

Session 3: Length Learning Trajectory
Instructional Activities

**Building
Blocks**



Overview of Session 3

- Your length assessments and anecdotal notes
 - Learning from Practice Protocol
- The third part of Learning Trajectories—Instructional Activities
 - Examples along the learning trajectory
 - Activities from your curriculum
- Review of the math of length measurement
- Classroom Connection Activity

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Test ourselves!

Length Unit Relater and Repeater

End-to-End...or earlier!

A protocol for anecdotal notes “workshop”

In small groups:

- “Presenters” share information about the performance of a student on a particular task
- “Colleagues” discuss
 - the tasks
 - learning trajectories
 - evidence of student thinking
 - anecdotal notes
- Reflect as a group on teaching practice
 - task selection
 - use of learning trajectory
 - anecdotal note taking

Discussing notes focused on assessments of length

- Share the task you used to assess length and your notes in grade-level small groups
- When everyone has shared consider the following
 - What levels of thinking were made visible by the tasks?
 - What new insights or questions do you have about the learning trajectory levels?
 - How could you enhance your next use of anecdotal notes?

Learning from practice protocol – Debriefing

Debrief in whole group:

- Insights into the learning trajectories
- Perspectives on talking with colleagues using information contained in the notes
- Ideas about anecdotal notes and the note-taking process
- Ways the protocol for anecdotal notes workshop could be enhanced

Learning from practice

Focusing on anecdotal note taking:

- Provides a tangible and practical way to learn from experience.
- Provides an opportunity to get better at creating notes about students as learners and using notes as a resource to support conversations with colleagues and personal reflection.

Learning trajectories in our sessions

- Goal
- Developmental Progression
- Instruction



What level is this developing?

Indirect Length Comparer (ILC)

Compares the length of two objects by representing them with a third object

Another example:

- Compares length of two objects with a piece of string.

Length riddles (Part 1)

Provide the students with more than 12 cubes and ask –

- I am 12 cubes long. I have bristles on me. What am I?
- Monitoring and formative assessment
 - Help: More obvious clues; guide measuring
 - Excel: More of each type of item

Length riddles (Part 2)

- Next time: Have at least two, but better 3-4, of every item, one of which is the cube length in the riddle
- I am 12 cubes long. You use me to write. What am I?
- Omit clues if children are doing well

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What level is this developing?

End-to-End Length Measurer (EE)

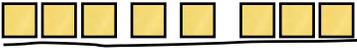
Lays units end-to-end. May not see the need for equal-length units.



Mr. Mixup's measuring mess (Part 1)

Mr. Mixup measures string with connecting cubes

- Gaps between cubes

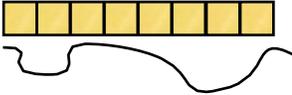


Mr. Mixup's measuring mess (Part 2)

- String end misaligned with cubes



- String or cubes not in a straight line



Not enough units!

- Children asked to measure their desks, but they don't have enough cubes!
- Video...

What level is this developing?

Length Unit Relater and Repeater (LURR)

- Relates size and number of units
 - “If you measure with centimeters instead of inches, you’ll need more of them, because each one is smaller”
- Repeats a single unit to measure. Sees need for identical units. Uses rulers with guidance
 - Measures a book’s length well with a ruler

Modify riddles and Mr. Mixup

- Make the two tasks require thinking at the “Relater and Repeater” level
- Think-pair-share
- Then we’ll share with the whole group

Activities supporting later learning trajectory levels

Later learning trajectory levels can involve tools such as rulers

- What kinds of activities involve students in meaningful use of rulers?
- What learning trajectory level(s) might those activities develop?

Measurement Club

- Learning about measurement through informal activities
- Examples of activities:
 - Measure the hallway two ways
 - Measurement Olympics
- What learning trajectory level(s) might measurement club activities develop?

What level is this developing?

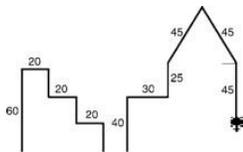
Consistent Length Measurer (CLM)

Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers.

- Begins to estimate
- Considers the length of a bent path as the sum of its parts (not the distance between the endpoints)
- "I used a meter stick three times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it's 3 meters, 14 centimeters in all"

1. Missing measures

Students figure out the measures of figures using given measures. This is an excellent activity to conduct on the computer using Logo's turtle graphics or Scratch.



2. Estimation

Students learn explicit strategies for estimating lengths, including

- Developing benchmarks for units (e.g., an inch-long piece of gum) and
- Composite units (e.g., a 6-inch dollar bill) and mentally iterating those units

What level are these tasks targeting?

Conceptual Ruler Measurer (CRM)

Possesses an "internal" measurement tool. Mentally moves along an object, segmenting it, and counting the segments.

- Estimates with accuracy
 - "I imagine one meter stick after another along the edge of the room. That's how I estimated the room's length is 9 meters"
- Operates arithmetically on measures ("connected lengths")

Length curriculum activity

In grade-level small groups, share the curriculum activities you brought in.

- What learning trajectory level(s) do they teach?
- Are they appropriate for your students—based on your assessments?
- How might you improve them?
- What is the activity doing (or not) to establish and maintain an environment that nurtures math learning, mathematical practices, and collective work on mathematics?
- What would you need to do to have the activity focus on an appropriate learning trajectory level

Review of the math

- Length is a characteristic of an object and can be found by quantifying how far it is between the endpoints of the object
- Distance refers to the empty space between two points

Measuring length

Measuring consists of two aspects:

- Identifying a unit of measure and subdividing (mentally and physically) the object by that unit, and
- Placing that unit end to end (iterating) alongside the object being measured

Measurement concepts – Length (Part 1)

Understanding of the attribute of length involves:

Conservation



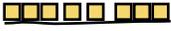
Transitivity

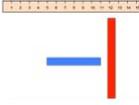


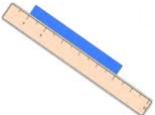
Equal partitioning

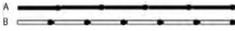


Measurement concepts – Length (Part 2)

Units and unit iteration 

Accumulation of distance and additivity 

Origin 

Relation between (discrete) number and measurement 

Summary

In this session you:

- Engaged in a workshop
 - Connecting students' performance on measurement tasks with the learning trajectories
 - Considering the nature of anecdotal notes
- Analyzed instructional activities in terms of the learning trajectories
