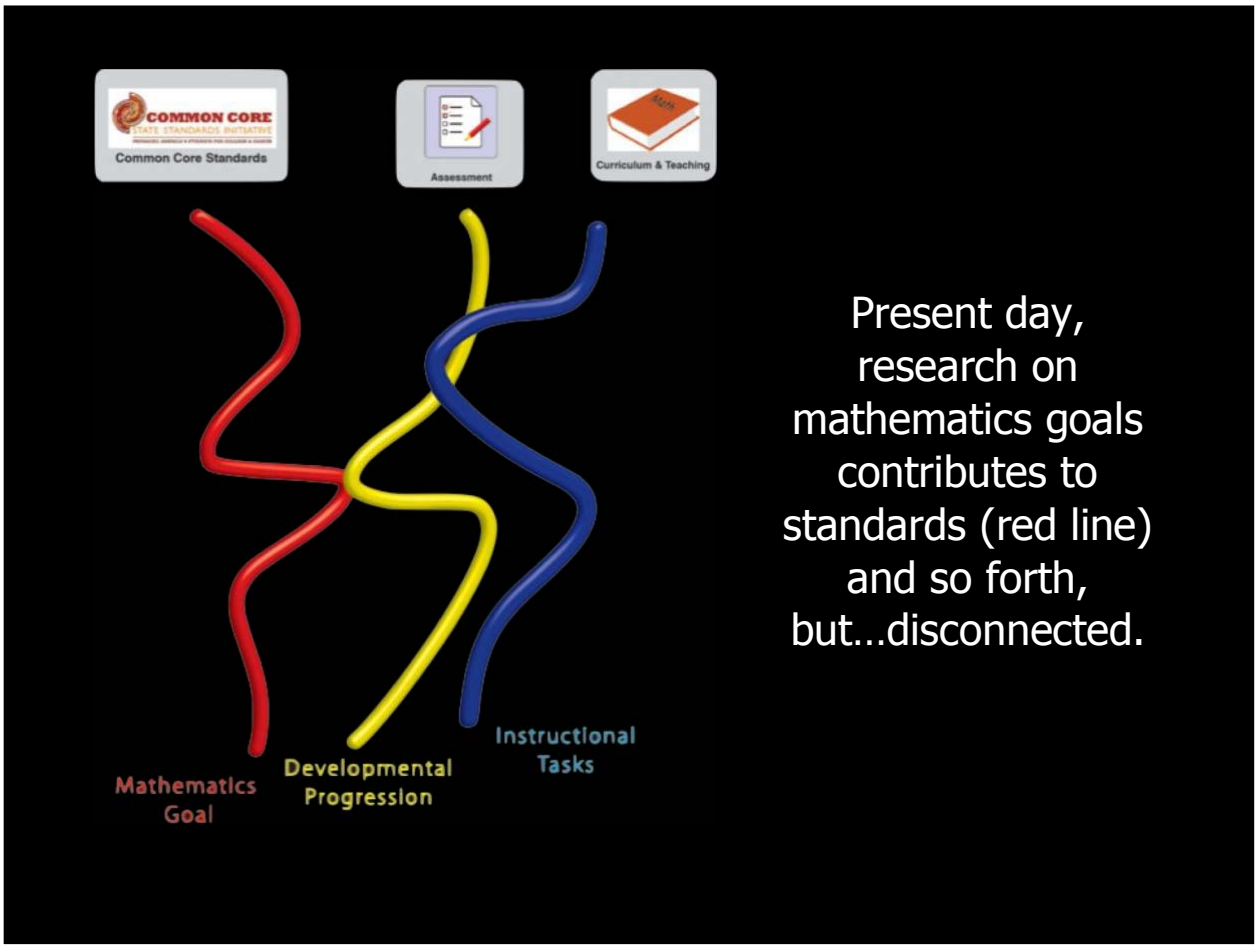
Session 2: Length Learning Trajectory – Developmental Progression

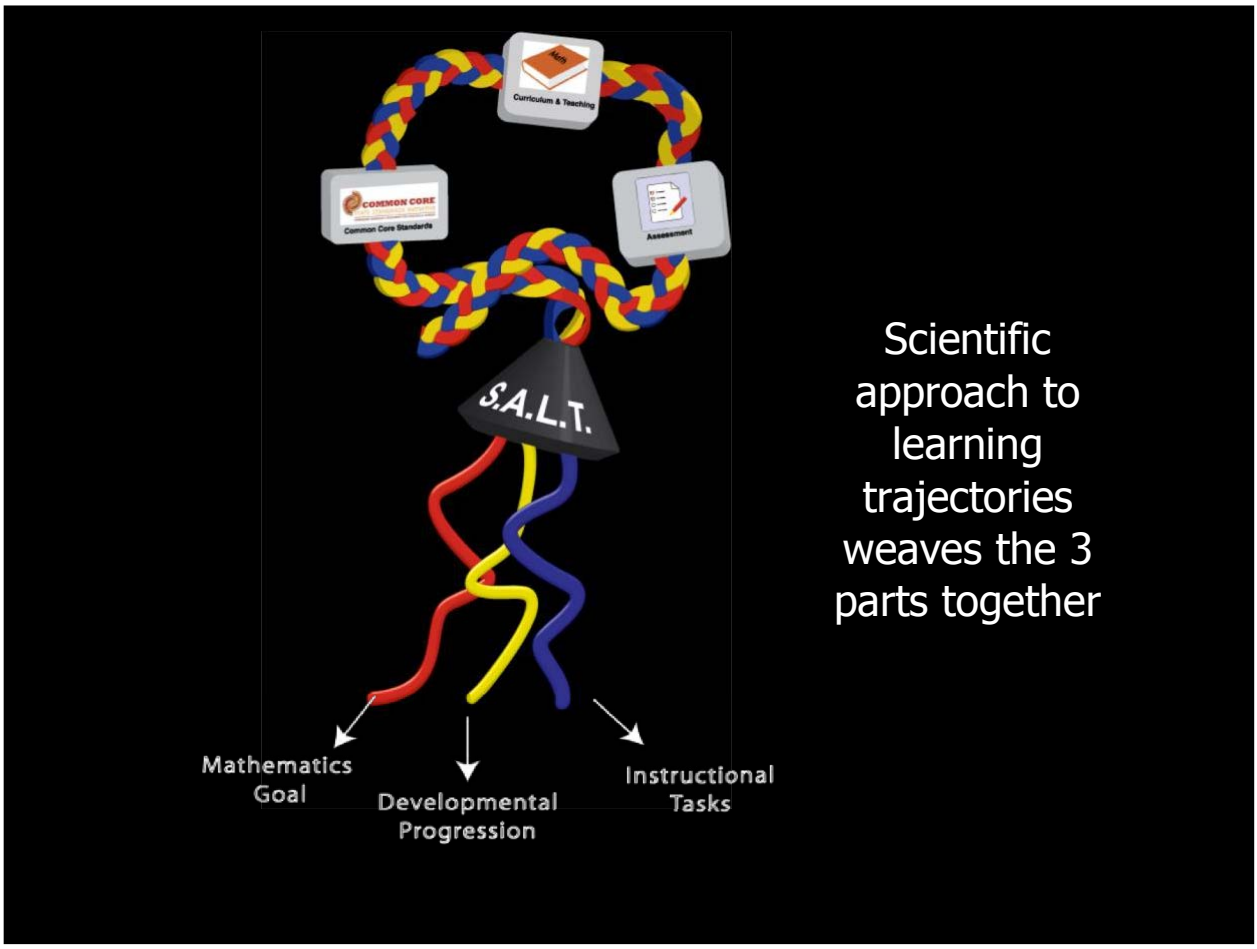
Overview of Session 2

- Unpacking learning trajectories for length by watching students measure
- Taking notes to support learning and note taking in teaching
- Applying learning trajectories to your work with students on the "Broken Ruler"

Learning trajectories approach

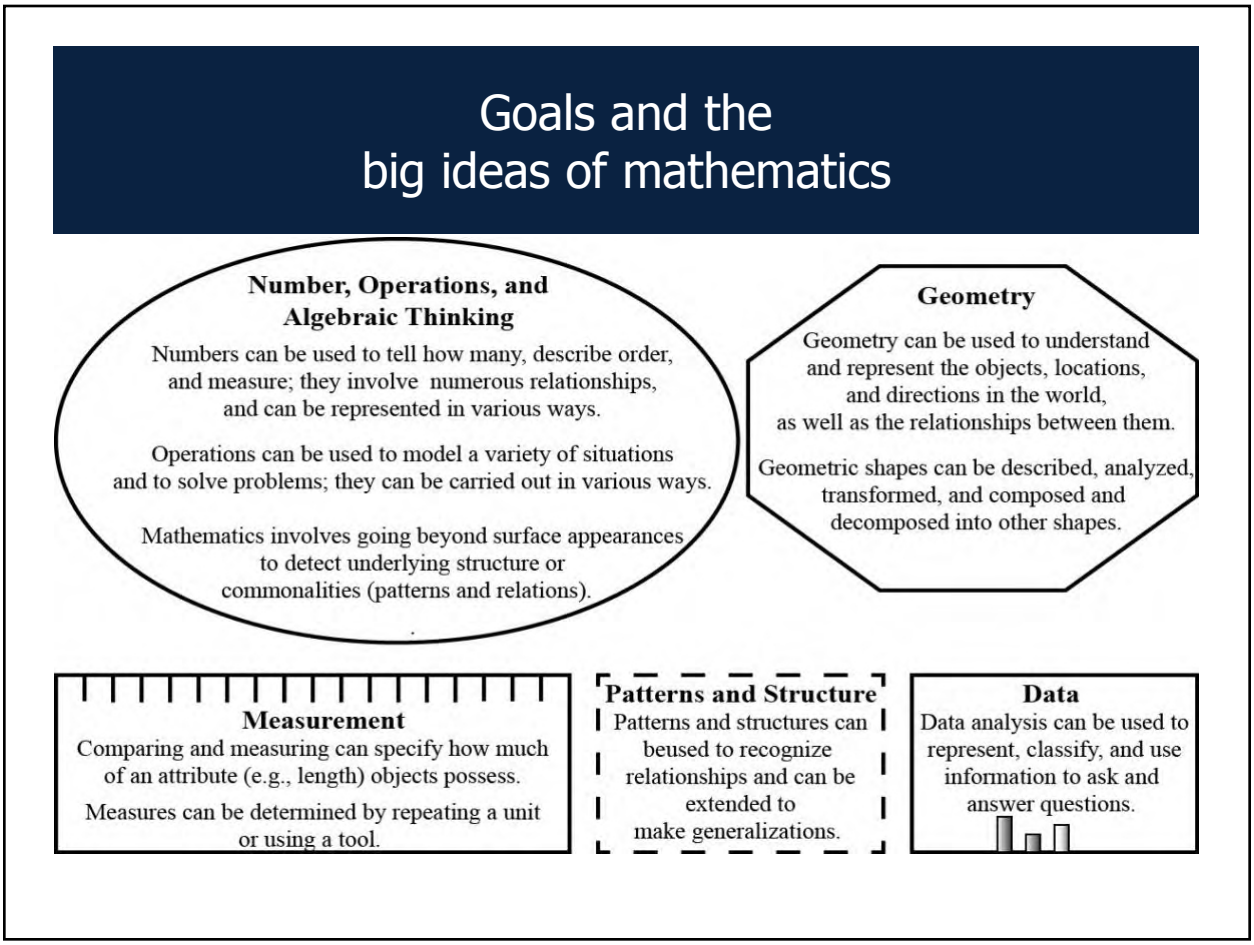
- Goal
- Developmental Progression
- Instruction





Learning trajectories in our sessions

- Goal (Session 1)
- Developmental Progression
- Instruction



Learning trajectories in our sessions

- Goal (Session 1)
- Developmental Progression (Session 2)
- Instruction

Today!

Length developmental progression

- Let's study students at different levels
- View and analyze each video
- What characterizes the students' thinking?

Learning trajectory levels (Length) – Length Quantity Recognizer

- **Length Quantity Recognizer**
- Length Comparer
- End-to-End Length Measurer

Length Quantity Recognizer (LQR)

- Identifies length as attribute
- Another example:
  - "I'm tall, see?"

Learning trajectory levels (Length) – Length Comparer

- Length Quantity Recognizer
- **Length Comparer**
- End-to-End Length Measurer

Length Direct Comparer (LDC)

- Physically aligns two objects to determine which is longer or if they are the same length
- Another example:
  - Stands two sticks up next to each other on a table and says, "This one's bigger"

Learning trajectory levels (Length) – Length Comparer

- Length Quantity Recognizer
- **Length Comparer**
- End-to-End Length Measurer

Indirect Length Comparer (ILC)

- Compares the lengths of two objects by representing them with a third object
- Another example:
  - Compares length of two objects with a piece of string

Learning trajectory levels (Length) – End-to-End Length Measurer

- Length Quantity Recognizer
- Length Comparer
- **End-to-End Length Measurer**

End-to-End Length Measurer (EE)

- Lays units end-to-end. May not see the need for equal-length units
- Another example:
  - Lays 9 inch cubes in a line beside a book to measure how long it is

End-to-End Length Measurer (EE)

- Lays units end-to-end. May not see the need for equal-length units
- Another example:
  - Lays 9 inch cubes in a line beside a book to measure how long it is

Learning trajectory levels (Length)

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Consistent Length Measurer
- Conceptual Ruler Measurer
- Integrated Conceptual Path Measurer
- Abstract Length Measurer

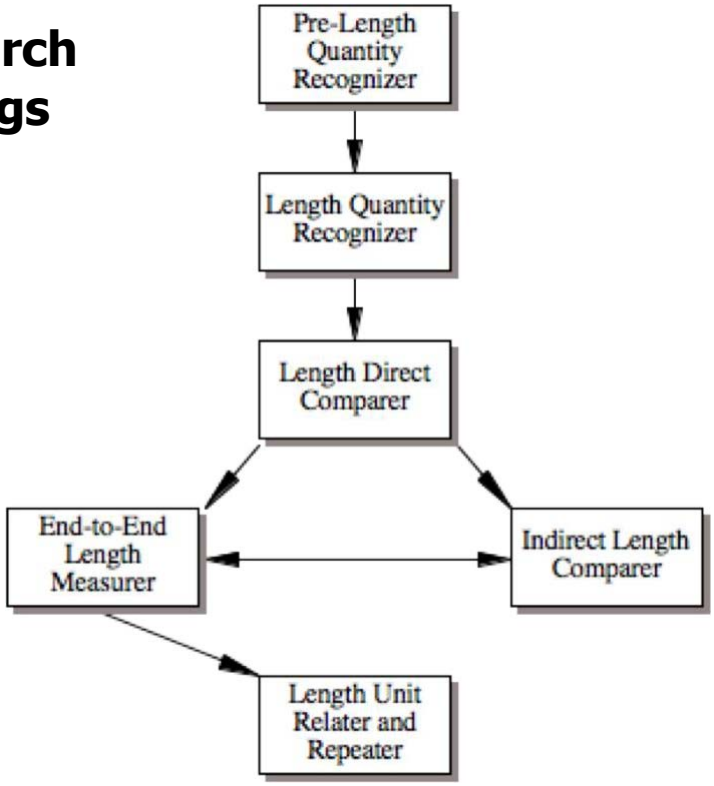
Learning trajectory levels (Length) – Length Unit Relater and Repeater

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- **Length Unit Relater and Repeater**
- Consistent Length Measurer
- Conceptual Ruler Measurer
- Integrated Conceptual Path Measurer
- Abstract Length Measurer

Length Unit Relater and Repeater (LURR)

- Relates size and number of units
  - "If you measure with centimeters instead of inches, you'll need more of them, because each one is smaller"
- Repeats or iterates a single unit to measure. Sees need for identical units. Uses rulers with guidance
  - Measures a book's length well with a ruler
- Uses rulers with guidance

Research findings



Learning trajectory levels (Length) – Consistent Length Measurer and Conceptual Ruler Measurer

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- **Consistent Length Measurer**
- **Conceptual Ruler Measurer**
- Integrated Conceptual Path Measurer
- Abstract Length Measurer

Consistent Length Measurer (CLM)

- Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers
  - Begins to estimate
  - Considers the length of a bent path as the sum of its parts (not the distance between the endpoints)
  - "I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it's 3 meters, 14 centimeters in all"

Conceptual Ruler Measurer (CRM)

- Possesses an "internal" measurement tool. Mentally moves along an object, segmenting it, and counting the segments.
  - Estimates with accuracy
    - "I imagine one meter stick after another along the edge of the room. That's how I estimated the room's length is 9 meters"
  - Operates arithmetically on measures ("connected lengths")

Learning trajectory levels (Length) – Integrated Conceptual Path Measurer

- Length Quantity Recognizer
- Length Comparer
- End-to-End Length Measurer
- Length Unit Relater and Repeater
- Consistent Length Measurer
- Conceptual Ruler Measurer
- **Integrated Conceptual Path Measurer**
- Abstract Length Measurer

Integrated Conceptual Path Measurer (ICPM)

- Computes perimeter of a polygon
- Changes one part of a figure and adjusts other sides to compensate for length changes to maintain overall lengths
- In selection of units, shows well-developed ideas of precision and accuracy

Abstract Length Measurer (ALM)

- Constructs derived units with linear measures, such as miles per hour, and make appropriate unit conversions
- Measures to the degree of precision allowed by a tool by estimating to a fraction of the smallest calibration mark provided on the instrument

Reflecting on the length developmental progression

- Read and discuss handout
- How does each level make sense in view of your assessments and our work here?
- Questions or comments?

Length Unit Relater and Repeater (LURR)

- Relates size and number of units
  - "If you measure with centimeters instead of inches, you'll need more of them, because each one is smaller"
- Repeats or iterates a single unit to measure. Sees need for identical units. Uses rulers with guidance
  - Measures a book's length well with a ruler
- Uses rulers with guidance

Test ourselves

- Let's see if we agree on these levels!
- Take notes as you watch:
  What do they do and why?
- Think-pair-share about what level can be seen in each video.
- Why?…

Why take notes?

- For us, good aids to memory and discussion
- In school, several particular purposes
  - Compiling main solution strategies
  - Deciding who to call on for class discussions
  - Considering what to focus on next in formative assessment
  - Accumulating information over time to support summative assessment (report cards or rich information to share at conferences)

Test ourselves: Focus questions

Two focus questions:

- How are students reasoning about measuring?
- How are students making sense of the length?

Notes and interpretations for example 1

- Did you agree on the level of the learning trajectory?
- What in your notes helped you in the interpretation and discussion?
- What might you do differently?

End-to-End Length Measurer

Notes and interpretations for example 2

- Share the notes you took
- Any particularly helpful insights to share with the whole group?
- Trying pictures or diagrams may help

Length Direct Comparer

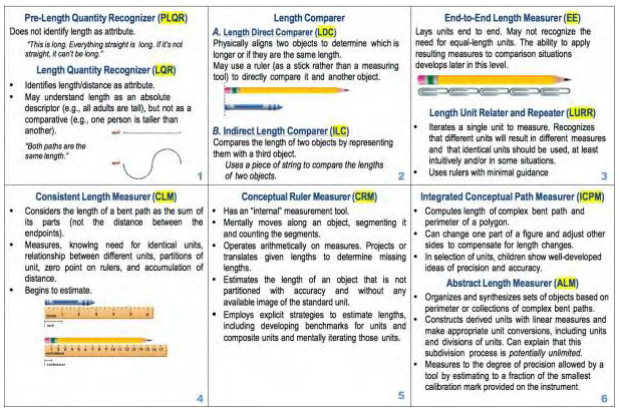
Revisiting the Broken Ruler

- Discuss your students responses to the broken ruler task
  - What was consistent or not with the learning trajectory (LT)?
  - How did or could the LT provide a framework for understanding their responses and strategies?
  - How could the LT help plan "next steps" (formative assessment)?

Using a form for taking anecdotal notes

Take notes on the performances of your students in ways that:

- Connect with the learning trajectory levels
- Support task selection and anticipation of student's thinking
- Structure space to record key information



Pre-Length Quantity Recognizer (PLQR)
Does not identify length as attribute.
"This is long. Everything straight is long. If it's not straight, it can't be long."

Length Quantity Recognizer (LQR)
- Identifies length/distance as attribute.
- May understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative (e.g., one person is taller than another).

"Both paths are the same length."

1

Length Comparer

A. Length Direct Comparer (LDC)
Physically aligns two objects to determine which is longer or if they are the same length.
May use a ruler (as a stick rather than a measuring tool) to directly compare it and another object.

B. Indirect Length Comparer (ILC)
Compares the length of two objects by representing them with a third object.
Uses a piece of string to compare the lengths of two objects.

2

End-to-End Length Measurer (EE)
Lays units end to end. May not recognize the need for equal-length units. The ability to apply resulting measures to comparison situations develops later in this level.

Length Unit Relater and Repeater (LURR)
- Iterates a single unit to measure. Recognizes that different units will result in different measures and that identical units should be used, at least intuitively and/or in some situations.
- Uses rulers with minimal guidance

3

Consistent Length Measurer (CLM)
- Considers the length of a bent path as the sum of its parts (not the distance between the endpoints).
- Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers, and accumulation of distance.
- Begins to estimate.

4

Conceptual Ruler Measurer (CRM)
- Has an "internal" measurement tool.
- Mentally moves along an object, segmenting it and counting the segments.
- Operates arithmetically on measures. Projects or translates given lengths to determine missing lengths.
- Estimates the length of an object that is not partitioned with accuracy and without any available image of the standard unit.
- Employs explicit strategies to estimate lengths, including developing benchmarks for units and composite units and mentally iterating those units.

5

Integrated Conceptual Path Measurer (ICPM)
- Computes length of complex bent path and perimeter of a polygon.
- Can change one part of a figure and adjust other sides to compensate for length changes.
- In selection of units, children show well-developed ideas of precision and accuracy.

Abstract Length Measurer (ALM)
- Organizes and synthesizes sets of objects based on perimeter or collections of complex bent paths.
- Constructs derived units with linear measures and make appropriate unit conversions, including units and divisions of units. Can explain that this subdivision process is *potentially unlimited*.
- Measures to the degree of precision allowed by a tool by estimating to a fraction of the smallest calibration mark provided on the instrument.

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Class Roster	LT code	Evidence (Notes/Images)

Session 2
Classroom Connection Activities (CCAs)

- Assessments of students
- Bring in a curriculum lesson or activity on length

Summary

In this session you:

- Analyzed examples of student engagement in measurement using the learning trajectory on length
- Considered the purposes and nature of note taking in teaching
- Connected the performance of your own students to the learning trajectories

Learning trajectories: Looking ahead

- Goal (Session 1)
- Developmental Progression (Session 2)
- Instruction (Session 3)