

***Description of the session***

In Session 3, participants will complete their work on the Learning Trajectory for length measurement by focusing on the third component of the Learning Trajectory: instructional tasks. At the beginning of the session, participants will review the levels of the developmental progression for length measurement by watching videos of students’ performance on tasks and identifying the level this performance represents. Then, they will engage in their first anecdotal notes workshop, which will provide them with an opportunity to examine their own students’ developmental levels as well as their methods of taking anecdotal notes. After that, participants will analyze a variety of instructional tasks to identify the developmental level they target. They will also discuss how to modify tasks to target a particular level and will then practice doing this with tasks from their own curriculum materials. The session will conclude with a summary of key concepts of length measurement.

***Activities and goals of the session***

| <b>Activities</b>                                    | <b>Times</b> | <b>Corresponding parts of the session</b> | <b>Goals</b>  |
|--|--------------|---|---|
| I. Review, overview, and anecdotal notes workshop    | 30 minutes   | Parts 1 & 2                               | <ul style="list-style-type: none"> <li>• Participants will be oriented to the work of the session</li> <li>• Participants will demonstrate understanding of the Learning Trajectory for length measurement.</li> <li>• Participants will be formally introduced to the Learning from Practice Protocol for anecdotal notes workshop.</li> <li>• Participants will understand the benefits and limits of their current use of the anecdotal notetaking form.</li> </ul>  |
| II. Instructional tasks for length measurement       | 50 minutes   | Parts 3, 4, & 5                           | <ul style="list-style-type: none"> <li>• Participants will recognize the Learning Trajectory level an instructional activity designed to target</li> <li>• Participants will understand how to modify/differentiate an instructional activity to target a different level of the learning Trajectory for length measurement</li> <li>• Participants will connect activities in their curriculum to the Learning Trajectory levels.</li> <li>• Participants will revise an activity to target the level of the students in their classroom.</li> </ul> |
| III. Wrap up including summary of length measurement | 10 minutes   | Part 6                                    | <ul style="list-style-type: none"> <li>• Participants will understand how understanding length measurement forms a foundation for other measurement domains.</li> <li>• Participants will understand ways of connecting the session content to their classroom.</li> <li>• Participants will recall the work they have done in the first three sessions.</li> <li>• Participants will understand the Classroom Connection Activity assignment.</li> </ul>   |

***Classroom Connection Activities***

| <b>Optional</b>   |
|---|
| Type of task: Anecdotal notes extension<br>Description: Facilitate students' work on a length measurement activity and use the anecdotal notes form to record observations. |
| Type of task: Preparation for Sessions 4-6<br>Description: Start looking for an activity or assessment focused on the measurement of area                                   |

***Preparing for the session***

- Make copies as needed: Handout: Content cube – Length Learning Trajectory (Parts 1, 3, 4, & 5); Handout: Anecdotal notes workshop protocol (Part 2); Handout: Anecdotal notes form – Length Learning Trajectories (Part 2)
- Customize and make copies of the Classroom Connection Activities
- Test technical setups: Internet connection, speakers, projector

***Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)***

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
  - a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
  - b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
  - c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards.

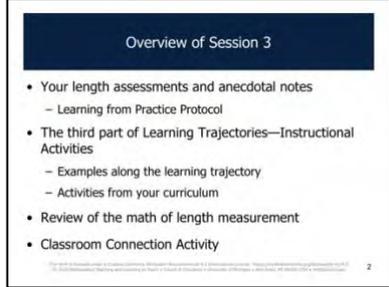
*Scope of the module (focal content of this session in **bold**)*

| Mathematics   | Student thinking   | Teaching practice   | Learning from practice   |
|---|--|---|--|
| <ul style="list-style-type: none"> <li>recognizing the mathematical goal as the first component of a complete Learning Trajectory</li> <li>understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement)</li> <li>understanding how measurement of length, area, and volume are represented and developed in the CCSS</li> <li>understanding how measurement connects with the CCSS standards for mathematical practice</li> <li><b>understanding concepts and skills involved in measuring length, area, and volume</b></li> <li>understanding connections between length, area, and volume measurement and between metric measurements for each</li> </ul> | <ul style="list-style-type: none"> <li>recognizing student development as the second component of a complete Learning Trajectory</li> <li>understanding children’s development of measurement through Learning Trajectories for length, area, and volume</li> <li>recognizing principles of measurement in student work</li> <li><b>interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement</b></li> <li>interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement</li> <li>interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement</li> </ul> | <ul style="list-style-type: none"> <li><b>recognizing instruction as the third component of a complete Learning Trajectory</b></li> <li>using anecdotal notes to document what students say and do when working on measurement tasks</li> <li><b>connecting measurement activities in curricula to measurement Learning Trajectory levels</b></li> <li><b>modifying measurement tasks to target different and/or particular Learning Trajectory levels</b></li> </ul> | <ul style="list-style-type: none"> <li><b>understanding the anecdotal notes workshop process</b></li> <li>using the anecdotal notes workshop to improve the practice of note taking</li> <li>using the anecdotal notes workshop to improve teaching</li> </ul> |

**Part 1: Test ourselves (~5 minutes)**

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| <p><b><u>Goals</u></b></p> <ul style="list-style-type: none"> <li>• Participants will be oriented to the work of the session.</li> <li>• Participants will demonstrate understanding of the Learning Trajectory for length measurement.</li> </ul> | <p><b><u>Instructional sequence</u></b></p> <ol style="list-style-type: none"> <li>1. Introduce the session by watching Video A</li> <li>2. Introduce the part and watch Video B.</li> <li>3. Have participants test their knowledge of the Learning Trajectory for length measurement by watching and discussing Videos C <b>Answer: Length Unit Relater and Repeater</b>) and D. Watch and discuss Videos E (<b>Answer: End-to-End Length Measurer or earlier</b>) and F to continue assessing participants’ understanding of the Learning Trajectory for length measurement.</li> </ol> | <p><b><u>Resources</u></b></p> <ul style="list-style-type: none"> <li>• Video A (01:32): Session overview</li> <li>• Video B (00:17): Introducing “test ourselves”</li> <li>• Video C (00:41): Test ourselves 1: Measuring string with tiles</li> <li>• Video D (01:14): Commentary on test ourselves 1</li> <li>• Video E (00:59): Test ourselves 2: Broken ruler</li> <li>• Video F (00:19): Commentary on test ourselves 2</li> <li>• Handout: Content cube – Length Learning Trajectory</li> </ul> <p><b><u>Supplements</u></b></p> <ul style="list-style-type: none"> <li>• Video (01:16): Responses to test ourselves 1</li> <li>• Video (02:26): Responses to test ourselves 2</li> </ul> |
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| Detailed description of activity   | Comments & other resources   |
|--|--|
| <p>1. Introduce the session by watching Video A. In this video, Dr. Sarama and Dr. Clements provide an overview of the activities in the session, including:</p> <ul style="list-style-type: none"> <li>• discussion of the length assessments and participants’ anecdotal notes</li> <li>• work on the third part of the Learning Trajectories—Instructional activities</li> <li>• review of the mathematics of length measurement</li> <li>• instructions for the Classroom Connection Activity associated with the session</li> </ul> | <p><i>This is the final session in a three-part sequence on the Learning Trajectory for length measurement. Each Learning Trajectory includes three parts:</i></p> <ul style="list-style-type: none"> <li>• <i>The mathematics (the focus of Session 1)</i></li> <li>• <i>Students’ development (the focus of Session 2)</i></li> <li>• <i>Instruction (the focus of Session 3)</i></li> </ul> |

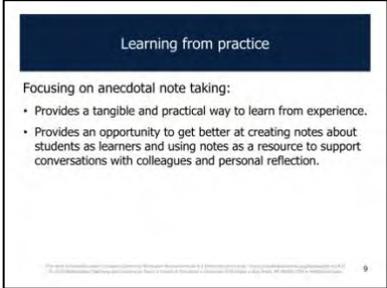


| Detailed description of activity   | Comments & other resources   |
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| <p>2. Introduce Part 1: In this part, participants will watch two videos of different children doing length tasks and will practice determining which level of the Learning Trajectory for length measurement is most appropriate to describe their behaviors.</p> <p>Watch Video B, where Dr. Clements sets up the activity in this part. Distribute Handout: Content cube – Length Learning Trajectory for participants to use as a resource in this part and in subsequent parts in this session.</p>   | <p><i>This part is designed to provide a <u>quick</u> review of what participants learned about the developmental progression for length measurement before considering strategies for teaching length measurement. Part 2 will launch the main work of this session and will provide an overview of the session.</i></p> <p><i>Because this session has many "first time" elements that will require time, be careful to limit discussion time during this part. If there are many questions about connecting a particular video to a level(s), it may be best to address those questions after the session or to encourage participants to watch the video in question again before the next session to think through reasons why the behaviors should be connected with a particular level.</i></p>   |
| <p>3. Have participants watch Video C, where the student uses a "leap frog" strategy to measure the length of a string using only two, inch tiles. Then, have participants discuss their observations and identify the level of thinking demonstrated in the video.</p> <p>If it would be helpful to support the discussion, consider showing Video: Test ourselves responses (to Video C).</p> <p>Conclude this discussion by having participants watch Video D, where Dr. Clements and Dr. Sarama clarify the difference between an End-to-End Length Measurer and a Length Unit Relater and Repeater.</p> | <p><i>Video C: Test ourselves 1: Measuring string with tiles</i></p> <p><i>The student in this video is demonstrating behaviors consistent with the "Length Unit Relater and Repeater" level because he is able to measure a string using only two tiles (rather than needing to line up tiles end to end across the entire length of the string).</i></p> <p><i>Supplementary Video: Test ourselves responses (to Video C) This video shows participants providing their rationales for saying this child might be an End-to-End Length Measurer or a Length Unit Relater and Repeater.</i></p> <p><i>Video D: Commentary on test ourselves 1</i></p> <p><i>In this video, Dr. Sarama emphasizes that End-to-End Length Measurers fill the entire length of the string end-to-end. Dr. Clements points out that the "leap frog" strategy is a behavior commonly observed at the Length Unit Relater and Repeater level.</i></p> |

| Detailed description of activity  | Comments & other resources   |
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| <p>4. Watch Video E and have participants discuss their observations and identify the level of thinking demonstrated in the video.</p> <p>If it would be helpful to support the discussion, consider showing Video: Test ourselves responses (to Video E).</p> <p>Conclude this discussion by having participants watch Video F, where Dr. Clements talks about the difficulty with truly identifying this student’s level of thinking. In this video, Dr. Clements points out that the student’s response to this task offers an invitation to ask further questions about his thinking.</p> | <p><i>Video E: Test ourselves 2: Broken ruler</i></p> <p><i>In this video, the student is presented with the Broken Ruler task, and he struggles to determine the answer. The student appears to show evidence of thinking about the zero point, as well as focusing on the endpoints, and he is possibly attempting to iterate. In the end, he appears to simply read the number on the ruler closest to the end of the strip</i></p> <p><i>Supplementary Video: Test ourselves responses (to Video E) This video shows Dr. Sarama asking participants to match what they see with the level descriptions. Participants talk about (1) looking for the zero point on the ruler, (2) using the span between two fingers, (3) possibly reading the number right off the ruler, and (4) possibly partitioning a unit. Dr. Sarama leads discussion about what levels participants’ observations relate to and why those may or may not fit this student’s understanding.</i></p> <p><i>Video F: Commentary on test ourselves 2</i></p> <p><i>In this video, Dr. Clements acknowledges that this student appears to have a “hodgepodge of knowledge about measurement that hasn’t really fallen together.”</i></p> |

**Part 2: Overview and anecdotal notes workshop (~25 minutes)**

| <u>Goals</u>   | <u>Instructional sequence</u>  | <u>Resources</u>   |
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| <ul style="list-style-type: none"> <li>Participants will be formally introduced to the Learning from Practice Protocol for anecdotal notes workshop.</li> <li>Participants will understand the benefits and limits of their current use of the anecdotal notetaking form.</li> </ul> | <ol style="list-style-type: none"> <li>Introduce the learning from practice piece of the module: focusing on anecdotal notes.</li> <li>Watch Video A and introduce the anecdotal notes workshop protocol; Distribute the handout.</li> <li>Have participants meet in small groups to engage in an anecdotal notes workshop. Watch Video B and debrief the workshop. Debrief the use of assessment tasks by watch Video C.</li> </ol> | <ul style="list-style-type: none"> <li>Video A (01:30): Introducing anecdotal notes workshop</li> <li>Video B (00:55): Anecdotal notes workshop debrief</li> <li>Video C (01:34): Debriefing the use of assessment tasks</li> <li>Handout: Anecdotal notes workshop protocol</li> <li>Handout: Anecdotal notes form – Length Learning Trajectory</li> </ul> <p><u>Supplements</u></p> <ul style="list-style-type: none"> <li>Video (01:27): Using a set of tasks to more fully see the Learning Trajectory</li> <li>Video (00:29): Selecting tasks and/or subsets of students</li> </ul> |

| Detailed description of activity  | Comments & other resources   |
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| <p>1. Introduce Part 2: Taking anecdotal notes during teaching and discussing them with colleagues can be used to study and improve teaching. This part launches the “learning from practice” element of the module by introducing participants to a method for focusing on the practice of anecdotal note taking.</p> <p>Explain that a focus on anecdotal note taking</p> <ul style="list-style-type: none"> <li>Provides a tangible and practical way to learn from experience.</li> <li>Provides an opportunity to get better at creating notes about students as learners and using notes as a resource to support conversations with colleagues and personal reflection.</li> </ul> |  <p><i>The reason this module includes work on note taking is because this practice is important in teaching, but teachers rarely get a chance to work on getting better at it.</i></p> |

| Detailed description of activity   | Comments & other resources  |
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| <p>2. Explain that, as a way of focusing on anecdotal note taking, participants will engage in an anecdotal notes “workshop.” During the workshop, participants will meet in small groups to discuss their examples of students’ thinking on the length tasks they posed as part of the Classroom Connection Activity for the last session. Participants should meet in groups of three with others who are teaching in similar grades levels.</p> <p>Watch Video A, in which Dr. Sarama explains the anecdotal notes workshop protocol. She explains that, during the anecdotal notes workshop,</p> <ul style="list-style-type: none"> <li>• The participant in the role of “presenter” will use his/her notes to support sharing information about the performance of <u>a student</u> on a particular task (and any key background information). (~3 minutes)</li> <li>• Participants in the role of “colleagues” will ask questions to better understand the task used, the student’s performance on the task, the connections between the student’s performance and the Learning Trajectories, and the presenter’s method of taking anecdotal notes. (~3 minutes)</li> <li>• Participants switch roles until all have shared about a student’s performance on a task that was used. With whatever time remains participants should reflect on what they learned, including discussing how the process of sharing with colleagues worked/didn’t work for them. (~5 minutes)</li> </ul> <p>Distribute and introduce the Handout: Anecdotal notes workshop protocol. Explain that the goal of the anecdotal notes workshop is for participants to use their anecdotal notes to support discussion with their colleagues about the mathematics that their students demonstrate and how students think about mathematics. A related goal is to discuss differences in students’ thinking.</p> <div data-bbox="877 459 1264 748" style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center; background-color: #002060; color: white; padding: 2px;">A protocol for anecdotal notes “workshop”</p> <p>In small groups:</p> <ul style="list-style-type: none"> <li>• “Presenters” share information about the performance of a student on a particular task</li> <li>• “Colleagues” discuss               <ul style="list-style-type: none"> <li>– the tasks</li> <li>– evidence of student thinking</li> </ul> </li> <li>• Reflect as a group on teaching practice               <ul style="list-style-type: none"> <li>– task selection</li> <li>– use of learning trajectory</li> <li>– anecdotal note taking</li> </ul> </li> </ul> </div> | <p><i>Grouping participants in groups of three will ensure that all we be able to share an instance of student thinking from their classrooms.</i></p> <p><i>Grouping participants with colleagues who teach similar grades (the same grade or a “close” grade) is likely to support “colleagues” in more quickly understanding the task and performance characteristics.</i></p> |

| Detailed description of activity  | Comments & other resources   |
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| <p>3. Have the groups of three begin their anecdotal notes workshop time. During this time, participants should share using their anecdotal notes what they learned about students from the tasks. They should discuss the following questions:</p> <ul style="list-style-type: none"> <li>• What levels of thinking were made visible by the tasks?</li> <li>• What new insights or questions do you have about the Learning Trajectory levels?</li> <li>• How could you enhance your next use of anecdotal notes?</li> </ul>  | <div data-bbox="879 302 1266 589" style="border: 1px solid black; padding: 5px;"> <p style="background-color: #333; color: white; padding: 2px;">Discussing notes focused on assessments of length</p> <ul style="list-style-type: none"> <li>• Share the task you used to assess length and your notes in grade-level small groups</li> <li>• When everyone has shared consider the following               <ul style="list-style-type: none"> <li>– What levels of thinking were made visible by the tasks?</li> <li>– What new insights or questions do you have about the learning trajectory levels?</li> <li>– How could you enhance your next use of anecdotal notes?</li> </ul> </li> </ul> </div> <p><i>The major focus of this workshop is on connecting the levels of the Learning Trajectory’s developmental progression to instructional tasks (including analyzing the content of tasks and videos). Also integrated with this is a focus on taking notes.</i></p> <p><i>While participants meet with their small group, circulate and record:</i></p> <ul style="list-style-type: none"> <li>• examples of student work that will be helpful in illustrating the different levels of the Learning Trajectory</li> <li>• ways that participants use the notetaking forms</li> </ul>  |
| <p>4. After participants have had time for small group discussion, return to the whole group to debrief the workshop. Watch Video B in which Dr. Sarاما frames the debriefing questions that include:</p> <ul style="list-style-type: none"> <li>• The notes produced and the note-taking process</li> <li>• The process of talking with colleagues to learn from the notes</li> <li>• The difference between the assessment they just used and the assessments that are a part of their own curriculum</li> </ul> <p>If time and interest permit, view one or more of the following videos:</p> <ul style="list-style-type: none"> <li>• Video: Using a set of tasks to more fully see the Learning Trajectory</li> <li>• Video: Selecting tasks and/or subsets of students</li> </ul> | <div data-bbox="879 695 1266 982" style="border: 1px solid black; padding: 5px;"> <p style="background-color: #333; color: white; padding: 2px;">Learning from practice protocol – Debriefing</p> <p>Debrief in whole group:</p> <ul style="list-style-type: none"> <li>– Insights into the learning trajectories</li> <li>– Perspectives on talking with colleagues using information contained in the notes</li> <li>– Ideas about anecdotal notes and the note-taking process</li> <li>– Ways the protocol for anecdotal notes workshop could be enhanced</li> </ul> </div> <p><i>It may help to project the anecdotal notes form as participants make comments about its use.</i></p> <p><i>Video: Using a set of tasks to more fully see the Learning Trajectory</i></p> <p><i>In this video, a teacher talks about how her student’s performance on a set of tasks helped her group pinpoint the student’s current level in the developmental progression for length measurement.</i></p> <p><i>Video: Selecting tasks and/or subsets of students</i></p> <p><i>In this video, Dr. Clements discusses different strategies for assessment—either assessing all students with one quick task or selecting a few students (who represent a range of achievement levels) and doing longer interviews with them to gain a sense of the different levels of understanding in the class.</i></p> |
| <p>5. Conclude this part by having participants watch Video C, where Dr. Sarاما concludes the whole group discussion, focusing on how the participants felt when doing this assessment with their children. In this video, Dr. Sarاما talks about how much children enjoy talking about what they know.</p>   |  |

**Part 3: Connecting instructional tasks with early Learning Trajectory levels (~20 minutes)**

| <b><u>Goals</u></b>  | <b><u>Instructional sequence</u></b>  | <b><u>Resources</u></b>   |
|--|---|---|
| <ul style="list-style-type: none"> <li>Participants will recognize the Learning Trajectory level an instructional activity is designed to target.</li> <li>Participants will understand how to modify/differentiate an instructional activity target a different level of the Learning Trajectory for length measurement.</li> </ul> | <ol style="list-style-type: none"> <li>1. Introduce Part 3 by watching Video A.</li> <li>2. Watch and discuss Videos B and C.</li> <li>3. Watch and discuss Videos D-F.</li> <li>4. Watch and discuss Video G.</li> <li>5. Watch and discuss Videos H and I.</li> <li>6. Watch Video J and think-pair-share about how to modify instructional tasks.</li> </ol> | <ul style="list-style-type: none"> <li>Video A (00:41): Selecting instructional tasks</li> <li>Video B (01:36): Measuring fish task</li> <li>Video C (00:30): Commentary on measuring fish Video D (01:44): Length riddles task</li> <li>Video E (00:50): Commentary on length riddles Video F (01:26): Building bridges task</li> <li>Video G (01:52): Mr. Mixup task Video H (03:39): Measuring desks task</li> <li>Video I (02:43): Commentary on measuring desks Video J (00:32): Modifying a task for a particular level</li> <li>Handout: Content cube – Length Learning Trajectory</li> </ul> <p><b><u>Supplements</u></b></p> <ul style="list-style-type: none"> <li>Video (00:36): An example of Indirect Length Comparer level</li> <li>Video (00:59): An example of End-to-End Length Measurer level</li> <li>Video (01:10): An example of Length Unit Relater and Repeater</li> </ul> |

| Detailed description of activity  | Comments & other resources |
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| <p>1. Introduce Part 3: This part introduces the third component of the Learning Trajectory for length measurement—instructional tasks.</p> <p>Watch Video A, where Dr. Clements explains that participants will watch videos of instructional tasks and identify the level of the Learning Trajectory that each is designed to target. Encourage participants to use Handout: Content cube – Length Learning Trajectory as a reference during this part.</p> |                            |

| Detailed description of activity  | Comments & other resources  |
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| <p>2. Have participants watch Video B, which shows two children working on a computer activity that involve using a piece of coral to compare the lengths of two different fish.</p> <p>After the video, discuss the question: “What level is this developing?”</p> <p>Then show Video C where Dr. Sarama and Dr. Clements explain why this instructional task helps develop students’ skills with indirect length comparison.</p>  | <p><i>Video B: Measuring fish task</i></p> <p><i>The activity in this video requires students to use indirect comparison to compare the lengths of the two fish. In this activity, students cannot compare the fish by placing them side-by-side but instead must use the piece of coral to compare them.</i></p> <p><i>The supplements section includes a video that illustrates the Indirect Length Comparer level (Video: An example of Indirect Length Comparer level). If needed, use this video to provide participants with a reminder of the features that characterize this level.</i></p>   |
| <p>3. Next, watch Video D, where Dr. Clements describes the length riddles task.</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="289 732 674 1019"> <p style="text-align: center;"><b>Length riddles (Part 1)</b></p> <p>Provide the students with more than 12 cubes and ask –</p> <ul style="list-style-type: none"> <li>• I am 12 cubes long. I have bristles on me. What am I?</li> <li>• Monitoring and formative assessment               <ul style="list-style-type: none"> <li>– <i>Help:</i> More obvious clues; guide measuring</li> <li>– <i>Excel:</i> More of each type of item</li> </ul> </li> </ul> </div> <div data-bbox="751 732 1136 1019"> <p style="text-align: center;"><b>Length riddles (Part 2)</b></p> <ul style="list-style-type: none"> <li>• Next time: Have at least two, but better 3-4, of every item, one of which is the cube length in the riddle</li> <li>• I am 12 cubes long. You use me to write. What am I?</li> <li>• Omit clues if children are doing well</li> </ul> </div> </div> <p>Again, give participants time to discuss the question: “What level is this developing?”</p> <p>After this discussion, watch Video E, where Dr. Sarama and Dr. Clements lead a discussion about why this activity is meant to target the End-to-End Length Measurer (rather than Length Unit Relater and Repeater, as one teacher suggests).</p> <p>Then, watch Video F, which shows a computer activity that is another example of a task targeted at the End-to-End Length Measurer level.</p> <div data-bbox="842 1084 1226 1372" style="margin-top: 20px;"> <p style="text-align: center;"><b>End-to-End Length Measurer (EE)</b></p> <p>Lays units end-to-end. May not see the need for equal-length units.</p> </div> | <p><i>Video D: Length riddles task</i></p> <p><i>The length riddles task is designed to develop students’ skill in using end-to-end length measurement. Because students are given multiple connecting cubes (which serves as the unit of measurement), they are able to place the cubes from one end of an object to the other to see how long the object is.</i></p> <p><i>The supplements section includes a video that illustrates the End-to-End Length Measurer level (Video: An example of End-to-End Length Measurer level). If needed, use this video to provide participants with a reminder of the features that characterize this level.</i></p> <p><i>For the length riddles task, it is important to either provide more than enough cubes for children to measure the lengths or, alternately, to pre-make lengths using connecting cubes (or other units), so that children have to first find the correct length and then use it to select the correct object.</i></p> <p><i>Video F: Building bridges task</i></p> <p><i>This video shows a computer activity called “Workin’ on the Railroad.” In this activity, students must place blocks end-to-end to repair a broken railroad and must then identify how many blocks were needed.</i></p> |

| Detailed description of activity  | Comments & other resources   |
|---|--|
| <p>4. Watch Video G, in which Dr. Sarama and Dr. Clements introduce the Mr. Mixup activity and then talk about research that shows that children can recognize and fix mistakes made by others even while they are still making these same mistakes in their own work.</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="296 407 680 699"> <p>Mr. Mixup's measuring mess (Part 1)</p> <p>Mr. Mixup measures string with connecting cubes</p> <ul style="list-style-type: none"> <li>Gaps between cubes</li> </ul> </div> <div data-bbox="758 407 1142 699"> <p>Mr. Mixup's measuring mess (Part 2)</p> <ul style="list-style-type: none"> <li>String end misaligned with cubes</li> <li>String or cubes not in a straight line</li> </ul> </div> </div>   | <p><i>In Video G, Dr. Sarama makes a joke about marriage, which is meant to tease Dr. Clements, who is her husband.</i></p> <p><i>The Mr. Mixup activity can be modified to support the development of all levels of the Learning Trajectory for length measurement. The focus should always be on something the students in your class are struggling with. Mr. Mixup, then, can be used to demonstrate that particular mistake.</i></p>  |
| <p>5. Next, watch Video H, in which Dr. Sarama introduces a video of herself facilitating a task in a first-grade classroom where she asks students to measure the lengths of their desks with counting cubes (but does not provide enough cubes for them to measure end to end).</p> <p>Again, take time to discuss the question: "What level is this developing?"</p> <p>Then, watch Video I, where Dr. Sarama discusses her attempt to target the Length Unit Relater and Repeater level with this task. Dr. Sarama and Dr. Clements follow up with a discussion about designing "challenging but achievable" tasks for students and making connections back to children's development</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="842 727 1226 1019"> <p>Not enough units!</p> <ul style="list-style-type: none"> <li>Children asked to measure their desks, but they don't have enough cubes!</li> <li>Video...</li> </ul> </div> <div data-bbox="842 1044 1226 1336"> <p>Length Unit Relater and Repeater (LURR)</p> <ul style="list-style-type: none"> <li>Relates size and number of units                             <ul style="list-style-type: none"> <li>"If you measure with centimeters instead of inches, you'll need more of them, because each one is smaller"</li> </ul> </li> <li>Repeats a single unit to measure. Sees need for identical units. Uses rulers with guidance                             <ul style="list-style-type: none"> <li>Measures a book's length well with a ruler</li> </ul> </li> </ul> </div> </div> | <p><i>The activity in Video H is designed to help students develop an understanding that units can be iterated in order to measure the length of an object (i.e., the Length Unit Relater and Repeater level). As she acknowledges in Video I, the students in the video were not yet able to understand this concept.</i></p> <p><i>The supplements section includes a video that illustrates the Length Unit Relater and Repeater level (Video: An example of Length Unit Relater and Repeater level). If needed, use this video to provide participants with a reminder of the behaviors and thinking that characterize this level.</i></p> |

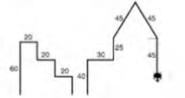
| Detailed description of activity   | Comments & other resources   |
|--|--|
| <p>6. Have participants watch Video J, where Dr. Clements asks teachers to take time to think about how to modify the length riddles and the Mr. Mixup tasks to target the Length Unit Relator and Repeater level.</p> <p>Give participants time to work on this activity— first by themselves and then with partners.</p> <p>Conclude this part by providing time for a few participants to share their modifications with the whole group.</p> | <div data-bbox="842 298 1226 586" data-label="Image"> </div> <p><i>Modifications of the length riddles task could involve changing the quantity of materials provided:</i></p> <ul style="list-style-type: none"> <li>• <i>only one cube/unit (to encourage iteration),</i></li> <li>• <i>only two cubes/units (to encourage "leap frogging"), or</i></li> <li>• <i>several cubes/units but not enough to reach end to end (to help children take the first step away from end-to-end length measurement).</i></li> </ul> <p><i>Modifications of Mr. Mixup should target mistakes a child who is not at the Length Unit Relater and Repeater level might make, such as</i></p> <ul style="list-style-type: none"> <li>• <i>sliding a single tile along the object and just counting, without connecting to unit size,</i></li> <li>• <i>given two tiles, placing one at each end of the object to be measured, or</i></li> <li>• <i>given several tiles, spacing them out so they reach from one end of the object to the other with gaps between them.</i></li> </ul> |

**Part 4: Connecting instructional tasks with later Learning Trajectory levels (~20 minutes)**

| <u>Goals</u>   | <u>Instructional sequence</u>   | <u>Resources</u>  |
|--|---|---|
| <ul style="list-style-type: none"> <li>Participants will recognize the Learning Trajectory level an instructional activity is designed to target.</li> </ul> | <ol style="list-style-type: none"> <li>Introduce the part by watching Video A.</li> <li>Watch and discuss Videos B and C.</li> <li>Watch and discuss Videos D and E.</li> </ol> | <ul style="list-style-type: none"> <li>Video A (00:49): Using rulers</li> <li>Video B (01:12): Measurement olympics task Video C (00:21): Commentary on measurement olympics</li> <li>Video D (00:46): Missing measures and estimation tasks</li> <li>Video E (01:07): Commentary on missing measures and estimation</li> <li>Handout: Content cube – Length Learning Trajectory</li> </ul> |
|  |   | <u>Supplements</u>  |
|  |   | <ul style="list-style-type: none"> <li>Video (02:09): An example of a Consistent Length Measurer level</li> </ul>   |

| Detailed description of activity  | Comments & other resources   |
|---|--|
| <p>1. Introduce Part 4: In this part, participants will discuss instructional tasks that are designed to target later levels of the Learning Trajectory for length measurement. As part of this work, they will discuss how to support students’ meaningful use of rulers.</p> <p>Launch this part by watching Video A, in which Dr. Clements and Dr. Sarama talk about how to introduce rulers into the classroom in ways that support students’ growth through the Learning Trajectory.</p> | <p><i>Participants can continue to use Handout: Content cube – Length Learning Trajectory as a reference during this part.</i></p> |



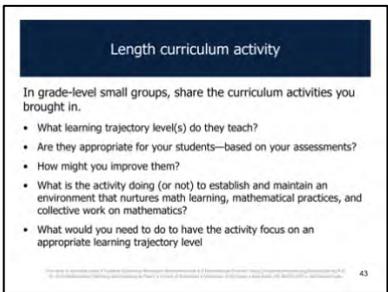
| Detailed description of activity   | Comments & other resources  |
|--|---|
| <p>2. Have participants watch Video B, in which Dr. Sarama introduces “measurement club” and talks about the straw javelin throw that was an activity within a measurement olympics held by the club. Then discuss the question: “What level is this developing?”</p> <div data-bbox="785 297 1167 586" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;"><b>Measurement Club</b></p> <ul style="list-style-type: none"> <li>• Learning about measurement through informal activities</li> <li>• Examples of activities:               <ul style="list-style-type: none"> <li>– Measure the hallway two ways</li> <li>– Measurement Olympics</li> </ul> </li> <li>• What learning trajectory level(s) might measurement club activities develop?</li> </ul> <p style="text-align: right; font-size: small;">34</p> </div> <p>After participants have had time to discuss, watch Video C, in which Dr. Sarama shares that this activity was designed to target the Consistent Length Measurer level because it requires students to estimate and involves accumulation of distance.</p> <div data-bbox="785 602 1167 891" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;"><b>Consistent Length Measurer (CLM)</b></p> <p>Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers.</p> <ul style="list-style-type: none"> <li>• Begins to estimate</li> <li>• Considers the length of a bent path as the sum of its parts (not the distance between the endpoints)</li> <li>• “I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it’s 3 meters, 14 centimeters in all”</li> </ul> <p style="text-align: right; font-size: small;">37</p> </div> | <p><i>The supplements section includes a video that illustrates the Consistent Length Measurer level (Video: An example of Consistent Length Measurer level). If needed, use this video to provide participants with a reminder of the features that characterize this level.</i></p>   |
| <p>3. Have participants watch Video D, in which Dr. Clements presents the missing measures activity and then talks about estimation activities.</p> <div data-bbox="296 995 678 1284" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;"><b>1. Missing measures</b></p> <p>Students figure out the measures of figures using given measures. This is an excellent activity to conduct on the computer using Logo’s turtle graphics or Scratch.</p>  <p style="text-align: right; font-size: small;">38</p> </div> <div data-bbox="726 995 1108 1284" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;"><b>2. Estimation</b></p> <p>Students learn explicit strategies for estimating lengths, including</p> <ul style="list-style-type: none"> <li>• Developing benchmarks for units (e.g., an inch-long piece of gum) and</li> <li>• Composite units (e.g., a 6-inch dollar bill) and mentally iterating those units</li> </ul> <p style="text-align: right; font-size: small;">39</p> </div> <p>Again, take time to discuss the levels that these tasks are designed to target.</p>  | <p>The key understandings associated with the Conceptual Ruler Measurer level are</p> <ol style="list-style-type: none"> <li>1. “Mentally moves along an object, segmenting it, and counting the segments”: This involves being able to see the individual segments of a path, as well as the whole path itself, in order to determine the length of a missing piece. This is targeted by the Missing Measures task.</li> <li>2. Estimates with accuracy: This is targeted by the Estimation task.</li> </ol> <p>The estimation slide suggests units that could be used to estimate distances/lengths. Tasks that would support estimation are long enough to discourage iterating to find exact measurements, while also being short enough to make formulation of estimates reasonable.</p> |

| Detailed description of activity   | Comments & other resources |
|--|----------------------------|
| <p>4. Conclude this part by having participants watch Video E, in which Dr. Clements and Dr. Sarama talk about the Conceptual Ruler Measurer level and developing an internal measurement tool from the concepts in the earlier levels of the Learning Trajectory.</p> <div data-bbox="787 354 1171 639" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center; background-color: #1a3d4d; color: white; margin: 0;">Conceptual Ruler Measurer (CRM)</p> <p>Possesses an "internal" measurement tool. Mentally moves along an object, segmenting it, and counting the segments.</p> <ul style="list-style-type: none"> <li>• Estimates with accuracy               <ul style="list-style-type: none"> <li>– "I imagine one meter stick after another along the edge of the room. That's how I estimated the room's length is 9 meters"</li> </ul> </li> <li>• Operates arithmetically on measures ("connected lengths")</li> </ul> <p style="font-size: small; margin-top: 10px;">© 2018 Mathematics Teaching and Learning to Teach • School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • mtl@umich.edu 41</p> </div> |                            |

**Part 5: Length in the school curriculum (~10 minutes)**

| <u>Goals</u>  | <u>Instructional sequence</u>   | <u>Resources</u>   |
|---|---|--|
| <ul style="list-style-type: none"> <li>Participants will connect activities in their curriculum to the Learning Trajectory levels.</li> <li>Participants will revise an activity to target the level of the students in their classroom.</li> </ul> | <ol style="list-style-type: none"> <li>Introduce Part 5 by showing Video A; have participants discuss their curriculum activities in grade-level groups.</li> <li>Share in whole group, watching Videos B-D as time and interest permit.</li> </ol> | <ul style="list-style-type: none"> <li>Video A (01:28): Sharing curriculum activities</li> <li>Video B (00:48): Using assessment to select tasks</li> <li>Video C (00:54): Thinking instructionally about levels</li> <li>Video D (02:03): Reconciling where students are at with what is in the curriculum</li> <li>Handout: Content cube – Length Learning Trajectory</li> </ul> |

| Detailed description of activity  | Comments & other resources   |
|---|--|
| <p>1. Introduce Part 5: In this part, participants will discuss activities in their curriculum and the ways in which they connect with the levels of the Learning Trajectory for length measurement. Show Video A, where Dr. Sarama and Dr. Clements introduce this activity.</p> <p>In grade-level small groups, have participants discuss the activities they brought, focusing on the following questions:</p> <ul style="list-style-type: none"> <li>What Learning Trajectory level(s) do they target?</li> <li>Are they appropriate for your students—based on your assessments?</li> <li>How might you improve them?</li> <li>What is the activity doing (or not) to establish and maintain an environment that nurtures math learning, mathematical practices, and collective work on mathematics?</li> <li>What would a teacher need to do to focus the activity on a different and more appropriate Learning Trajectory level for his/her students?</li> </ul> | <p><i>Participants can continue to use Handout: Content cube – Length Learning Trajectory as a reference during this part.</i></p> |



| Detailed description of activity   | Comments & other resources   |
|--|--|
| <p>2. Invite participants to share what they discussed in their grade-level groups with the whole group.</p> <p>If it would be useful to support the discussion, show one or more of the following videos:</p> <ul style="list-style-type: none"> <li>• Video B: Using assessment to select task</li> <li>• Video C: Thinking instructionally about levels</li> <li>• Video D: Reconciling where students are at with what is in the curriculum</li> </ul> | <p><i>Video B: Using assessment to select task</i></p> <p><i>In this video, a participant suggests that the majority of her students are not yet at the level that the lesson she selected is targeting. In response, Dr. Sarama emphasizes the importance of identifying students' levels of thinking and then starting from there to help them move forward.</i></p> <p><i>Video C: Thinking instructionally about levels</i></p> <p><i>In this video, Dr. Clements discusses with a group how they have identified a task in their curriculum that is targeted above the End-to-End Length Measurer level, which is the level their students are currently working toward.</i></p> <p><i>Video D: Reconciling where students are at with what is in the curriculum</i></p> <p><i>In this video, Dr. Sarama begins a discussion about how students in their classrooms may not at the levels targeted by the activities in the curriculum. She continues with how to make small adjustments in the curriculum to build understanding, as well as to use the assessment to help identify students' levels of thinking to better target instruction.</i></p> |

**Part 6: Wrap up (~10 minutes)**

**Goals**

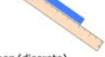
- Participants will understand how understanding length measurement forms a foundation for other measurement domains.
- Participants will understand how understanding length measurement forms a foundation for other measurement domains.
- Participants will recall the work they have done in the first three sessions.
- Participants will understand the Classroom Connection Activity assignment.

**Instructional sequence**

1. Introduce Part 6 and review key concepts of length measurement.
2. Summarize the work of the session and preview the focus of the upcoming sessions.
3. Explain and distribute the Classroom Connection Activities.

**Resources**

- Video A (03:20): Reconnecting with principles of measurement
- Classroom Connection Activity 3 – Length
- Handout: Measurement concepts – Length

| Detailed description of activity   | Comments & other resources  |
|--|---|
| <p>1. Introduce Part 6: This part provides a review the key concepts of length measurement and summarizes the session.</p> <p>Watch Video A, where Dr. Clements and Dr. Sarama review the mathematics of length measurement.</p> <div data-bbox="268 587 655 878" style="border: 1px solid black; padding: 5px;"> <p>Measurement concepts – Length (Part 1)</p> <p><b>Understanding of the attribute of length involves:</b></p> <p>Conservation </p> <p>Transitivity </p> <p>Equal partitioning </p> </div> <div data-bbox="697 295 1104 586" style="border: 1px solid black; padding: 5px;"> <p>Measuring length</p> <p>Measuring consists of two aspects:</p> <ul style="list-style-type: none"> <li>Identifying a unit of measure and subdividing (mentally and physically) the object by that unit, and</li> <li>Placing that unit end to end (iterating) alongside the object being measured</li> </ul> </div> <div data-bbox="697 587 1083 878" style="border: 1px solid black; padding: 5px;"> <p>Measurement concepts – Length (Part 2)</p> <p>Units and unit iteration </p> <p>Accumulation of distance and additivity </p> <p>Origin </p> <p>Relation between (discrete) number and measurement </p> </div> | <p><i>It is possible to extend the discussion of the ideas on the "measuring length" slide by generalizing to measurement in general. So for instance, you could foreshadow the work that participants will be doing with area measurement in the next sessions and even into volume by discussing:</i></p> <ol style="list-style-type: none"> <li><i>that area is quantifying how much 2-D space is covered and volume is quantifying how much 3-D space is filled.</i></li> <li><i>how identification of a unit runs across all three domains – length, area, and volume; the unit in area is a square unit and the unit in volume is a cubic unit.</i></li> <li><i>how end-to-end measurement in length parallels completely covering an area or completely filling a container with units.</i></li> </ol> <p><i>These principles are also key in measuring other attributes such as area and volume.</i></p> <p><i>There are key questions that can be asked with respect to each principle.</i></p> <ul style="list-style-type: none"> <li><i>Attribute- What is being measured?</i></li> <li><i>Conservation- Does the measurement change if I move what is being measured?</i></li> <li><i>Transitivity- How could I know how the measurements of these objects relate without directly comparing them?</i></li> </ul> <p><i>There are key questions that can be asked with respect to each principle.</i></p> <ul style="list-style-type: none"> <li><i>Equal partitioning- How can we partition this into equal sized parts?</i></li> <li><i>Units and unit iteration- How can this small unit be used to measure something so large?</i></li> <li><i>Accumulation- How many copies of this unit were used to measure this attribute?</i></li> <li><i>Origin- Where could I start the process of measuring?</i></li> <li><i>Relation between number and measurement- How does the number of units change when I use a larger unit of measure?</i></li> </ul> |

| Detailed description of activity  | Comments & other resources |
|---|----------------------------|
| <p>2. Summarize the session by emphasizing that participants:</p> <ul style="list-style-type: none"> <li>• Engaged in a workshop</li> <li>• Connecting students’ performance on measurement tasks with the Learning Trajectories</li> <li>• Considering the nature of anecdotal notes</li> <li>• Analyzed instructional activities in terms of the Learning Trajectories.</li> </ul> <p>Note that participants have now completed sessions focused on length related to the three components of the Learning Trajectory: the mathematics, the developmental progression of students’ thinking, and instructional activities designed to advance students’ thinking about length measurement.</p> <p>Explain that the focus of the next three sessions will be on area measurement. The content in those sessions will be organized in the same three-part structure as the length-focused sessions.</p> |                            |
| <p>3. Distribute the handout you customized with the Classroom Connection Activities.</p> <ul style="list-style-type: none"> <li>• Use a length measurement activity (or some portion of it) with your students             <ul style="list-style-type: none"> <li>○ Could be something from your curriculum (even the activity you brought in and modified) or something completely different</li> <li>○ Use the anecdotal notes form</li> </ul> </li> <li>• In preparation for our upcoming sessions, start looking for an activity or assessment focused on the measurement of area.</li> </ul>  |                            |

