

Session 4: Area Learning Trajectory  
Mathematical Goals

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Discussing the CCA  
from the previous session

- What did you do?
- What mathematical practices did you see?
- If you engaged students with length...
  - What levels of the LT did you see?
  - Did the activity promote new levels of thinking?
- Notes: Did you use text, pictures? Did you write using note cards, charts, technology, or other means?

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Overview of Session 4

- Measuring the room in a new way
- Comparing and applying measures of area
- Analyzing the ways that area appears in standards for students' learning
- Unpacking the mathematics involved in measuring area

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**Learning trajectories approach**

- Goal
- Developmental Progression
- Instruction

The diagram illustrates the components of a learning trajectory. It consists of four colored boxes arranged horizontally, connected by mathematical symbols. From left to right: a red box labeled 'Mathematical Goal', a plus sign, a green box labeled 'Developmental Path', a plus sign, a purple box labeled 'Set of Activities', an equals sign, and a blue box labeled 'Learning Trajectory'.

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**Area focused sessions**

- Mathematical goal: Session 4 (today!)
- Developmental progression: Session 5
- Instructional tasks: Session 6

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**Measuring the area of the room**

- Here's the rub: You have to do it two different ways and get two answers using different units!
- Work individually for 10 minutes.
- Discuss your results with a partner for 5 minutes.
- Share methods and results with whole group.

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Different answers –  
Are they all correct?

- Are all the measurements correct? How can we relate them?
- What is the ratio between your two measures and the ratio between the units used in them?
- Why did we get different answers? What differences are or are not acceptable?

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Applying measures

- Now that we have a measure, let's put it to work.
- We're refinishing the floor; pick a problem:
  - Rug: Comes in 4' widths on a roll. Where should the factory 'cut'?
  - Tiles: 18" by 18" sizes. How many tiles?
  - Painting: How many quarts? One quart covers 85 square feet. Requires 2 coats.

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Discussion

- How did different selections and methods affect the resulting measurements?
- How did you deal with partial units?
- What about non-rectilinear area?

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**Reflection**

What did you learn  
or what are you now thinking about area?

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**Area in the Common Core (Part 1)**

- Where is area in the Common Core?
- What is the mathematical progression?
- What "connective tissue" may be appropriate in grades in between the explicit standards?
  - What should be done at each grade so area understandings develop?
  - e.g., Confrey's "Bridging standards"

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**Area in the Common Core (Part 2)**

- What mathematical practices?
- Reason abstractly and quantitatively (MP #2)
- Model with mathematics (MP #4)
- Use appropriate tools strategically (MP #5)
- Attend to precision (MP #6)
- Look for and make use of structure (MP #7)

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The mathematics of area measurement

- Measuring consists of two aspects:
  - Identifying a unit of measure and subdividing (mentally and physically) the object by that unit, and
  - Iterating, covering the object being measured without gaps or overlaps.
- Compare with length measurement

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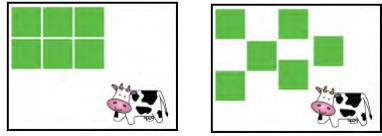
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Measurement concepts with examples from area (Part 1)

Understanding the attribute of area 

Conservation 

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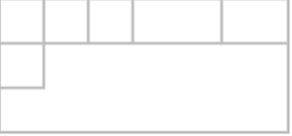
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Measurement concepts with examples from area (Part 2)

Transitivity 

Equal partitioning 

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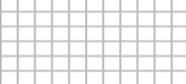
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Measurement concepts with examples from area (Part 3)

Units and unit iteration 

Accumulation of area and additivity 

Relation between number and measurement 

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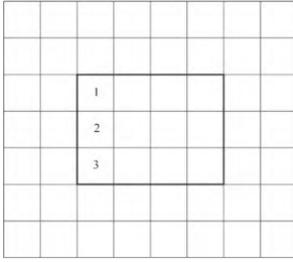
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Exploring measurement concepts: Perimeter and area (Part 1)



My students understand the difference between area and perimeter.

I drew this rectangle on a grid. To figure the area, one girl counted down like this...

Do you agree?

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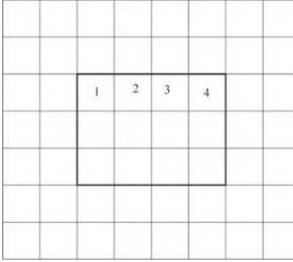
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Exploring measurement concepts: Perimeter and area (Part 2)



...then she counted across like this. Then she multiplied three times four and got twelve.

Do you agree?

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Exploring measurement concepts:  
Perimeter and area (Part 3)

|  |    |    |    |    |    |    |
|--|----|----|----|----|----|----|
|  |    |    |    |    |    |    |
|  | 1  | 2  | 3  | 4  | 5  | 6  |
|  | 18 |    |    |    |    | 7  |
|  | 17 |    |    |    |    | 8  |
|  | 16 |    |    |    |    | 9  |
|  | 15 | 14 | 13 | 12 | 11 | 10 |
|  |    |    |    |    |    |    |

So, I asked her what the perimeter was. She said it was "the squares around the outside." She counted like this. She understood the perimeter, she just counted wrong. She was always off by four."

Do you agree?

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Exploring perimeter and area

- In pairs, using tiles: Make several shapes with area of 12 square inches
- Find the perimeters
- Compare with others
- Examine: Largest and smallest perimeters
  - What do they have in common?
  - Predict largest/smallest for 16 square inches

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Summary

In this session you:

- Determined, compared and applied different measures of area
- Analyzed the ways that area appears in standards for students learning
- Unpacked the mathematics involved in measuring area

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