Session 4: Area Learning Trajectory – Mathematical goals

Description of the session

In Session 3, participants concluded their work on the Learning Trajectory for length measurement. In this session, participants begin a sequence of work on area measurement, focusing specifically on the mathematics of area measurement. They start by measuring the area of the room using two different units. They will calculate the ratio between the two units as well as the ratio between the two measurements of the room and will discuss the issues that arise. Participants then explore how area measurement is addressed in the Common Core State Standards, and they examine a Learning Trajectory for area measurement. After that, participants consider the core concepts involved in area measurement and discuss an example of student thinking. The session closes with an overview of the Classroom Connection Activities that will be completed prior to the next session.

*Activities and goals of the session**

Activities	Times	Corresponding parts of the session	Goals
I. Overview	10 minutes	Part 1	• Participants will reflect on their students' thinking and on the notetaking practices they used when facilitating a measurement activity in their classrooms. • Participants will be oriented to the work of the session.
II. Studying the math of area measure	45 minutes	Parts 2, 3, & 4	• Participants will be able to describe their approaches to measuring area • Participants will begin to recognize and understand the concepts and skills involved in measuring area through measuring the area of the room. • Participants will recognize the issues involved with using different units to measure area. • Participants will become aware of challenges associated with area measurement.
III. Studying mathematical goals for students' learning about area measurement	30 minutes	Parts 5 & 6	• Participants will recognize and identify area measurement within the Common Core State Standards. • Participants will understand connections between the CCSS standards for area measurement across the grade levels. • Participants will recognize foundational mathematical ideas of area measurement. • Participants will recognize the principles of measurement in student work.
IV. Wrap up	5 minutes	Part 7	• Participants will understand ways of connecting the session content to their classroom.

*A conversation about a CCA from the last session is integrated into this session (in Part 1).

Classroom Connection Activities

Required
Type of task: Practice and extension of in-class work Description: Complete two area assessment tasks with 2-3 students of different (hypothesized) achievement levels, and ask students to record how they solved the tasks. Video record your interactions with students around these tasks.

Preparing for the session

- ☐ Gather materials: rulers (with both inches and centimeters) and yard/meter sticks
- ☐ Make copies as needed: Handout: Problem cards – Area (Part 4); Handout: Content Cubes – Common Core State Standards area (Part 5); Handout: Learning Trajectory display of measurement standards (Part 5); Handout: The mathematics of area measurement (Part 6)
- ☐ Customize and make copies of the Classroom Connection Activities
- ☐ Test technical setups: Internet connection, speakers, projector, document camera

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Developing an understanding of the principles of measurement

Foundational concepts of measurement include: understanding of the attribute, conservation, transitivity, equal partitioning, iteration of a standard unit, accumulation, origin, and relation between measurement and number.

- **Attribute –** understanding what is being measured
  - Key question: What is being measured?
- **Conservation –** understanding that an attribute being measured does not change when moved
  - Key question: Does the measurement change if I move what is being measured?
- **Transitivity –** understanding that a third object can be used to compare the measures of two other objects
  - Key question: How could I know how the measurements of these objects relate without directly comparing them?
- **Equal partitioning –** understanding that an attribute to be measured can be partitioned into the same-sized units
  - Key question: How can we partition this into equal sized parts?
- **Units and unit iteration –** understanding that an attribute can be measured with a smaller unit without gaps or overlaps
  - Key question: How can this small unit be used to measure something so large?
- **Accumulation –** understanding that as you iterate a unit the count represents the total of all units used
  - Key question: How many copies of this unit were used to measure this attribute?
- **Origin –** the notion that any point on a ratio scale can be used as the origin. Young children who lack this understanding often begin a measurement with "1" instead of zero.
  - Key question: Where could I start the process of measuring?
- **Relation between number and measurement –** Understanding that there is an inverse relation between the size of the unit and the number of those units in a given measure.
  - Key question: How does the number of units change when I use a larger unit of measure

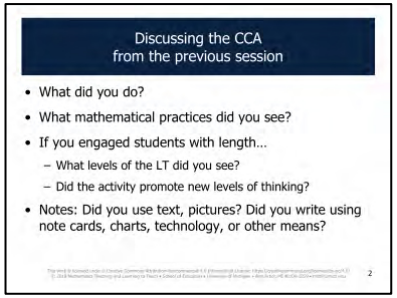
Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **recognizing the mathematical goal as the first component of a complete learning trajectory** • **understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement)** • **understanding how measurement of length, area, and volume are represented and developed in the CCSS** • **understanding how measurement connects with the CCSS standards for mathematical practice** • **understanding concepts and skills involved in measuring** length, **area**, and/or volume • understanding connections between length, area, and volume measurement and between metric measurements for each	• recognizing student development as the second component of a complete learning trajectory • understanding children's development of measurement through learning trajectories for length, area, and volume • **recognizing principles of measurement in student work** • interpreting student work on measurement tasks using the levels of the learning trajectory for length measurement • interpreting student work on measurement tasks using the levels of the learning trajectory for area measurement • interpreting student work on measurement tasks using the levels of the learning trajectory for volume measurement	• recognizing instruction as the third component of a complete learning trajectory • using anecdotal notes to document what students say and do when working on measurement tasks • connecting measurement activities in curricula to measurement learning trajectory levels • modifying measurement tasks to target different and/or particular learning trajectory levels	• understanding the anecdotal notes workshop process • using the anecdotal notes workshop to improve the practice of note taking • using the anecdotal notes workshop to improve teaching

Part 1: Overview (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will reflect on their students' thinking and on the notetaking practices they used when facilitating a measurement activity in their classrooms. • Participants will be oriented to the work of the session.	1. Discuss the CCA from the previous session. 2. Watch Video A and preview the series of sessions focused on area measurement. 3. Provide an overview of this session.	• Video A (01:47): Launching the area-focused sessions

Detailed description of activity	Comments & other resources
1. Direct the participants to have a conversation about the CCA from the previous session. After they have had a chance to talk with a partner, ask a few participants who tried a whole-class measurement activity to share their experiences. Use the following questions to guide the discussion: • What did you do? • What mathematical practices did you see? • If you engaged students with length, what levels of the learning trajectory did you see? Did the activity promote new levels of thinking? • How did you take notes?	*This portion of the session is only meant to provide participants with a chance to check in with colleagues about their CCA and to hear about some options for note taking when the teaching is happening at a whole class level. You may want participants to sit with colleagues teaching different grade levels.* *There will be other opportunities to discuss and develop these ideas through the CCAs in sessions 6 and 9.*



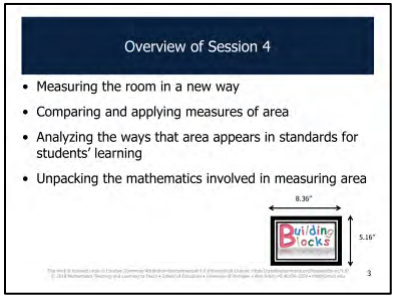
Detailed description of activity	Comments & other resources

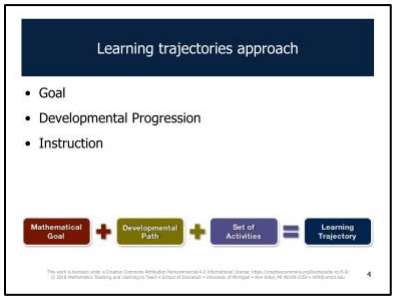
2. Introduce Part 1: In this part, participants will be oriented to the series of sessions on area measurement, and introduced to the work they will do in this session. Watch *Video A*, in which Dr. Sarama and Dr. Clements introduce area measurement and provide an overview of the session. In this session, participants will

   - Measure the room in a new way
   - Compare and apply measures of area
   - Analyze the ways that area appears in the standards for students' learning
   - Unpack the mathematics involved in measuring area

   These sessions, which are focused on area measurement will follow the same format as the sessions on length measurement, paralleling the three components of the learning trajectories:

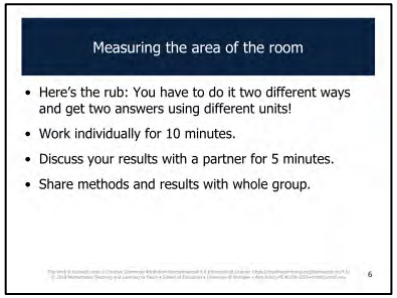
   - Mathematical goal (Session 4)
   - Developmental progression (Session 5)
   - Instructional tasks (Session 6)





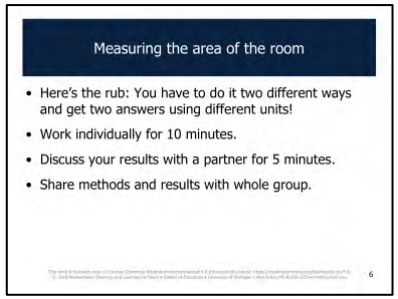
Part 2: Measuring the Area of the Room – Set Up (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will begin to recognize and understand the concepts and skills involved in measuring area. • Participants will recognize the issues involved with using different units to measure area.	1. Watch Video A and have participants work independently to measure the area of the room. 2. Have participants discuss their results with a partner.	• Video A (01:08): Measuring the area of the room

Detailed description of activity	Comments & other resources
1. Introduce Part 2 by watching *Video A* describing the work that participants will do to measure the area of the room using two different units of measure. Allow participants about 10 minutes to work individually to find the area of the room using two different units. 	*In this task, participants find the area of the room using two different units because later in the session, they will calculate the ratio between the two units as well as the ratio between the two measurements they have taken.* *Prior to the session, examine the room where the sessions will be held and consider the challenges of measuring its area and anticipating what participants might do or encounter when measuring the area.*
2. Have participants discuss their methods and results with a partner for about five minutes. Encourage participants to compare and contrast their approaches. What are the benefits, drawbacks, limitations and/or challenges they encountered?	*As they talk with partners, be listening for the ways in which participants describe their approaches and the challenges/benefits of those approaches. You will be able to draw on what you hear as you lead the discussion in Part 3.*

Part 3: Measuring the Area of the Room – Discussion (~25 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will begin to recognize and understand the concepts and skills involved in measuring area. • Participants will be able to describe their approaches to measuring area. • Participants will begin recognize the issues involved with using different units to measure area.	1. Introduce the part by watching Video A. 2. Have participants share their methods and solutions for solving the problem with the whole group; Watch Videos B-J as time and interest permit.	• Video A (00:21): Launching a discussion of methods • Video B (00:48): Method 1: Using carpet squares • Video C (02:08): Description of Method 1: Standard unit answer • Video D (01:06): Description of Method 1: Nonstandard unit answer • Video E (00:43): The challenge of irregular spaces • Video F (00:14): Method 2: Pacing the distance • Video G (00:22): Method 3: Iterating a meter stick • Video H (00:23): Method 4: Iterating an object of known length • Video I (00:28): Method 5: Using wall sections • Video J (00:11): Method 6: Using ceiling tiles

Detailed description of activity	Comments & other resources
1. Introduce Part 3 by watching *Video A*, in which Dr. Sarama invites teachers to share their methods and results for solving this problem with the whole group. 	*The following themes may emerge in this discussion:* • *Square feet/square yards/rectangular units – We often say units like "square feet" or "square yards" without really picturing a unit that actually is a square with sides of length one foot or one yard. The fact that the unit is a square is important. What would happen if the unit was rectangular, say 1 foot by 1 yard? We could say that our unit is a "foot-yard" but more commonly we convert so that both dimensions are the same linear unit, in this case feet.* • *Fractions of units – What do you do when a full unit won't fit?* • *Static versus dynamic measurement – Most commonly, we present area as a static measurement; for example, a rectangle filled with squares. There are contexts, though, were a dynamic view of area makes sense; for example, using a paint roller to paint a wall. In this case, you're not placing tiles to fill an area, but rolling a roller across and as you do so, the area becomes covered.*

Detailed description of activity	Comments & other resources
2. Ask pairs of participants to share their methods and the results of their measurement activities. Facilitate the sharing and encourage both partners to contribute to the explanation. As time and interest permit, watch a subset of *Videos B-J* (described in the right-hand column of this guide): • Video B: Method 1: Using carpet squares • Video C: Description of Method 1: Standard unit answer • Video D: Description of Method 1: Non-standard unit answer • Video E: The challenge of irregular spaces • Video F: Method 2: Pacing the distance • Video G: Method 3: Iterating a meter stick • Video H: Method 4: Iterating an object of known length • Video I: Method 5: Using wall sections • Video J: Method 6: Using ceiling tiles	*Video B: Method 1: Using carpet squares* *In this video, teachers use carpet squares as way to measure the room. They debate whether they can consider each carpet square to be 24 inches long or whether they should use the more precise measurement of 23 ¾ inches.* *Video C: Description of Method 1: Standard unit answer* *In this video, a teacher describes how she and her partner measured the room using a tile as the unit of measurement and then converted this to square centimeters.* *Video D: Description of Method 1: Non-standard unit answer* *In this video, teachers describe how they used tiles to measure the room. One considered the room to be a rectangle (and did not include the irregular parts of the room in her measurement) while the other teacher attempted to measure the area of the entire room. Dr. Sarama comments that treating the room as a rectangle may not be the best approach to use when trying to calculate its area.* *Video E: The challenge of irregular spaces* *Dr. Sarama acknowledges that it is important not to ignore the irregular nature of the room when attempting to measure its area. She then talks about the implications of this for designing area measurement tasks for students.* *Video F: Method 2: Pacing the distance* *A teacher paces the distance of the room and then records the number of steps he took.* *Video G: Method 3: Iterating a meter stick* *A teacher iterates a meter stick along the wall of the room and mentally keeps track of the number of iterations.* *Video H: Method 4: Iterating an object of known length* *A teacher iterates a set of markers along the wall of the room and mentally keeps track of the units.* *Video I: Method 5: Using wall sections* *In this video, teachers measure the length of the room by measuring the length of one section of the wall and then counting how many sections there are altogether.* *Video J: Method 6: Using ceiling tiles* *In this video, teachers examine the ceiling tiles in an attempt to use them as a unit for measuring the area of the room.*

Part 4: Making Sense of Measures of Area (~10 minutes)

Goals

- Participants will become aware of challenges associated with area measurement.

Instructional sequence

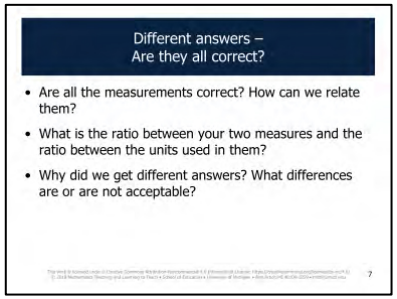
1. Watch Video A and give participants time to work individually or with partners.
2. Discuss participants' answers and watch Videos B and C if time and interest permit.
3. Give participants time to reflect on their learning; share responses and watch Videos D and E if applicable.
4. *Optional:* Have participants work on the "Applying Measures" activity.

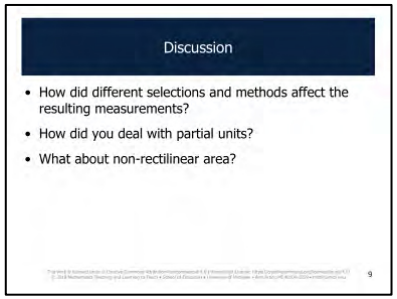
Resources

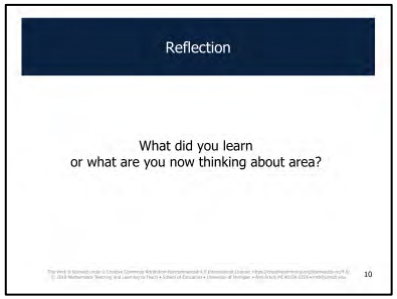
- Video A (01:00): Finding the ratio between area measurements
- Video B (01:36): The challenge of converting between metric and standard units
- Video C (01:41): Making sense of discrepant measurements
- Video D (01:05): Reflection: Substantial mathematics in area
- Video E (00:24): Reflection: Thinking in squares, not in lines

Supplements

- Slide: Applying measures
- Handout: Problem cards – Area

Detailed description of activity	Comments & other resources
1. Introduce Part 4 by watching *Video A,* where Dr. Clements and Dr. Sarama ask teachers to determine how the ratio between two measures of area they calculated relates to the ratio between the two units used in those measures.  Give participants time to work individually or with a partner to answer the following questions: • Are all the measurements correct? How can we *relate* them? • What is the ratio between your measures and the ratio between the units? • Why did we get different answers; what differences are and are not acceptable?	*This purpose of this activity is to help participants recognize the connection between linear units and area units. For example, some participants might have measured the room using feet and inches. The ratio between inches and feet is 12:1, but the ratio between square inches and square feet is 144:1. This is something we usually do not think about, but it important in really understanding the connection between length and area. We often use the area formula to calculate the area using two lengths, but don't think about how that affects the ratio between the units.* *The ratios of the actual measurements are likely to be different from the true ratio between the units. This often occurs due to choices that participants make when measuring with the different units (e.g. partial units). They may also not be aware of the true ratio between units or factor that into their thinking when finding the ratio of actual measurements. This could be suggested to participants who complete the task early or brought out in the whole group discussion later.*

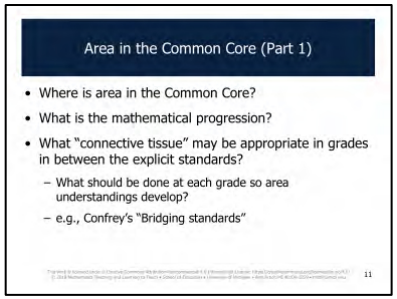
Detailed description of activity	Comments & other resources
2. After participants have had an opportunity to calcualate the ratio between their measures and the ratio between the units, facilitate a whole-group discussion of this work. Focus the discussion on the following questions:  • How did different selections and methods affect the resulting measurements? • How did you deal with partial units? • What about non-rectilinear area? As time and interest permit, consider showing *Video B* and/or *Video C* to support this discussion.	*The slide that includes the discussion questions will appear in the right side of the viewer at the end of Video A. After watching the video, click on the slide to project it while you are facilitating a discussion of these questions.* *The purpose of this discussion is, again, the help participants think more deeply about area.* • **How did the different selections and methods affect the resulting measurement?** We all measured the same room, but did we all arrive at the same area? This is a good way to come back to measurement error. Did we leave gaps? Did we accurately measure the object we were using to measure with? Additionally, some participants will measure in feet/inches and then convert to centimeters as their second measure. Did we use an exact conversion ratio (e.g., 1 inch = 2.54 cm)? What happens when we estimate the conversion ratio (e.g., 1 inch = 2.5 cm)? With each inch, our conversion is off 0.04 cm, so the error propagates multiplicatively. • How did you deal with partial units? • **What about non-rectilinear area?** Students aren't always sure what to do with a circle, for example. Does it really make sense for the unit used to measure the area of a circle to be a "square" unit? Children, when presented with a circle, will often want circular units to cover, but don't think about the gaps that will be left if we try to cover with circles. *Video B: The challenge of converting between metric and standard units* *A teacher explains that she and her partner converted feet into centimeters and tried to compare that ratio with the ratio between the two squared measurements of the room that they had calculated (square feet and square centimeters). There is a large discrepancy between the ratios, and the teachers immediately think that they must have done something wrong. Dr. Sarama then points out that the ratio between inches and feet is not the same as the ratio between square inches and square feet.* *Video C: Make sense of discrepant measurements* *The teacher from Video B predicts that, if her students encountered the type of discrepancy she and her partner encountered, they may not recognize this is an issue. Dr. Sarama and Dr. Clements respond to this comment, explaining that, in the third session on area measurement, they will discuss the kind of teaching that can support students in making sense of discrepant measurements. They also point out this problem is designed for use with adults and that they would provide more support and structure if using this type of problem with students.*

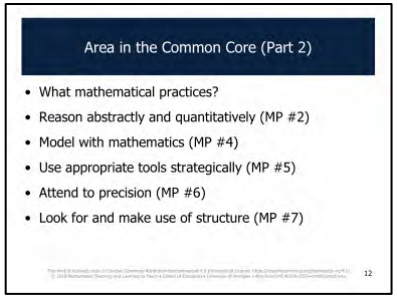
Detailed description of activity	Comments & other resources
3. After the whole-group discussion, give participants two or three minutes to reflect individually about what they are learning about area measurement. Encourage participatants to write down their thinking.  Then, take a few minutes to allow participants to share their reflections. If it would be useful, watch *Video D* and/or *Video E.*	*The slide with the reflection question is available in the resources section. It is also available in the right-hand side of the viewer at the end of Videos D and E. Videos D and E provide examples of what teachers may take out of the activity and might be interesting for participants to hear.* *Video D: Reflection: Substantial mathematics in area* *In this video, a teacher comments about the scope of mathematical understandings and skills that are involved in measuring area, including knowledge of standard units, conversion, multiplication, division, linear measurement, and squaring.* *Video E: Reflection: Thinking in squares, not in lines* *In Video E, a teacher acknowledges that he had difficulty with the task because he slipped into linear thinking rather than thinking about squared units.*
4. *Optional:* If there is an interest in the group for further exploring the measurement of area (probably at a later time), there is a supplemental slide called "Applying measures" that introduces this activity. A handout is also available in the supplements to be used with this activity.	*These activities illustrate the idea of dynamic measurement. Participants take the measurements they know and think about how those measurements interact with real life scenarios in which people create areas as they unroll carpet or paint walls.*

Part 5: Analyzing Area in Standards for Student Learning (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will recognize and identify area measurement within the Common Core State Standards (CCSS). • Participants will understand connections between the CCSS standards for area measurement across the grade levels.	1. Introduce the part by watching Video A. 2. Have participants explore how Common Core State Standards address area measurement. 3. Discuss in whole group, using Videos B-F as time and interest permit.	• Video A (01:07): The measurement of area in the Common Core • Video B (00:58): The number and operation aspects of measurement • Video C (01:12): Connecting measurement standards with grade levels • Video D (00:26): Using standards to support readiness • Video E (01:45): Connecting standards and learning trajectories • Video F (02:20): Working on mathematical practices through measurement • Handout: Content cubes – Common Core State Standards area • Handout: Learning trajectory display of measurement standards

Detailed description of activity	Comments & other resources
1. Introduce Part 5: In this part, participants explore the development of area measurement in the Common Core State Standards. Watch *Video A,* where Dr. Clements introduces the activity and poses questions participants should consider as they examine these documents, including: • Where is area in the Common Core? • What is the mathematical progression? • What "connective tissue" may be appropriate in grades in between the explicit standards?	*During this part, consider showing another representation of the Common Core State Standards that was developed by Jere Confrey, Kenny Nguyen, and colleagues. This representation, which uses a hexagon map to organize learning trajectories around the Common Core State Standards, can be found on the following webpage:* • Homepage and descriptors: Confrey, J., Nguyen, K. H., Lee, K., Panorkou, N., Corley, A. K., and Maloney, A. P. (2012). Turn-On Common Core Math: Learning Trajectories for the Common Core State Standards for Mathematics. URL: http://www.turnonccmath.net/ *The TurnOnCCMath website is provided here as an interesting resource for enrichment, not as an integral part of the session. Dr. Confrey and her team started with the Common Core State Standards and developed the hexagon map on the website as a way of showing connection among the Standards. In addition, Dr. Confrey and her team identified "holes" in the progression within the Standards. That is, they looked at the children's development research and found areas where what was identified in the research as being developmental steps in children's understanding were missing from the CCSS. With this in mind, they developed what they call "Bridging Standards" to fill in these holes for educators. These can be found on the website for each progression.*





Detailed description of activity	Comments & other resources
2. Distribute *Handout: Learning trajectory display of measurement standards* and *Handout: Content Cubes – Common Core State Standards Area.* Encourage participants to explore how area measurement is addressed in the Common Core State Standards and to write down any questions, comments, observations, or concerns they have as they examine these documents.	*The Handout: Learning trajectory display of measurement standards was also used in Session 1, so participants may already have a copy of this handout.* *If participants need more focus, suggest they example one of the following:* • *How is area treated in Grade 3? Focus on the treatment of formula and transitions from counting squares to formulas* • *How are and perimeter treated in Grade 4? Focus on the relative importance of unit conversion and the connection of area to the standards for length measurement.*

Detailed description of activity	Comments & other resources
3. After participants have had time to review the handouts and record their ideas, encourage them to share what they noticed in the Common Core State Standards. Toward the end of this discussion, perhaps in connection with *Video F*, explicitly identify mathematical practices that can be worked on in the context of area measurement, including • Reason abstractly and quantitatively (MP #2) • Model with mathematics (MP #4) • Use appropriate tools strategically (MP #5) • Attend to precision (MP #6) • Look for and make use of structure (MP #7) If applicable, show one or more of the following *videos* (described in the right-hand column of this guide) to support the discussion: • *Video B*: The number and operation aspects of measurement • *Video C*: Connecting measurement standards with grade levels • *Video D*: Using standards to support readiness • *Video E*: Connecting standards and learning trajectories • *Video F*: Working on mathematical practices through measurement	*One of the purposes of this discussion is to help participants recognize how work on area measurement (and measurement more generally) can provide rich opportunities to focus on several of the mathematical practices. Working with students on measurement is not only important because it corresponds with particular content standards, but it also provides an important context for work on the standards of mathematical practice. (Dr. Clements gives specific examples of this in Video F.)* *It may help to dig into one of these practices more specifically. For instance, try focusing on MP #2 Reasoning abstractly and quantitatively and link back to the measuring of the area of the room. In particular note the quantities that were involved and why the units are changing.* *Video B: The number and operation aspects of measurement* *In this video, a teacher talks about how the Grade 4 standard that asks students to apply the formula for area (CCSS.Math.Content.4.MD.A.3) seems to focus more on number and operations than on measurement. Dr. Clements talks about how students' experiences prior to their work on this standard may influence the extent to which they connect the formula to a conceptual understanding of area measurement.* *Video C: Connecting measurement standards with grade levels* *In this video, Dr. Clements talks about how the Common Core State Standards came to be organized the way they are. He also makes the point that students' levels of thinking depend on their learning experiences. In other words, children's "readiness" to learn certain concepts and skills does not depend on their age or stage of natural development, but rather on the prior learning experiences they have had.* *Video D: Using standards to support readiness* *Dr. Clements explains that the Common Core Standards are designed to support students' "readiness" to learn certain concepts at certain grade levels by providing them (in the previous grade levels) with the foundational experiences they will need.* *Video E: Connecting standards and learning trajectories* *Dr. Sarama explains how the learning trajectories can be a resource for teachers as they seek to support students in their class who are at different developmental levels.* *Video F: Working on mathematical practices through measurement* *Dr. Clements gives examples of how work on measurement connects to specific standards for mathematical practice.*

Part 6: The Mathematics of Area Measurement (~15 minutes)

Goals

- Participants will recognize foundational mathematical ideas of area measurement.
- Participants will recognize the principles of measurement in student work.

Instructional sequence

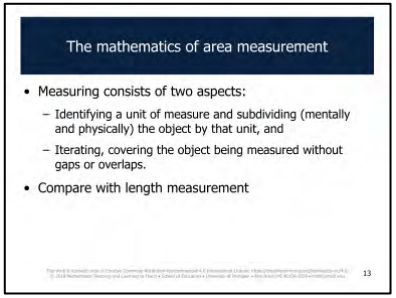
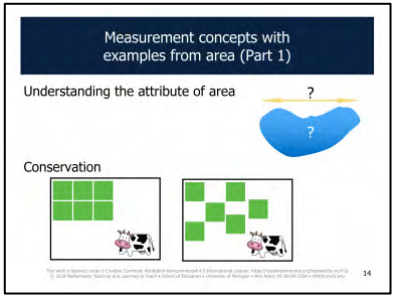
1. Introduce Part 6 by watching Video A and distributing the Handout.
2. Play Video B and discuss it in partners.
3. Discuss Video B in whole group and show Video C if time permits.
4. *Optional:* Have participants examine the perimeters of shapes with an area of 12 square units.

Resources

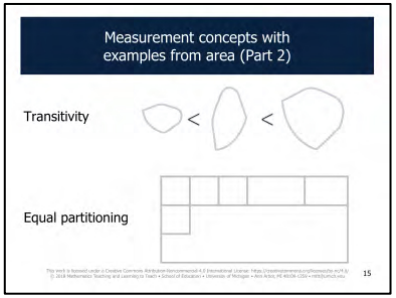
- Video A (03:05): The mathematics of area
- Video B (00:53): A student's thinking about perimeter and area
- Video C (01:21): Response to the student's thinking: Counting squares
- Handout: The mathematics of area measurement

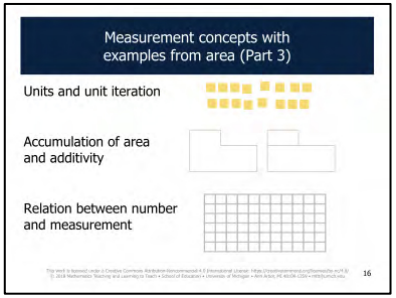
Supplements

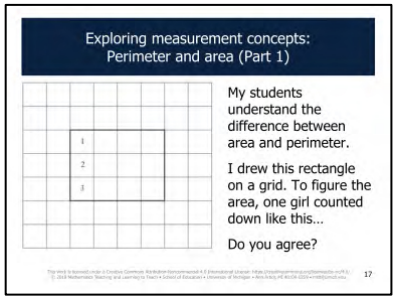
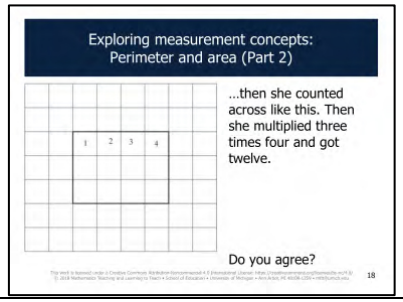
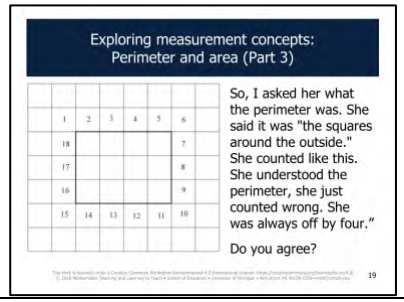
- Video (00:19): Perimeters of shapes with an area of 12 square units

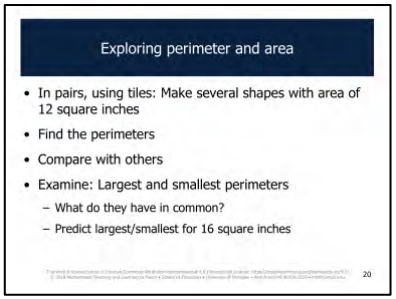
Detailed description of activity	Comments & other resources
1. Introduce Part 6: In this part, participants will examine how the principles of measurement apply to area measurement. Watch *Video A*, where Dr. Clements explains the mathematics of area measurement and compares it with length measurement. Also distribute *Handout: The mathematics of area measurement* at this time. In this video, Dr. Clements discusses how the following concepts relate to area measurement: • Attribute • Conservation of area	 	*Measurement, in general, involves (1) identifying a unit and subdividing the object by that unit, and (2) iterating that unit without gaps or overlaps. In length, only one dimension is being considered, so the object and unit are linear. In area, the same two aspects of measurement are involved, but in the case of area, two dimensions are being considered.* *Descriptions of these principles of measurement are included at the beginning of this guide.* *Consider pausing the video after the points on each slide are addressed to see if participants have comments or questions about those ideas.* *Understanding the attribute of area: Area is not the distance across an object, but instead the amount of surface or space covered by an object.* *Conservation: Recognizing that moving the "units" does not change the amount of total area covered.*

Detailed description of activity	Comments & other resources
• Transitivity • Equal partitioning • Units and unit iteration • Accumulation of area and additivity • The relation between number and measurement	*Transitivity: If one area is less than a second and the second area is less than a third, then the first area is less than the third area.* *Equal partitioning: Understanding that an area must be partitioned into equal-size units for the measurement to make sense is an important concept.* *Units and unit iteration: To measure area, the units must completely fill the area; there can be no gaps between units or overlaps of units.* *Accumulation of area and additivity: Understanding that an area can be broken into pieces, the area of each piece can be found, and those areas added together to equal the area of the whole object. This is important when finding the area of irregular shapes, such as non-rectangular rooms.* *Relation between number and measurement: The relationship between area and multiplication is valuable. Area is a good way to understand multiplication, but also multiplication is a good way to understand area. The second piece is often overlooked.*



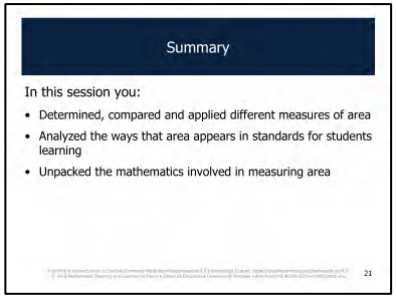


Detailed description of activity	Comments & other resources
2. Play *Video B*, where Dr. Clements describes a student's strategy for solving an area and perimeter task. He then describes a teacher's interpretation of the student's understanding and invites teachers to think-pair-share about whether or not they agree. After watching this video, give participants an opportunity to think about this question and talk with a partner about it.    	
3. Discuss the teacher's interpretation of the student's work in whole group. If time and interest permit, show *Video C* to support the discussion.	*Video C: Response to the student's thinking: Counting squares* *In this video, a teacher wonders whether asking students to use square tiles (or squares) to measure length can lead to confusion. Dr. Sarama points out that, in this particular case, the student could have used squares to measure length if she had started and stopped counting at the ends of each side of the rectangle. Instead, she appears to be trying to use areas (i.e., the areas of the squares surrounding the rectangles) to solve a perimeter problem. This is a key point because tiles are often used to measure side lengths without accompanying language that makes clear that the side of the square is the feature that is being attended to, and not its area.*

Detailed description of activity	Comments & other resources
4. *Optional:* If time permits, have participants work on the following task. In pairs, participants should use tiles to make different shapes with an area of 12 square inches. They should then • Find the perimeters • Compare with colleagues • Examine which shapes have the largest and smallest perimeters to o Determine what they have in common o Predict what the largest and smallest shapes would be if the area were 16 square inches 	*In Video: Perimeters of shapes with an area of 12 square inches, Dr. Sarama gives instructions for this task. This video is included as a supplement.*

Part 7: Wrap Up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom.	1. Summarize the work of the session. 2. Introduce the assessment tasks included in the Classroom Connection Activities by watching Video A. 3. Explain and distribute the Classroom Connection Activities.	• Video A (02:37): Session Summary • Handout: Session 4 Classroom Connection Activity

Detailed description of activity	Comments & other resources
1. Summarize the session by emphasizing that participants • Determined, compared, and applied different measures of area • Analyzed the ways that are appears in standards for student learning • Unpacked the mathematics involved in measuring area 	
2. Distribute the *handout* you customized with the Classroom Connection Activities described below. Watch *Video A,* where Dr. Clements and Dr. Sarama introduce the two area assessment tasks that are included in the handout: • Piaget's area conservation task (and the suggested variation of this task) • The incomplete array task Also explain that participants may want to start looking for a lesson in their curriculum materials that is relevant to area. They will need this for Session 6.	
3. Give instructions for the Classroom Connection Activity that participants should complete before the next session. <u>Required:</u> • Complete the two provided tasks with 2-3 students of different (hypothesized) achievement levels. • Ask the students to write down and/or draw to show their thinking. • Video record this to compare the video with what students wrote or drew.	