

## Session 3: Length Learning Trajectory Instructional Activities

**Building  
Blocks**



**DTE@**  
**MATHEMATICS**



## Overview of Session 3

- Your length assessments and anecdotal notes
  - Learning from Practice Protocol
- The third part of Learning Trajectories—Instructional Activities
  - Examples along the learning trajectory
  - Activities from your curriculum
- Review of the math of length measurement
- Classroom Connection Activity

## Session 3: Length Learning Trajectory Instructional Activities

**Building  
Blocks**



**DTE@**  
**MATHEMATICS**



Test ourselves!

## Length Unit Relater and Repeater

End-to-End...or earlier!

## A protocol for anecdotal notes “workshop”

In small groups:

- “Presenters” share information about the performance of a student on a particular task
- “Colleagues” discuss
  - the tasks
  - evidence of student thinking
  - learning trajectories
  - anecdotal notes
- Reflect as a group on teaching practice
  - task selection
  - use of learning trajectory
  - anecdotal note taking

## Discussing notes focused on assessments of length

- Share the task you used to assess length and your notes in grade-level small groups
- When everyone has shared consider the following
  - What levels of thinking were made visible by the tasks?
  - What new insights or questions do you have about the learning trajectory levels?
  - How could you enhance your next use of anecdotal notes?

## Learning from practice protocol – Debriefing

Debrief in whole group:

- Insights into the learning trajectories
- Perspectives on talking with colleagues using information contained in the notes
- Ideas about anecdotal notes and the note-taking process
- Ways the protocol for anecdotal notes workshop could be enhanced

## Learning from practice

Focusing on anecdotal note taking:

- Provides a tangible and practical way to learn from experience.
- Provides an opportunity to get better at creating notes about students as learners and using notes as a resource to support conversations with colleagues and personal reflection.

## Learning trajectories in our sessions

- Goal
- Developmental Progression
- Instruction



What level is this developing?

## Indirect Length Comparer (ILC)

Compares the length of two objects by representing them with a third object

Another example:

- Compares length of two objects with a piece of string.

## Length riddles (Part 1)

Provide the students with more than 12 cubes and ask –

- I am 12 cubes long. I have bristles on me. What am I?
- Monitoring and formative assessment
  - Help: More obvious clues; guide measuring
  - Excel: More of each type of item

## Length riddles (Part 2)

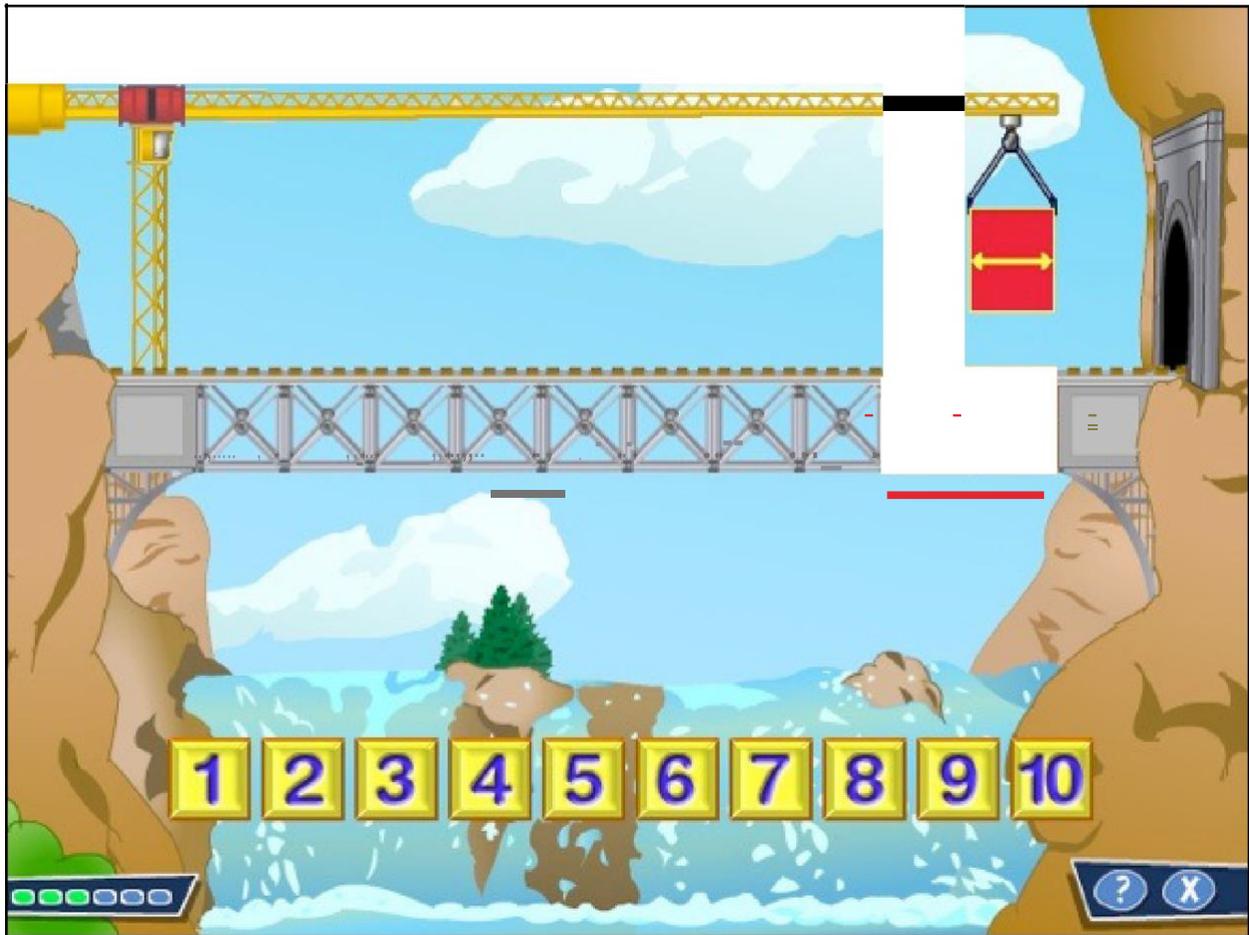
- Next time: Have at least two, but better 3-4, of every item, one of which is the cube length in the riddle
- I am 12 cubes long. You use me to write. What am I?
- Omit clues if children are doing well

This work is licensed under a Creative Commons Attribution-Noncommercial-4.0 International License: <https://creativecommons.org/licenses/by-nc/4.0/>  
© 2018 Mathematics Teaching and Learning to Teach School of Education • University of Michigan • Ann Arbor, MI 48109-1259 • [mtlt@umich.edu](mailto:mtlt@umich.edu)

What level is this developing?

## End-to-End Length Measurer (EE)

Lays units end-to-end. May not see the need for equal-length units.



## Mr. Mixup's measuring mess (Part 1)

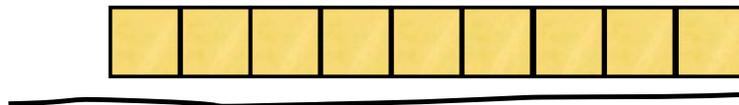
Mr. Mixup measures string with connecting cubes

- Gaps between cubes

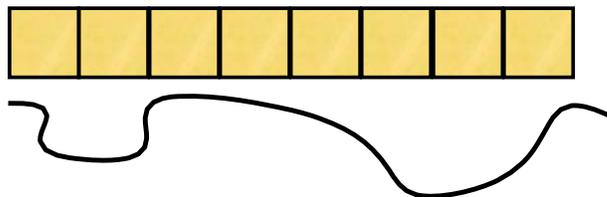


## Mr. Mixup's measuring mess (Part 2)

- String end misaligned with cubes



- String or cubes not in a straight line



## Not enough units!

- Children asked to measure their desks, but they don't have enough cubes!
- Video...

What level is this developing?

## Length Unit Relater and Repeater (LURR)

- Relates size and number of units
  - “If you measure with centimeters instead of inches, you’ll need more of them, because each one is smaller”
- Repeats a single unit to measure. Sees need for identical units. Uses rulers with guidance
  - Measures a book’s length well with a ruler

## Modify riddles and Mr. Mixup

- Make the two tasks require thinking at the “Relater and Repeater” level
- Think-pair-share
- Then we’ll share with the whole group

## Activities supporting later learning trajectory levels

Later learning trajectory levels can involve tools such as rulers

- What kinds of activities involve students in meaningful use of rulers?
- What learning trajectory level(s) might those activities develop?

## Measurement Club

- Learning about measurement through informal activities
- Examples of activities:
  - Measure the hallway two ways
  - Measurement Olympics
- What learning trajectory level(s) might measurement club activities develop?

What level is this developing?

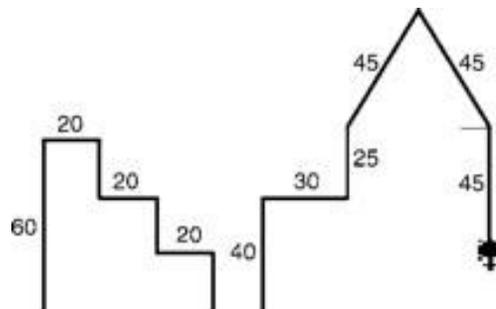
## Consistent Length Measurer (CLM)

Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers.

- Begins to estimate
- Considers the length of a bent path as the sum of its parts (not the distance between the endpoints)
- "I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it's 3 meters, 14 centimeters in all"

## 1. Missing measures

Students figure out the measures of figures using given measures. This is an excellent activity to conduct on the computer using Logo's turtle graphics or Scratch.



## 2. Estimation

Students learn explicit strategies for estimating lengths, including

- Developing benchmarks for units (e.g., an inch-long piece of gum) and
- Composite units (e.g., a 6-inch dollar bill) and mentally iterating those units

What level are these tasks targeting?

## Conceptual Ruler Measurer (CRM)

Possesses an “internal” measurement tool. Mentally moves along an object, segmenting it, and counting the segments.

- Estimates with accuracy
  - “I imagine one meter stick after another along the edge of the room. That’s how I estimated the room’s length is 9 meters”
- Operates arithmetically on measures (“connected lengths”)

## Length curriculum activity

In grade-level small groups, share the curriculum activities you brought in.

- What learning trajectory level(s) do they teach?
- Are they appropriate for your students—based on your assessments?
- How might you improve them?
- What is the activity doing (or not) to establish and maintain an environment that nurtures math learning, mathematical practices, and collective work on mathematics?
- What would you need to do to have the activity focus on an appropriate learning trajectory level

## Review of the math

- Length is a characteristic of an object and can be found by quantifying how far it is between the endpoints of the object
- Distance refers to the empty space between two points

## Measuring length

Measuring consists of two aspects:

- Identifying a unit of measure and subdividing (mentally and physically) the object by that unit, and
- Placing that unit end to end (iterating) alongside the object being measured

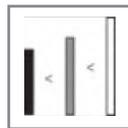
## Measurement concepts – Length (Part 1)

**Understanding of the attribute of length involves:**

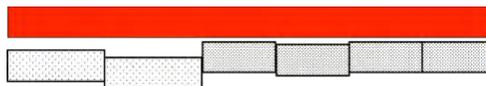
Conservation



Transitivity



Equal partitioning

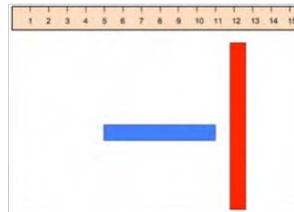


## Measurement concepts – Length (Part 2)

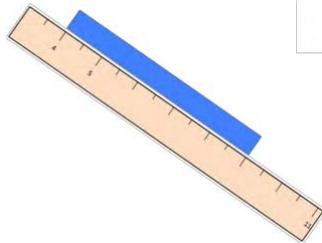
Units and unit iteration



Accumulation of distance  
and additivity



Origin



Relation between (discrete)  
number and measurement



## Summary

In this session you:

- Engaged in a workshop
  - Connecting students' performance on measurement tasks with the learning trajectories
  - Considering the nature of anecdotal notes
- Analyzed instructional activities in terms of the learning trajectories