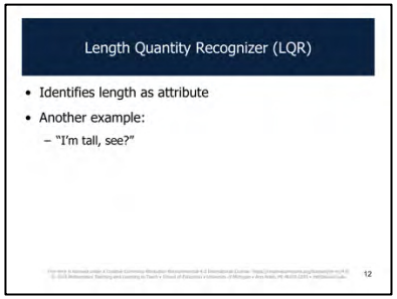
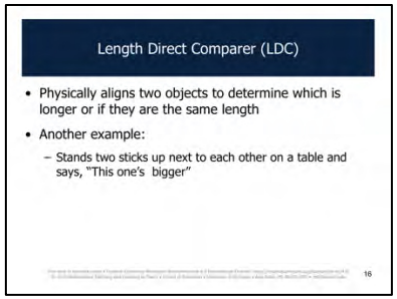
Resources

- Video A (00:51): Length developmental progression
- Video B (00:24): Interview: Pre-Length Quantity Recognizer – White and gray ants
- Video C (00:46): Length Quantity Recognizer (LQR)
- Video D (00:30): Interview: Length Quantity Recognizer – Strip in opening Video E (00:19): Interview: Length Direct Comparer – Which is shortest? Video F (01:54): Length Direct Comparer (LDC)
- Video G (00:27): Interview: Indirect Length Comparer – Which is longest? Video H (01:12): Indirect Length Comparer (ILC)
- Video I (00:59): End-to-End Length Measurer – Blue strips and yellow strips Video J (01:18): End-to-End Length Measurer (EE)
- Handout: Content cube – Length Learning Trajectory

Supplements

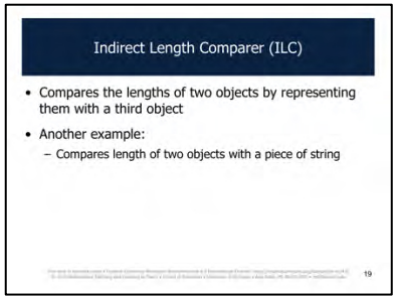
- Video (00:46): Interview: Indirect Length Comparer – Which is longer? (Alternative)
- Video (01:02): Interview: End-to-End Length Measurer – How many firetrucks long? (Alternative)

Detailed description of activity	Comments & other resources
1. Introduce Part 2: In this part, the early levels of the Learning Trajectory for length will be introduced and illustrated using video examples of students who are working on various measurement tasks. Distribute Handout: Content cube – Length Learning Trajectory, and give participants time to skim the descriptions of the different levels. Have participants watch Video A where Dr. Clements and Dr. Sarama introduce the idea of viewing and analyzing videos with a focus on what characterizes the student's thinking. In this video, Dr. Sarama directs participants to a handout that describe the levels of the developmental progression. She explains that these levels are meant to serve as a useful resource for teachers when observing students; however, the focus should be on what characterizes the student's thinking rather than on trying to learn all of the names of the levels. 	*When using the names of the Learning Trajectory levels, it is important to keep in mind that the **student** is not the level; instead, the student's **performance in this case** corresponds with a particular level.*

Detailed description of activity	Comments & other resources
2. Have participants watch Video B, which a child who identifies the longer path and says it is longer because of the color of the ant.  Following the video, discuss the question: • What characterizes the student's thinking? Then, watch Video C, where Dr. Sarama talks about the child in this video and introduces the Length Quantity Recognizer level of the developmental progression for length measurement. Allow time for any follow-up questions/discussion as time and need permit. After that, watch Video D and discuss: • Why is this child an example of a Length Quantity Recognizer? • What characterizes the student's thinking?	*Video B shows an example of a "Pre-Length Quantity Recognizer." The child in the video does not seem to understand yet that length is an attribute that can be measured.* *Although this child identifies the color of the ant as the reason he chose the bottom line, he does trace his finger along the line, which suggests that he may be beginning to think about length.* *Stress that even though the behavior of a student reflects a particular level, that doesn't mean that this is the limit of the student's thinking. It means he or she is <u>at least</u> at this level.* *Direct participants to the Content cube handout to support their recall and later use of the ideas that are discussed.* *In Video D, the child puts a foam strip into a cutout and says that it does not fit because "it's not longer." This statement suggests that she recognizes that length is an attribute that can be measured, which indicates that she has reached the "Length Quantity Recognizer" level of the developmental progression.*
3. Next, watch and discuss Video E, an example of a Length Direct Comparer.  Ask: • Why is this child an example of a Length Direct Comparer? What characterizes the student's thinking? Then, have participants watch Video F, where Dr. Sarama and Dr. Clements discuss the video of the child and talk about the description of the Length Direct Comparison level, focusing on aligning two objects and comparing endpoints. Allow time for any follow-up questions/discussion as time and need permit.	*The child in this video is asked to find the shortest object from a collection, and he physically aligns two objects (i.e., uses direct comparison), to determine which of two objects is shorter.* *Participants can sometimes go down the rabbit hole of talking about what students <u>cannot</u> do. It's important to steer the conversation toward thinking about what students <u>can</u> do.*

4. Continue watching videos that illustrate the early levels of the developmental progression for length measurement by viewing Video G, an example of an Indirect Length Comparer. Discuss:
   - Why is this child an example of an Indirect Length Comparer?
   - What characterizes the student's thinking?

   After discussing these questions, have participants watch Video H where Dr. Sarama talks about what an Indirect Length Comparer does, gives a couple of examples, and describes this sub-level of the "Length Comparer" level. Allow time for any follow-up questions/discussion as time and need permit.



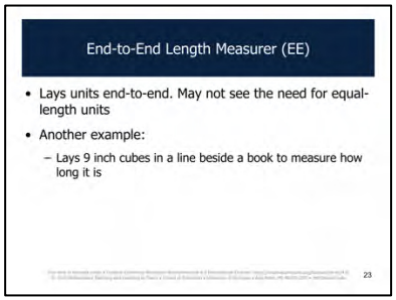
The child in this video uses the length of a pencil to indirectly compare the lengths of two line segments. This involves transitive reasoning.

It is not clear that Length Direct Comparer and Indirect Length Comparer develop independently in some children. Research shows some children (those with solid transitive reasoning) developing these concurrently, while other children (who lack transitive reasoning) are able to directly compare, but do not indirectly compare until much later. We incorporate both comparisons into a single level called "Length Comparer" but understand that the indirect comparison doesn't always happen in the sequence listed in this developmental progression.

As time and interest permit, have participants watch Video: Interview: Indirect Length Comparer – Which is longer? (Alternative), where the child uses the length of connecting cubes to indirectly compare the lengths of a piece of string and a pencil.

5. Next, watch Video I and discuss:
   - Why is this child an example of an End-to-End Length Measurer?
   - What characterizes this student's thinking?

   Finally, have participants watch Video J, where Dr. Sarama stresses that an End-to-End Length Measurer needs to "fill the space" and stresses the importance of providing children at this level with different-length "units" to see whether they recognize the need for equal-size units. Allow time for any follow-up questions/discussion as time and need permit.



In this video, the child is asked to determine which blue strip is the same length as four yellow strips. Notice how, after this child measures the left-most strip, she still needs to lay out yellow strips along the length of the middle strip before she is certain that it is four yellow strips long.

During the discussion, consider noting that some students move to end-to-end measurement before indirect comparison.

Part 3: Learning trajectories – Later levels of the developmental progression (~25 minutes)

Goals

- Participants will recognize and understand the later levels of the Learning Trajectory for length measurement.

Instructional sequence

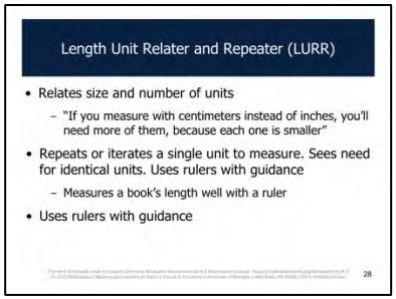
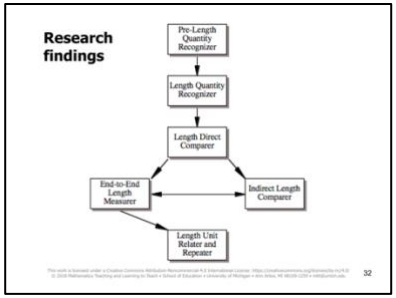
1. Introduce Part 3 and watch and discuss Videos A and B to introduce the Length Unit Relater and Repeater level.
2. Watch Video C to introduce research findings on the Learning Trajectory for length measurement.
3. Watch and discuss Videos D and E to introduce the Consistent Length Measurer/Conceptual Ruler Measurer levels.
4. Watch and discuss Videos F and G to introduce the Integrated Conceptual Path Measurer level.
5. Introduce the Abstract Length Measurer level by watching Video H and conclude this part by watching Videos.

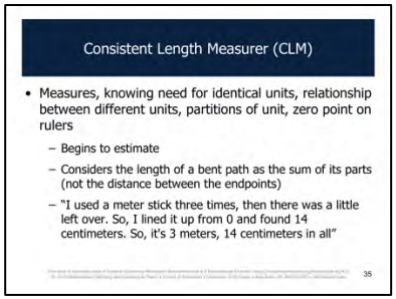
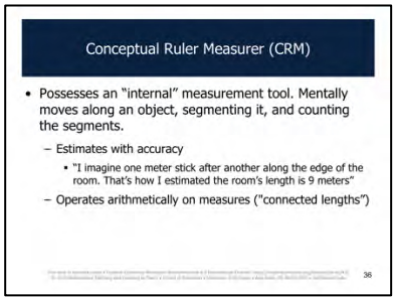
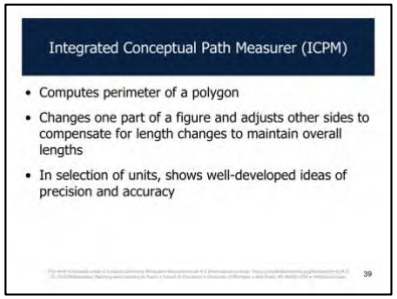
Resources

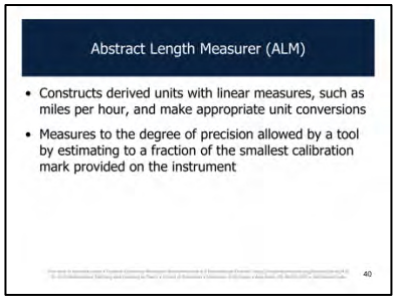
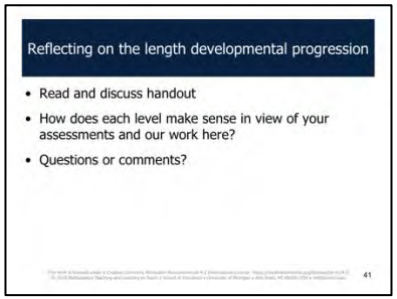
- Video A (01:10): Interview: Length Unit Relater and Repeater – Measuring gray strip with boxes
- Video B (02:40): Length Unit Relater and Repeater (LURR)
- Video C (00:35): Overview of research findings: flowchart
- Video D (01:56): Interview: Consistent Length Measurer/Conceptual Ruler Measurer
- – Measuring and drawing paths
- Video E (03:17): Consistent Length and Conceptual Ruler Measurers (CLM and CRM)
- Video F (01:17): Interview: Integrated Conceptual Path Measurer – Rectangles with perimeter of 24
- Video G (01:44): Integrated Conceptual Path Measurer (ICPM)
- Video H (01:18): Abstract Length Measurer (ALM)
- Video I (00:14): Reflecting on the length developmental progression
- Handout: Content cube – Length Learning Trajectory
- Handout: Research findings flowchart

Supplements

- Video (01:20): Interview: (NOT) Length Unit Relater and Repeater (Measuring with blue and yellow strips)
- Video (00:41): Interview: Length Unit Relater and Repeater (Leap frog)
- Video (01:19): Interview: Length Unit Relater and Repeater (Mental iteration and sliding)

Detailed description of activity	Comments & other resources
1. Introduce Part 3: In this part, participants will be introduced to the later levels of the Learning Trajectory for length measurement and will learn about some of the research base underlying these levels. Have participants watch Video A, which illustrates the Length Unit Relater and Repeater level of the Learning Trajectory for length measurement. Discuss**:** • Why is this child an example of a Length Unit Relater and Repeater? • What characterizes the student's thinking? Then have participants watch Video B, where Dr. Sarama describes behaviors that characterize the Length Unit Repeater Level. Following the video, allow time for follow-up questions or discussion as time and need permit. 	*In this video, a student is asked to figure out "how many boxes would be as long as the gray strip?" She is given a gray strip that has two (non-adjacent) white rectangles underneath it. She is able to mark off units that are the same size as those boxes (using her fingers to measure) and then iterate them until she has measured the entire length of the gray strip.* *If time allows it may be useful to show participants one or more of the supplemental videos that show variations on the LURR level:* • *a student NOT quite at the LURR level* • *a student using a "leap frog" approach to repeating the unit* • *a student using mental iteration and sliding*
2. Distribute Handout: Research findings flowchart and have participants watch Video C, where Dr. Clements talks about what his and Dr. Sarama's research showed regarding the development of the early levels and the variations that occur at the point of length comparison. 	*Dr. Clements and Dr. Sarama developed their initial Learning Trajectory for length measurement on previous and current research at the point their books were published in 2009. This previous research demonstrated that the developmental progression following a path from direct comparison to indirect length measurement to end-to-end length measurement. Their research since then shows that the evidence supporting indirect comparison (involving transitive reasoning) is inconsistent. As a result, they've combined length comparison into a single level called "Length Comparer" and, thus, the developmental progression is not necessarily linear at this point. More research is currently underway to help fully determine the placement of indirect length comparison.* *More information on levels LURR to ALM is available through the following weblinks:* *http://www.childrensmeasurement.org/length.html*

Detailed description of activity	Comments & other resources
3. Next, view Video D, which illustrates an example of a Consistent Length Measurer. Discuss**:** • Why is this child an example of a Consistent Length Measurer? • What characterizes the student's thinking? Then, watch Video E where Dr. Clements and Dr. Sarama lead a discussion about the Consistent Length Measurer video and then introduce the Conceptual Ruler Measurer level, which is distinct from the Consistent Length Measurer level in that objects can be measured using a "mental measuring tool." After this video, allow time for follow-up questions and discussion as time and need permit.  	*In this video, the student is asked to draw a path that is longer than bent path B (which is 5 inches), but not as long as bent path A (which is 6 inches). He uses a ruler to measure the lengths of each path before drawing. The path he draws is 4.5 inches.* *In Video E, Dr. Sarama points out that learning how to measure a bent path is an important step towards learning how to measure a perimeter.*
4. Have participants watch Video F, an example of an Integrated Conceptual Path Measurer. Discuss**:** • Why is this child an example of an Integrated Conceptual Path Measurer? • What characterizes the student's thinking? Then, view Video G, where Dr. Clements and Dr. Sarama talk about the ICPM level and introduce how this thinking leads to an understanding of fractions. Allow time for follow-up questions and discussion as time and need permit. 	*In this video, a child creates a new rectangle with the same perimeter as a previous rectangle, by adjusting the length of the sides (making some shorter and others longer).*

Detailed description of activity	Comments & other resources
5. Introduce the final level of the developmental progression of length measurement watching Video H, where Dr. Clements describes the Abstract Length Measurer level. Following the video, allow time for follow-up questions and discussion as time and need permit. Conclude this part by having participants look at all of the levels in the length developmental progression. Watch Video I and discuss: • How do the levels make sense in view of your assessments and our work here? Encourage participants to think about how these levels relate to the measurement skills of the students in their classrooms.	 

Part 4: Test ourselves – Connecting students' thinking with learning (~15 minutes)

Goals

- Participants will demonstrate understanding of the Learning Trajectory for length measurement.
- Participants will be introduced to notetaking and begin to use notetaking in observation.

Instructional sequence

1. Introduce part 4 by watching Video A.
2. Watching Video B; discuss the level of the student's thinking and the process of notetaking **(Answer: End-to-End Length Measurer)**. Watch Video C; discuss the level of the student's thinking and the process of notetaking **(Answer: Length Direct Comparer).**
3. Conclude by watching and discussing Video D.

Resources

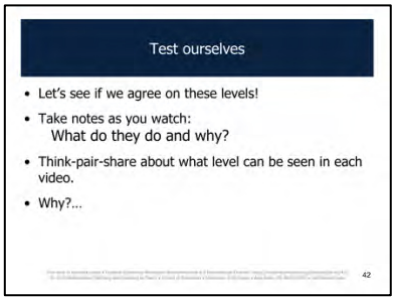
- Video A (03:09): Introduction to "Test Ourselves"
- Video B (02:28): Test yourself 1: Measuring a strip with tiles
- Video C (00:28): Test yourself 2: Which is longest?
- Video D (01:52): Gaining Familiarity with the Learning Trajectories
- Handout: Content Cube – Length Learning Trajectory

Supplements

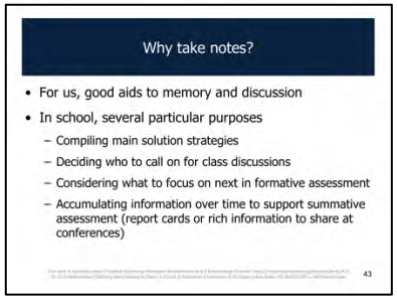
- Video (03:23): Test yourself example discussion
- Video (01:38): Test yourself example 2 discussion

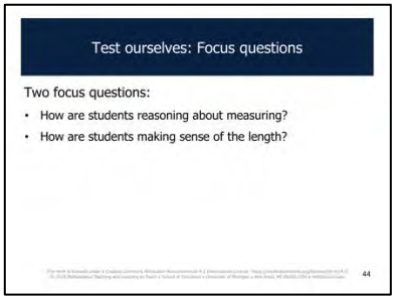
Detailed description of activity	Comments & other resources

1. Introduce Part 4 by watching Video A. In this video, Dr. Sarama explains that participants will have the opportunity to test themselves to see if they can characterize students' responses to measurement tasks using the levels of the developmental progression for length that were introduced in Parts 2 and 3. She also encourages them to take notes and explains why notetaking is a useful teaching practice. As participants engage in this activity, she asks them to focus on two questions:

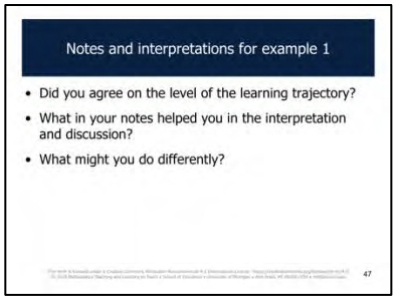


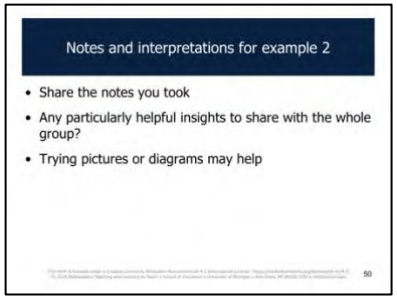
   - How are students reasoning about measuring?
   - How are students making sense of length?





Detailed description of activity	Comments & other resources
2. Encourage participants to focus on the questions listed above as they watch the first video. Then have participants watch Video B, taking notes about what they are noticing. Afterwards, give participants time to talk with a partner about what they noticed and the level at which they would characterize the student. After participants have had time to talk with their partners, return to the whole group to debrief this process, focusing on the following questions: • Did you agree on the level of the Learning Trajectory? • What in your notes helped you in the interpretation and discussion? • What might you do differently?	*As participants discuss the levels of the Learning Trajectory that they believe characterize the students' behaviors in the videos, encourage them to use specific evidence from the video to support their claims.* *Video B: Test yourself 1: Measuring a strip with tiles* *In this video, a student attempts to iterate a single tile, and then two tiles, to determine the length of a green strip. The student then demonstrates end-to-end length measurement when given more than enough tiles.* *Notice how the child initially attempts to use only red tiles and needs a little bit of guidance to also use the blue.* *Many children use some sort of visual patterning when laying out "units" that may or may not relate to measuring length. Also note that, when four tiles are replaced with two toy firetrucks, the child is not bothered by this and counts tiles and firetrucks as the same.* *Also note that, often at this level, children are not bothered by getting different answers on repeated measurements of the same object.* *<u>Optional:</u> The Video: Test yourself 1 discussion (in the supplements), can be used here to enhance discussion of why this student is an End-to-End Length Measurer. This video includes a discussion of the fact that the student is okay with using different "units" and is not iterating correctly. In the video, a teacher talks about how the student recognizes that the firetrucks are not the same size as the tiles. This leads to a discussion about how it appears that the student to be a little bothered that she gets different answers when measuring in different ways.*



Detailed description of activity	Comments & other resources
3. Repeat the process of watching and taking notes on a video by having participants watch Video C. Again, have participants talk with a partner about what they noticed about the student's reasoning and the level at which they would classify the student's behaviors. 	*Video C: Test yourself 2: Which is longest?* *In this video, a student is asked to find the longest object, and she uses direct comparison repeatedly to determine which object is the longest. This is evidence that she is a Length Direct Comparer.* *<u>Optional:</u> The "Video: Test yourself 2 discussion" (in the supplements) can be used here to enhance discussion of why this student is a Length Comparer (specifically, a Length Direct Comparer). This video includes a discussion about how to use the Learning Trajectory levels to identify what a student can do, but not necessarily to label a child as **only** a Length Direct Comparer.*
4. Conclude this part by having participants watch Video D, where Dr. Sarama and Dr. Clements talk to participants about using Learning Trajectories in their own classrooms with their own students. This video includes a discussion of how the names of the levels were developed, and it emphasizes that being able to identify the description/understanding of what the student is doing is more important than memorizing the names of the levels. After watching the video, allow time for follow-up questions and discussion as time and need permit.	*As teachers discuss these videos, it is helpful to keep in mind that they are "teachers, not just testers". They may discuss aspects of the videos – beyond the developmental level – that they find interesting or meaningful, even if these do not directly relate to the focus questions.* *As participants gain experience analyzing students' responses according to the levels of the developmental progression, they should also develop an orientation that is focused towards teaching. In other words, they should be thinking about what "next steps" they might want to take with the student based on what the student is currently doing.*

Part 5: Connecting the Learning Trajectories with the Broken Ruler Task (~10 minutes)

Goals

- Participants will discuss their own students' responses to the Broken Ruler Task.
- Participants will use the Learning Trajectory for length measurement to describe their students understanding.
- Participants will be introduced to particular note taking form.

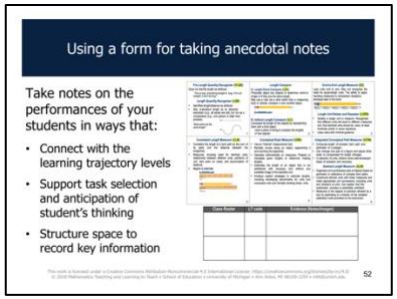
Instructional sequence

1. Introduce Part 5 and watch Video A.
2. Discuss students' responses to the Broken Ruler Task in small groups.
3. Watch Video B and share responses in whole group.
4. Introduce the anecdotal notes form by watching Video C.

Resources

- Video A (00:42): Setting up the Broken Ruler discussion
- Video B (01:06): Learning trajectories as a lens
- Video C (01:10): Using a form for taking anecdotal notes
- Handout: Anecdotal notes form – Length Learning Trajectories

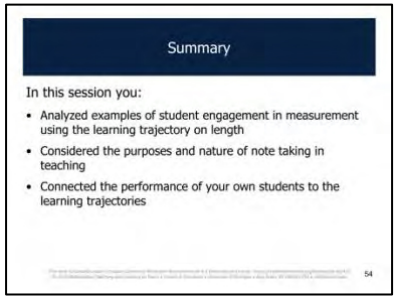
Detailed description of activity	Comments & other resources
1. Introduce Part 5: In this part, participants use the Learning Trajectory for Length to analyze their students' responses to the Broken Ruler Task. Have participants watch Video A, where Dr. Sarama brings the teachers in the professional development back to the Broken Ruler Task and invites them to look at what their students did through the lens of the Learning Trajectory for length.	*Participants will routinely draw on the work that they do in the Classroom Connection Activities as the basis for work in subsequent sessions. This will typically happen at the outset of the session or when participants are engaged in the "learning from practice" activity.* *The use of the Broken Ruler responses here is to allow participants to look at what their students did through the lens of developmental progressions that have just been introduced.* *If Sessions 1 and 2 are being done together, then move onto step 3 (below).*
2. In groups of 2-4, have participants discuss their students' responses to the Broken Ruler Task, using the following questions to focus the discussion: • What was consistent or not with the Learning Trajectory? • How did or could the Learning Trajectory provide a framework for understanding their responses and strategies? • How could the Learning Trajectory help plan "next steps" (formative assessment)?	

Detailed description of activity	Comments & other resources
3. After participants have had an opportunity to analyze and discuss their students' work in small groups, have them watch Video B in which Dr. Sarama and Dr. Clements talk more about how to use Learning Trajectories as a lens to understand student thinking in the classroom. Use this video to launch a whole group discussion about what participants noticed about their students' responses to the Broken Ruler Task and how they related to the Learning Trajectory framework.	
4. Distributed Handout: Anecdotal notes form – Length Learning Trajectories and explain that participants will use this form as part of their CCA for the next session. Watch Video C, in which Dr. Sarama introduces the different parts of the form. 	*It is important for participants to try using the form as they work with their students. It might be strange using such a structured form to take notes, but encourage participants to view the work as experimenting that can result in new insights into the work of teaching, as well as insights into students' thinking.* *These notes will be the main artifact from teaching that will support sharing what happened when the tasks were used with students. Taking complete notes will make sharing with colleagues more manageable for those sharing and more meaningful for those listening.* *The section on predicting Learning Trajectory levels is meant to help participants have these at the forefront of their thinking when working with students as it may help them more quickly and easily make connections with what students are doing in the moment.* *It may be helpful to share with participants that the table will not be filled in left to right. Participants will probably record the student's names, then record notes on what the student says and does, and then, perhaps later, use those notes to determine the Learning Trajectory level that is most aligned with what the student said and did.*

Part 6: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom. • Participants will recognize instruction as the third component of a Learning Trajectory.	1. Distribute and explain the Classroom Connection 2. Activities and watch Video A. 3. Preview the next session by watching Video B. Summarize the work of the session.	• Video A (03:08): Session 2 Classroom Connection Activities • Video B (00:15): Looking ahead • Handout: Session 2 Classroom Connection Activity • Handout: Anecdotal notes form – Length Learning Trajectories

Detailed description of activity	Comments & other resources
1. Introduce the Classroom Connection Activities for the next session. Distribute the handout you customized with selected Classroom Connection Activities and accompanying documents described below. Required: • Length Assessment Task: Complete tasks (the Broken Ruler Task which participants have already tried with a few students and also other new tasks) with 3-4 students of different (hypothesized) achievement levels. Ask the students to write down or draw how they measured. Use the anecdotal notes form to record how students engage in the task. Respond to the reflection questions listed on the CCA handout. • Select a lesson or a short sequence of instructional activities from your curriculum that focus on measuring length. Bring these materials with you to the next session. Watch Video A, where Dr. Sarama and Dr. Clements introduce the tasks participants can use with students as part of their CCA. Then, introduce the other part of the CCA—selecting instructional activities from the curriculum that are focused on measuring length.	*Make sure to explain it is key to try using the anecdotal notes form so that the group can work together in the next session on understanding its affordances and limitations.* *Make clear to participants that they are only going to use a subset of the tasks that are available and only doing those tasks with a subset of students. This makes task selection worth considering so that it will provide insight into the thinking of the students that are chosen.* *If you would like to have participants submit the notes, question responses, etc. prior to the next session, then make sure that they understand the method by which the should submit those.*

Detailed description of activity	Comments & other resources
2. Summarize this session by emphasizing that participants: • Analyzed examples of student engagement in measurement in terms of Learning Trajectory on length • Considered the purposes and nature of note taking in teaching • Connected the performance of your own students to the Learning Trajectory for length 	
3. Preview the next session by having participants watch Video B, where Dr. Sarama explains that the focus of the next session will be on instruction. The focus of Session 3 is on the importance of instruction as the "Now what do I do?" component of a Learning Trajectory.	