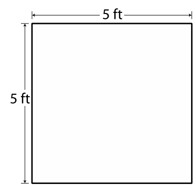
Math Notes: Ratios Comparing Length and Area Units

A ratio is a relation between two numbers. It is a comparison showing the number of times one value contains or is contained within another. Ratios are used in many situations, such as recipes, pricing, fuel economy, etc. Some ratios are fixed ratios between different but related units of measurement in a measurement system, such as the number of feet in a yard or the number grams in a kilogram. There are also ratios between standard units in different systems, like the number of centimeters in an inch or the relationship of one degree Celsius to one degree Fahrenheit.

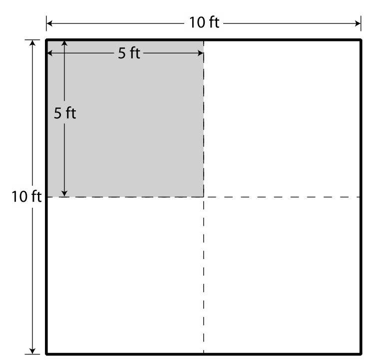
There are also ratios (fixed numerical relationships) between measurements in different dimensions. For instance, let's suppose we have the square room pictured below:



Length of a wall measured in Linear Units	Space inside the room measured in Area Units
5 ft	5 ft x 5 ft = 25 square feet

What happens if the walls in the room are doubled in length?

Linear Units	Area Units
10 ft	10 ft x 10 ft = 100 square feet



Notice the ratio of these measurements: Multiplying the linear dimensions by 2 results in a corresponding change in the area by a factor of 4. More generally, the proportion of the change in linear measurement (P) is squared when considering the proportion of the effect on the area (P^2).

Conversion of Measurements

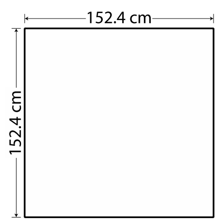
It is possible to use ratios to convert measures in one measurement system to a related measure in another system. For instance, a ratio connects the number of inches to the number of centimeters (1 inch equals 2.54 cm). This ratio can determine the number of centimeters that are equal in length to any number of inches. For instance:

Linear Units

5 ft. = 60 in x 2.54 cm/in = 152.4 cm

It is also possible to use those converted measurement to determine the area of a region, such as the room discussed earlier, in square centimeters.

Linear Units	Area Units
5 ft. = 60 in x 2.54 cm/in = 152.4 cm	152.4 cm x 152.4 cm = 23,225.76 cm^2



Errors in Measurement

How is the precision of an area measurement effected when the linear measurement is imprecise? Suppose the ratio between inches and centimeters is “rounded” to 1 in = 2.5 cm when converting a length measurement. How would this affect the precision of the length measurement?

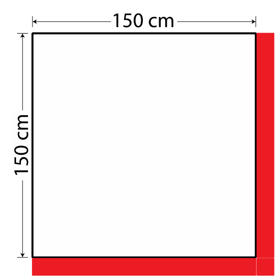
Linear Units

5 ft. = 60 in x 2.5 cm/in = 150 cm

Notice that the relatively minor rounding of the length of a centimeter by 4 hundredths of a centimeter results in the linear distance of 5 ft being 2.4 cm smaller than the actual length (imprecise by *about* 1 inch total length). This may seem relatively minor, but what would happen to the accuracy of an area measurement when the linear measurements of sides are off by as little as 2.4 cm?

Linear Units	Area Units
5 ft. = 60 in x 2.5 = 150 cm	150 cm x 150 cm = 22,500 cm^2

The effect on the total area when the linear unit measurement is off by just a little results in a pronounced difference of 725.76 cm^2. Notice in the diagram below how the errors compound in the area measurement (red indicates the region of the measurement error).



More generally, small errors in linear measurements can quickly propagate into greater imprecision when they are used to calculate areas (and an even larger greater when calculating volume!) because the imprecise linear measurement is used for each dimension of the area or volume.