Session 5: Area Learning Trajectory – Developmental Progression

Description of the session

Session 5 extends the work from Session 4 that focused on studying the mathematics of area measurement. This session focuses on the second component of the Learning Trajectory for area measurement: the developmental progression of learning to measure area. First, participants will interpret their own students' performance on area tasks using what they already know about area measurement. Then, they will be introduced to the levels of the developmental progression in the area Learning Trajectory. After that, they will practice observing students' performance on measurement tasks and identifying the level at which they are performing. During this activity, they will practice using a form to structure their anecdotal note taking, and they will talk about the benefits of writing descriptive notes about a student's performance and then later going back to interpret the student's work. Finally, participants will be introduced to the Classroom Connection Activities they will complete prior to Session 6.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
I. Overview of session and discussion of CCAs	10 minutes	Parts 1 & 2	• Participants will be oriented to the work of the session. • Participants will describe their students' thinking on the area and array tasks.
II. Studying student thinking: Introduction to the developmental progression for area	60 minutes	Parts 3 & 4	• Participants will recognize and understand the early levels of the Learning Trajectory for area measurement. • Participants will recognize and understand the later levels of the Learning Trajectory for area measurement.
III. Taking anecdotal notes to identify students' Learning Trajectory levels	15 minutes	Part 5	• Participants will demonstrate understanding of the Learning Trajectory for area measurement. • Participants will practice and understand the benefit of taking descriptive notes during an interaction with a student and then using those to later interpret the student's thinking.
IV. Wrap up	5 minutes	Part 6	• Participants will understand ways of connecting the session content to their classroom.

Classroom Connection Activities

Required
Type of task: Assessment task/Collecting records of practice Description: Complete area tasks with 3-4 students of different (hypothesized) achievement levels. Ask the students to write down or draw how they measured. Use the anecdotal notes form to record how students engage in the task and the Learning Trajectory level that is associated with the students' performances. Respond to the reflection questions listed on the CCA handout.
Type of task: Analysis of curriculum materials Description: Select a lesson or a short sequence of instructional activities from your curriculum that focus on measuring area. Bring these materials with you to the next session.

Preparing for the session

☐ Make copies as needed:

- *Resources:* Handout: Comment Cubes – Common Core State Standards Area (Part 2); Handout: Content cube – Area Learning Trajectory (Parts 3 & 4); Handout: Anecdotal notes form – Area Learning Trajectories (Parts 4 & 5)
- *Supplemental resources:* Math notes: Instances of equivalence in mathematics (Part 6)

☐ Customize the Classroom Connection Activities and make copies as needed

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

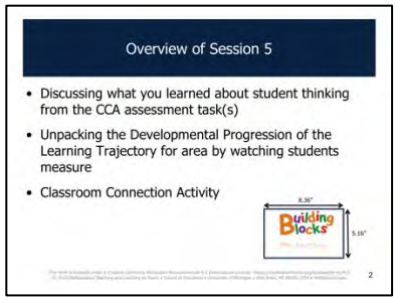
1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learning from their own teaching.
6. Help participants understand connections between module content and the Common Core Standards for School Mathematics.

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• recognizing the mathematical goal as the first component of a complete learning trajectory • understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement) • understanding how measurement of length, area, and volume are represented and developed in the CCSS • understanding how measurement connects with the CCSS standards for mathematical practice • understanding concepts and skills involved in measuring length, area, and/or volume • understanding connections between length, area, and volume measurement and between metric measurements for each	• **recognizing student development as the second component of a complete learning trajectory** • **understanding children's development of measurement through learning trajectories for length, area, and volume** • recognizing principles of measurement in student work • interpreting student work on measurement tasks using the levels of the learning trajectory for length measurement • **interpreting student work on measurement tasks using the levels of the learning trajectory for area measurement** • interpreting student work on measurement tasks using the levels of the learning trajectory for volume measurement	• recognizing instruction as the third component of a complete learning trajectory • **using anecdotal notes to document what students say and do when working on measurement tasks** • connecting measurement activities in curricula to measurement learning trajectory levels • modifying measurement tasks to target different and/or particular learning trajectory levels	• understanding the anecdotal notes workshop process • using the anecdotal notes workshop to improve the practice of note taking • using the anecdotal notes workshop to improve teaching

Part 1: Module preview (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the session.	1. Introduce the session by watching Video A.	• Video A (01:06): Session overview.

Detailed description of activity	Comments & other resources
1. Introduce the session by watching Video A. This session continues work on area measurement by focusing on the developmental progression portion of the Learning Trajectory for area measurement. In this session, participants will • Discuss what they learned about student thinking from the Classroom Connection Activity assessment task(s) • Unpack the developmental progression of the Learning Trajectory for area by watching students measure • Learn about the Classroom Connection Activity that they will complete before the next session 	

Part 2: Sharing experiences from the Classroom Connection Activity (~5 minutes)

Goals

- Participants will describe their students' thinking on the area and array tasks.

Instructional sequence

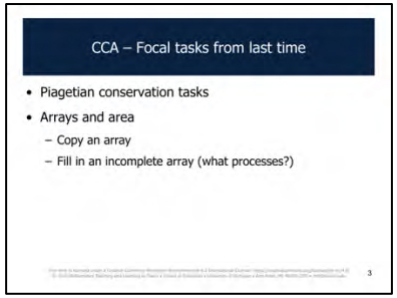
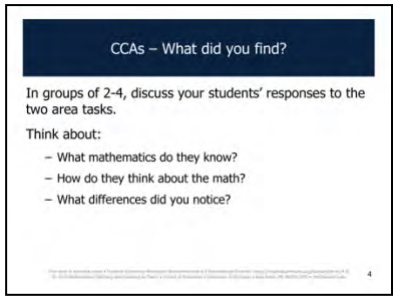
1. Watch Video A and have participants share their students' responses on the area and array tasks.
2. Watch Video B and facilitate a whole-group discussion of students' thinking about these tasks.

Resources

- Video A (00:50): Discussing the mathematics revealed through the CCA
- Video B (02:27): Focusing on what students are thinking
- Handout: Content cubes – Common Core State Standards area

Supplemental

- Math notes: Instances of equivalence in mathematics

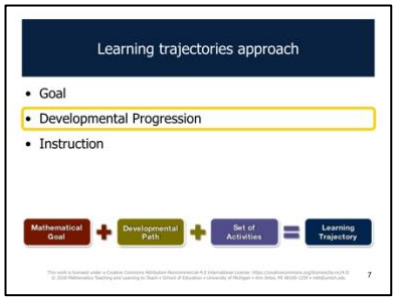
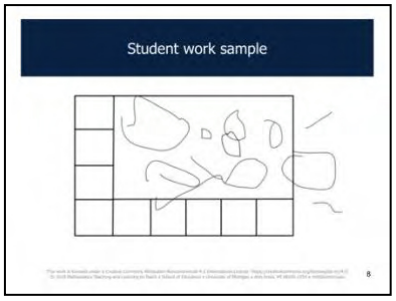
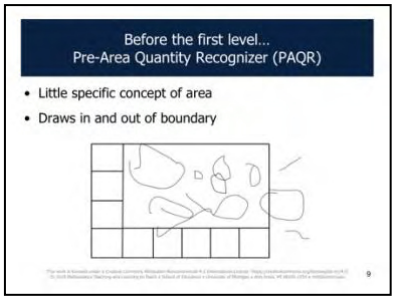
Detailed description of activity	Comments & other resources
1. Introduce Part 2 by showing Video A, in which Dr. Sarama reviews the tasks included in the Classroom Connection Activity from the previous session and launches a discussion of students' work on these tasks. Allow participants a few minutes to discuss their students' thinking in small groups.   	*This is meant to be a quick launch activity for work in this session to understand the developmental progression in area, so do not take time for an in-depth discussion. The main goal is for participants to connect their own students to the idea of area measurement.* *Consider giving participants time to share one of their videos or an example of a students' work in their small groups.* *Grouping teachers in cross-grade level groups could be productive for this portion of the session to provide an opportunity to see that grade may not be a determining factor in what students do, but also to hear how different aspects of the task connect to different grade level content.* *Listen to participants describe their implementation of tasks and the extent to which they were ensuring that students had opportunities to grapple with the ideas in the tasks. It is key for participants to provide students' space to try out their ideas or formulate ideas when engaging with the measurement tasks. It can be difficult for teachers to avoid jumping in too early to "support the student", but it is necessary in order to gauge the students own ideas about measurement.*

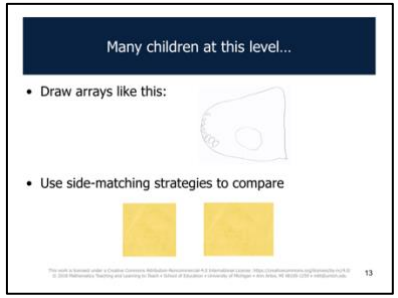
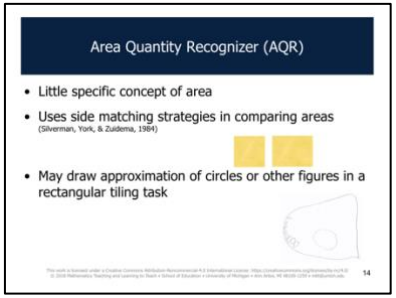
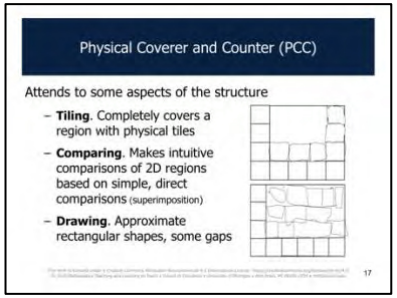
Detailed description of activity	Comments & other resources

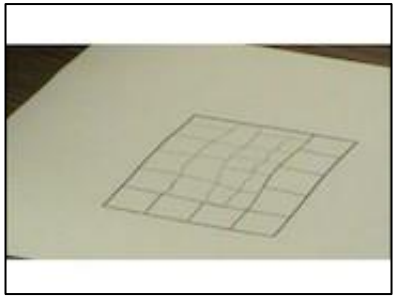
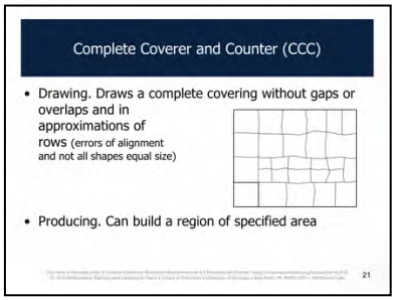
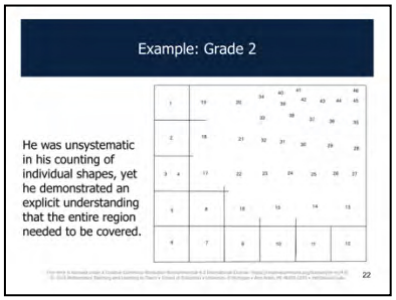
2. Watch Video B, in which Dr. Sarama summarizes some of the discussion that the teachers in the professional development had about their students' thinking. She then previews the work participants will do during this session. As time and interest permit, watch one or both of the videos included in the supplements: • Video: Unexpected student approaches • Video: Students' readiness to think about arrays	*In Video B, Dr. Sarama points out that teachers in the professional development seemed to have shifted from a focus on evaluating what their students did not understand to a focus on inquiring into what students were thinking about and how to make sense of students' work on these tasks. This disposition of inquiry is one that can be cultivated among participants throughout the module.* *Video: Unexpected student approaches* *In this video, a teacher comments on being surprised that, on the arrays and area task, her students attempted to recreate the figure without counting the number of squares in the array. Dr. Sarama comments that students who did this did not see the task as a "quantity" task.* *Video: Students' readiness to think about arrays* *A teacher noted that the arrays and area task was a good formative assessment of her students' understanding of arrays.*

Part 3: Learning trajectories – Early levels (30 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will recognize and understand the early levels of the Learning Trajectory for area measurement.	1. Introduce Part 3 by watching Video A and distributing the handout; introduce the Pre-area Quantity Recognizer level. 2. Introduce the Area Quantity Recognizer level by watching and discussing Videos D and E. 3. Introduce the Physical Coverer and Counter level by watching and discussing Videos D and E. 4. Introduce the Complete Coverer and Counter level by watching and discussing Videos F and G.	• Video A (01:10): Students' thinking about arrays and areas • Video B (02:18): Comparing shapes and copying arrays • Video C (00:48): Commentary on "comparing shapes and copying arrays" • Video D (02:04): Covering a rectangle 1 • Video E (01:17): Commentary on "covering a rectangle 1" Video F (00:51): Covering a rectangle 2 • Video G (03:13): Commentary on "covering a rectangle 2" • Handout: Content cube – Area Learning Trajectory.

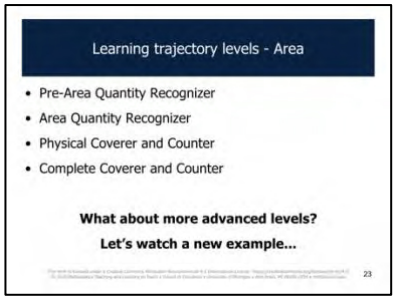
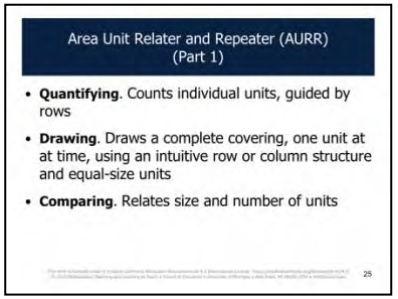
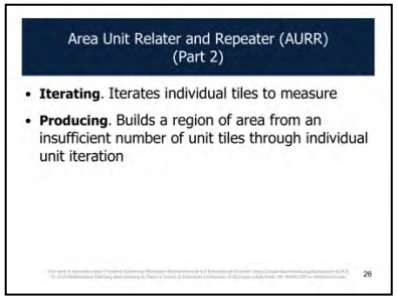
Detailed description of activity	Comments & other resources
1. Introduce Part 3: In this part, the early levels of the Learning Trajectory for area will be introduced and illustrated using video examples of students who are working on various measurement tasks. Have participants watch Video A where Dr. Clements explains what prompted him (and Dr. Michael Battista) to investigate students' thinking about arrays and area.   	*This part introduces the early levels of the developmental progression for area measurement:* • *Pre-Area Quantity Recognizer (PAQR)* • *Area Quantity Recognizer (AQR)* • *Physical Coverer and Counter (PCC)* • *Complete Coverer and Counter (CCC)* *When using the names of these levels, continue to keep in mind that the **student** is not the level; instead, the student's **behavior in this case** corresponds with a particular level.* *Direct participants to the Content cube handout to support their recall and later use of the ideas that are discussed.* *During Parts 3 & 4 of this session, help participants keep their focus on children's development of ideas about area. This may be challenging as the discussions might lead to thinking about the tasks that are featured in the videos. Later in Session 6 there will be opportunities for focusing on tasks and curriculum that support students' development of ideas about area.*

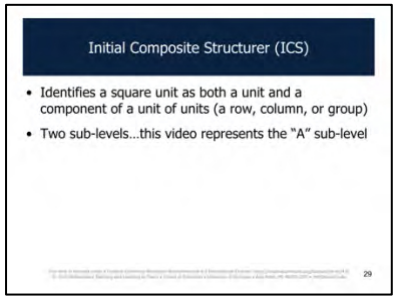
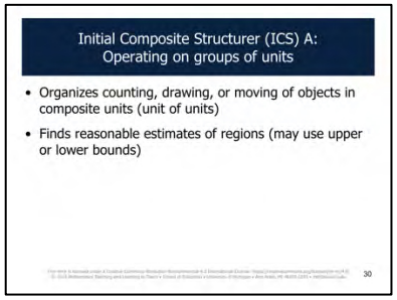
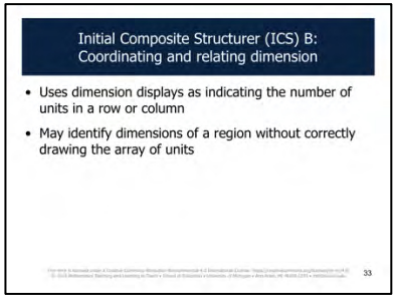
Detailed description of activity	Comments & other resources
2. Have participants watch Video B, which shows two examples of the Area Quantity Recognizer (AQR) level. Following the video, discuss the question: **What characterizes the students' thinking?** After that, watch Video C, where Dr. Sarama describes the Area Quantity Recognizer level.  	*Video B begins with an example of a student who is given three pieces of paper (two of which are congruent) and asked to figure out which one would help him paint the biggest picture. Looking at only one dimension, he responds that two of the papers are the same size. Dr. Clements points out that he compares the area of the papers to answer the question. Then, another clip is shown, in which a student is asked to copy a given array. The student ends up drawing an outline with shapes inside, however the shapes are arranged in a line, not an array.* *Stress that even though the behavior of a student reflects a particular level, that doesn't mean that this is the limit of the student's thinking. It means he or she is <u>at least</u> at this level.*
3. Next, watch and discuss Video D, an example of the Physical Coverer and Counter level. Ask: **Why is this child an example of a Physical Coverer and Counter? What is the same or different about this example and the previous examples?** Then, have participants watch Video E, where Dr. Clements discusses the video of the child and talks about the description of the Physical Coverer and Counter level.  Students at this level attend to some aspects of the spatial structure, but not all. As a result, they can use tiles to physically cover a rectangular area, but they have difficulty drawing an array of squares to cover a rectangular area. Allow time for any follow-up questions/discussion as time and need permit.	*Video D: The child in this video is asked to complete a partially drawn array by drawing in the remaining squares to completely cover the rectangle. She draws shapes along the edges of the rectangle that do not have the same (or similar) size as the square that was already been drawn. The shapes the student draws are also not always adjacent to one another.* *This example and others could be connected with the key measurement concepts shared in Session 4 like equal partitioning, iterating, the presence of gaps and overlaps.*

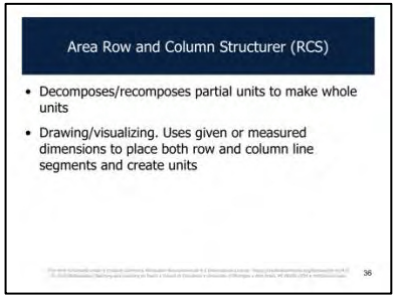
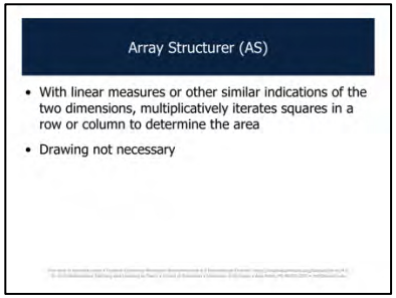
Detailed description of activity	Comments & other resources
4. Continue watching videos that illustrate the early levels of the developmental progression for area measurement by viewing Video F, an example of a Complete Coverer and Counter. Then, show the slide that has the student's work and discuss: **Why is this child an example of a Complete Coverer and Counter? What characterizes the student's thinking?**  After discussing these questions, have participants watch Video G, in which Dr. Clements discusses characteristics of the Complete Coverer and Counter level and presents another example. Dr. Sarama follows up by saying that students at this level often sense that their work is not completely adequate. After watching this video, allow time for any follow-up questions/discussion as time and need permit.  	*Video F: In this video, a student is asked to finish drawing in squares to completely cover a rectangle. The student covers the entire rectangle without leaving any gaps, but doesn't maintain the size of the square in his drawing.*

Part 4: Learning trajectories – Later levels (~30 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will recognize and understand the later levels of the Learning Trajectory for area measurement.	1. Introduce Part 4 by watching Video A; watch and discuss Videos B and C to introduce the Area Unit Relater and Repeater level. 2. Watch and discuss Videos D-H to introduce the Initial Composite Structurer levels (including sub- levels A and B). 3. Watch and discuss Videos I and J to introduce the Area Row and Column Structurer level. 4. Watch and discuss Videos K-M to introduce the Array Structurer level. 5. Discuss how these levels fit with what participants have seen with their students.	• Video A (00:17): Considering later Learning Trajectory levels • Video B (01:07): Covering a rectangle 3 • Video C (01:12): Commentary on "covering a rectangle 3" • Video D (00:58): Tiling an area 1 • Video E (01:37): Commentary on "Tiling an area 1" • Video F (00:51): Setting up "Tiling Area 2" • Video G (01:02): Tiling an area 2" • Video H (02:22): Commentary on "Tiling an area 2" • Video I (01:08): Tiling an area 3 • Video J (00:48): Commentary on "Tiling an area 3" • Video K (00:36): Setting up "Tiling an area 4" • Video L (01:32): Tiling an area 4 • Video M (00:35): Commentary on "Tiling an area 4" • Handout: Content cube – Area Learning Trajectory • Handout: Anecdotal notes form – Area Learning Trajectories

Detailed description of activity	Comments & other resources
1. Introduce Part 4 by watching Video A: In this part, participants will be introduced to the later levels of the Learning Trajectory for area measurement. Have participants watch Video B, which illustrates the Area Unit Relater and Repeater level. Discuss**: Why is this child an example of an Area Unit Relater and Repeater?** **What characterizes the student's thinking?** Then have participants watch Video C, where Dr. Clements describes behaviors that characterize the Area Unit Relater and Repeater Level. Following the video, allow time for follow-up questions or discussion as time and need permit.    	*During Part 4 of this session, help participants keep their focus on children's development of ideas about area. This may be challenging as the discussions might lead to thinking about the tasks that are featured in the videos. Later in Session 6 there will be opportunities for focusing on tasks and curriculum that support students' development of ideas about area.* *Video B shows an example of a student who is able to fill in the remaining part of an incomplete array by drawing in additional squares. Dr. Clements asks teachers in the professional development what geometric knowledge would lead to this kind of behavior on this task (as opposed to the kinds of behaviors illustrated in Part 3 of this session).* *Refer participants back to the measurement principles of (a) equal partitioning, (b) units and unit iteration, (c) accumulation, and (d) relation between number and measurement.* *Prior to showing Video C, provide participants the opportunity to discuss what they noticed in the video featuring the student.*

Detailed description of activity	Comments & other resources
2. Next, view Video D, which illustrates an example of the first sub-level of the Initial Composite Structurer (ICS) level. Discuss**: Why is this child an example of an Initial Composite Structurer? What characterizes the student's thinking?** Then, watch Video E, where Dr. Clements describes the distinguishing characteristics of the Initial Composite Stucturer (ICS) and explains one of the sub-levels of this level (which was illustrated in Video D). He then introduces the next video, which will show an example of the slightly more advanced sub-level of the Initial Composite Structurer level.   Watch Video F, where Dr. Clements and Dr. Sarama explain what participants should pay attention to as they watch the next video. Have participants watch Video G, an example of an Initial Composite Structurer (ICS) B. Discuss: **Why is this child an example of an Initial Composite Structurer (ICS) B? What about this example is different from the previous example?** Then, view Video H, where Dr. Clements and Dr. Sarama talk about this sub-level of the Initial Composite Structurer and how it differs from the "A" sub-level. Allow time for follow-up questions and discussion as time and need permit.  	*In Video D, given an incomplete array, the student is able to tell the interviewer how many squares it would take to completely fill the array. She is able to do this without drawing in the extra squares, using the little square to determine that there are 4 squares in one row and then using that "unit of units" to arrive at a total of 12.* *In Video E, Dr. Clements talks about how children at this level see a row or column of an array as a "unit of units", which is an important idea for students to come to understand when working with area. In other words, children can see the whole row/column as a composite, but can also visualize the individual squares that make up that row/column. This parallels what children need to be able to do in number with seeing a 10 as both a ten and 10 ones.* *In video G, a child fills in one row of an incomplete array and then is able to determine how many squares are in the entire array without drawing in any other squares. She states that there are three squares in each row, then uses skip counting to determine the total.* *In Video H, Dr. Sarama points out that this student was able to see the multiplicative structure in the array. A teacher talks about how the student seems to have more of an "abstract understanding". Dr. Clements then discusses the terms "concrete" and "abstract" and explains that he and Dr. Sarama refer to the thinking in this video as "integrated concrete" because it involves some abstraction/generalization but is still visualizing concrete things (as opposed to something that is even more abstract, such as using the formula "length x width").*

Detailed description of activity	Comments & other resources
3. Have participants watch Video I, an example of an Area Row and Column Structurer (ARCS). Discuss**: Why is this child an example of an Area Row and Column Structurer?** Then, view Video J, where Dr. Clements explains that the conceptualization of an array as row and column line segments (as students at the Area Row and Column Structurer level do) is an understanding that students must construct over time. Allow time for follow-up questions and discussion as time and need permit. 	*In Video I, a student completes an incomplete array by extending the horizontal and vertical lines that have already been drawn. Dr.* *Clements then poses the question: "What does it mean when kids start drawing the lines all the way across?"* *As children develop a more sophisticated understanding of area, they move from drawing shapes inside to drawing individual units to drawing rows OR columns (but not both) of units. When children start drawing line segments all the way across both horizontally, it means then have moved to seeing rows AND columns, demarcating both as units of units, rather than relying on a strategy of drawing individual units.*
4. Watch Video K, where Dr. Clements and Dr. Sarama explain the task that the student will be doing in the next video. Then, watch Video L, which illustrates the last level of the developmental progression of area measurement that will be discussed (the Array Stucturer level). Following the video, allow time for follow-up questions and discussion as time and need permit. Then, watch Video M, in which Dr. Clements explains that what distinguishes students at this level is that they can see the structure of the entire array (rather than just one row or one column). They can superimpose the entire array structure onto a figure. 	*In Video L, a student is given a larger rectangle (with the dimensions, 9 cm x 8 cm, marked) and a smaller rectangle (with the dimensions, 2 cm x 3 cm, marked). Note that the small rectangle has to be mentally rotated 90 degrees for the dimensions to align correctly. He solves the problem mentally by multiplying the number of small rectangles that would fill a row by the number of small rectangles that would fill a column of the larger rectangle (using what he knows about the dimensions of both rectangles).*

Detailed description of activity	Comments & other resources
5. Conclude this part by having participants look at all of the levels in the area developmental progression. Discussion: **How do the levels make sense in view of your assessments and our work here?** Encourage participants to think about how these levels relate to the measurement skills of the students in their classrooms	*Discussion points might include:* • *Pointing participants back to the measurement principles;* • *Tiling vs. drawing: How often do we ask students to completely tile an area? If we do, what happens if we ask then to draw a representation of the tiled area?* • *Enumerating vs. drawing: Even if a child can correctly tell you the area, how do they draw a representation of that area? Is the number important in how they draw? Do they pay attention to dimensions?*

Part 5: Test ourselves – Connecting students' thinking with learning trajectories (~15 minutes)

Goals

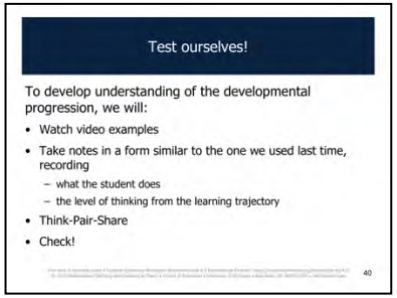
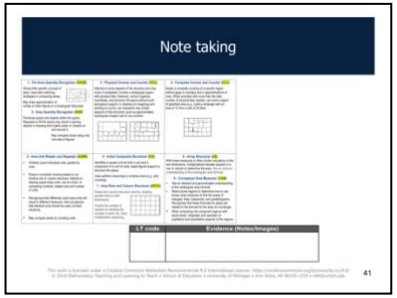
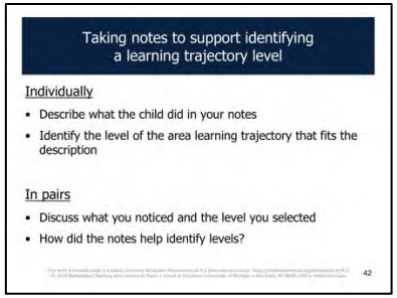
- Participants will demonstrate understanding of the Learning Trajectory for area measurement.
- Participants will practice and understand the benefit of taking descriptive notes during an interaction with a student and then using those to later interpret the students' thinking.

Instructional sequence

1. Introduce Part 5 by watching Video A.
2. Discuss the reasons for using the Anecdotal notes from when watching and analyzing the videos.
3. Think-pair-share about Video B **(Answer: Physical Cover and Counter)**; watch Video C during the whole-group discussion.
4. Think-pair-share about Video D **(Answer: Unit Relater and Repeater)**; watch Video E during the whole-group discussion.
5. Think-pair-share about Video F **(Answer: Area Quantity Recognizer)**; watch Video G during the whole-group discussion.
6. Think-pair-share about Video H **(Answer: Physical Coverer and Counter)**; watch Video I during the whole-group discussion.
7. Think-pair-share about Video J **(Answer: Initial Composite Structurer B)**; watch Video K during the whole-group discussion.

Resources

- Video A (00:33): Introducing "test ourselves"
- Video B (01:26): Test ourselves 1: Tilling an area
- Video C (01:15): Commentary on "Test ourselves 1"
- Video D (01:06): Test ourselves 2: Tilling with two squares
- Video E (02:21): Commentary on "Test ourselves 2"
- Video F (01:55): Test ourselves 3: Copying a diagram
- Video G (02:19): Commentary on "Test ourselves 3"
- Video H (01:52): Test ourselves 4: Tiling with envelopes
- Video I (01:33): Commentary on "Test ourselves 4"
- Video J (00:21): Test ourselves 5: Partially filled picture
- Video K (01:05): Commentary on "Test ourselves 5"
- Handout: Anecdotal notes form – Area Learning Trajectories

Detailed description of activity	Comments & other resources
1. Introduce Part 5 by watching Video A, in which Dr. Sarama explains that participants will watch videos of students working on tasks and then think, pair, and share about the Learning Trajectory levels that seem to be demonstrated by each student. Explain that, as participants watch the videos, they should focus on two questions: • How are students reasoning about measuring? • How are students making sense of the area?  	*During this part, encourage participants to use specific evidence from the video to support their claims.* *As participants analyze students' responses according to the levels of the developmental progression, keep in mind that they should also be developing an orientation that is focused towards teaching. Allow them opportunities to think about what "next steps" they might want to take with the student based on what the student is currently doing.*
2. Distribute Handout: Anecdotal notes form – Area Learning Trajectories. Explain that, as participants watch the videos, they should use this form to take notes about what they are noticing. For the purpose of practicing, the participants will only be using the section of the table where teachers write the student name, LT code, and evidence.  After each video, participants will use these notes to identify the level of the area Learning Trajectory that was demonstrated in the video. They will then talk with a partner about what they noticed and whether/how the notes they took helped them	*Because participants will see many activities and because they are just learning the names of the trajectories, it does not make sense at this point to fill in the "tasks" and "focal Learning Trajectories" sections.* *When participants are using this note taking form in their own classrooms, they will be selecting particular tasks to use. The note taking sheet has a section for noting which task(s) are being used and what parts of the Learning Trajectory for area that those tasks are likely to surface. This approach will help participants zoom in on key facets of the Learning Trajectory and hopefully make them easier to reference when taking notes as the students engage in the activity.*

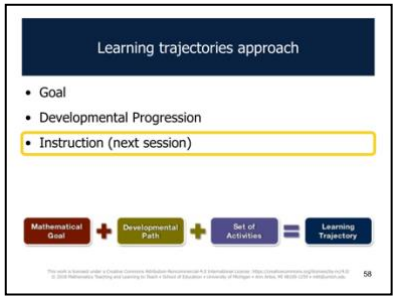
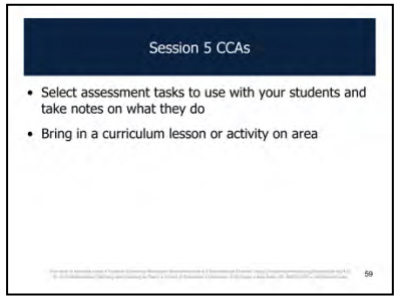
Detailed description of activity	Comments & other resources
3. Have participants watch Video B, using the Anecdotal notes form to record what they are noticing. Afterwards, give participants time to identify the Learning Trajectory level represented in the video. Then, have participants talk with a partner about the topics listed on Slide: Taking notes to support identifying a Learning Trajectory level: • Discuss what you noticed and the level you selected • How did the notes help identify levels? After participants have had time to talk with their partners, return to the whole group watch Video C, where Dr. Sarama facilitates a whole group discussion of these questions with teachers in the professional development. Dr. Sarama emphasizes that it is difficult to interpret the student's level while watching the video, but it is possible to record descriptions of what the student is doing. After watching the video, continue this discussion—debriefing the process of using the form to take anecdotal notes and using these notes to identify the level of the Learning Trajectory.	*In Video B, a student is asked to use tiles to completely cover a rectangle. He covers the rectangle physical tiles, but doesn't structure his tiling well and also leaves some gaps. This performance reflects the Physical Coverer and Counter level.*
4. Repeat the process of taking notes about a student's performance on an area task by watching Video D. Again, have participants talk with a partner about the topics listed on Slide: Taking notes to support identifying a Learning Trajectory level. After participants have had time to talk with their partners, return to the whole group watch Video E, in which Dr. Sarama facilitates a discussion about how teachers' notes helped them determine the level of the Learning Trajectory that was demonstrated. Dr. Sarama and Dr. Clements then explain why they classified this performance as the "Unit Relater and Repeater" level. They argue that, even though the student was not totally accurate, her overlaps seemed to be more of a problem of coordination than a lack of internal structuring of the array. Dr. Clements also points out that students are not at a single level at a single time; rather, their performances on different tasks might reflect various levels, depending on the context, the complexity of the task, and so forth. After watching the video, continue this discussion—debriefing the process of using the form to take anecdotal notes and using these notes to identify the level of the Learning Trajectory.	*In Video D, a student is given two tiles and is asked how many of those tiles will be needed to cover a given rectangle. She uses a "leap frog" technique to iterate the tiles across a row (and counts as she does this). Although she is not precise in terms of how she places the tiles when moving to each new column (and therefore has "overlaps" that cause her to count an extra column), she does seem to be intuitively attending to the column structure as she iterates the tiles. This performance reflects the Area Unit Relater and Repeater Level.*

Detailed description of activity	Comments & other resources
5. Repeat the process of watching and taking notes on a video by watching Video F. Again, have participants talk with a partner about the topics listed on Slide: Taking notes to support identifying a Learning Trajectory level: • Discuss what you noticed and the level you selected • How did the notes help identify levels? After participants have had time to talk with their partners, return to the whole group and watch Video G., in which Dr. Sarama leads a discussion about aspects of the performance that reflect the Area Quantity Recognizer level as well as those that suggest that the student may be beginning to develop competencies of the "Physical Coverer and Counter" level as well. After watching the video, continue this discussion—debriefing the process of using the form to take anecdotal notes and using these notes to identify the level of the Learning Trajectory.	*In Video F, a student is asked to copy a 5 x 4 array. She starts by drawing the perimeter of the rectangle and then draws several squares (and rectangles) of different sizes within it. Her shapes do not start at the edges of the large rectangle and they only cover a portion of it. While she is drawing, the student acknowledges that she "messed up." This performance is an example of the "Area Quantity Recognizer" level.*
6. Repeat the process of watching and taking notes on a video by watching Video H. Again, have participants talk with a partner about the topics listed on Slide: Taking notes to support identifying a Learning Trajectory level: • Discuss what you noticed and the level you selected • How did the notes help identify levels? After participants have had time to talk with their partners, return to the whole group and watch Video I, in which Dr. Clements and Dr. Sarama clarify the prompt that was given to the student and discuss why this is an example of the Physical Coverer and Counter level. They also make the connection between the Physical Coverer and Counter level of the Learning Trajectory for area and the End-to-End Measurer level of the Learning Trajectory for length. After watching the video, allow time for follow-up questions and discussion as time and need permit.	*In Video H, a student uses both square tiles and rectangular envelopes to fill the area of a rectangular section of carpet. The prompt the student was given— which is not included in this video—was to "find the area of the carpet". The student was given two tiles, but she then found the envelopes in the room and used them to help her solve the problem. This performance reflects the Physical Coverer and Counter level.* *Note: Needing enough objects to completely cover and not recognizing the need for equal-size units is parallel to thinking at the End-to-End Length Measurer level in length.*

Detailed description of activity	Comments & other resources
7. Watch the final example in this part, Video J, and repeat the process of watching and taking notes. Again, have participants talk with a partner about the topics listed on Slide: Taking notes to support identifying a Learning Trajectory level: • Discuss what you noticed and the level you selected • How did the notes help identify levels? After participants have had time to talk with their partners, return to the whole group and watch Video K, in which teachers discuss their interpretations of the video. Different teachers classified the performance in this video in different ways, including Area Row Column Structurer and Array Structurer. Dr. Clements explains why he and Dr. Sarama classified it at the Initial Composite Structurer B because she is using additive rather than multiplicative reasoning. However, he acknowledges that the student did use composite units to compute the area, which is a characteristic of all three levels that were being discussed. After watching the video, continue this discussion—debriefing the process of using the form to take anecdotal notes and using these notes to identify the level of the Learning Trajectory.	*In Video J, the student is presented with a partially filled array. She determines the area of the array by figuring out the number of square units in each row (5) and then skip-counts by fives as she points to each row of the array. This is an example of an Initial Composite Structurer B because the student uses additive reasoning to compute the area.*

Part 6: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connection the session content to their classroom.	1. Watch Video A and explain and distribute the Classroom Connection Activities. 2. Summarize the work of the session.	• Video A (02:39): Previewing the next session and the CCA • Handout: Classroom Connection Activity 5 – Area

Detailed description of activity	Comments & other resources
1. Produce the next session by having participants watch Video A, where Dr. Sarama explains that the next section will be focused on instruction and introduces the Classroom Connection Activities.   After watching the video, distribute the handout you customized with selected Classroom Connection Activities and accompanying documents described below. <u>Required:</u> • Area Assessment Task: Select area tasks to complete with 3-4 students of different (hypothesized) achievement levels. Ask the students to write down or draw how they measured. Use the anecdotal notes form to record how students engage in the task and the Learning Trajectory level that is associated with the students' performances. Respond to the reflection questions listed on the CCA handout. • Select a lesson or a short sequence of instructional activities from your curriculum that focus on measuring area. Bring these materials with you to the next session.	*Explain the Classroom Connection Activity and the materials participants need to bring to the next session.* *Make sure to explain that it is important to try using the anecdotal notes form so that the group can work together on understanding its affordances and limitations.* *If you would like to have participants submit the notes, question responses, etc. prior to the next session, then make sure they understand the method by which they should submit those.* *The items included in the CCA roughly correspond with the following grade levels:* • *Pre-K to Grade 2 – Items 1 through 3* • *Grades 3 and 4 – Items 2 through 4* • *Grades 5 and 6 – Items 4 through 6* *Take time to briefly review each of the tasks.*

Detailed description of activity	Comments & other resources
2. Summarize this session by emphasizing that participants: • Analyzed examples of student engagement in measurement in terms of Learning Trajectory for area measurement • Used note taking to describe student work and student thinking before trying to interpret what the student was doing