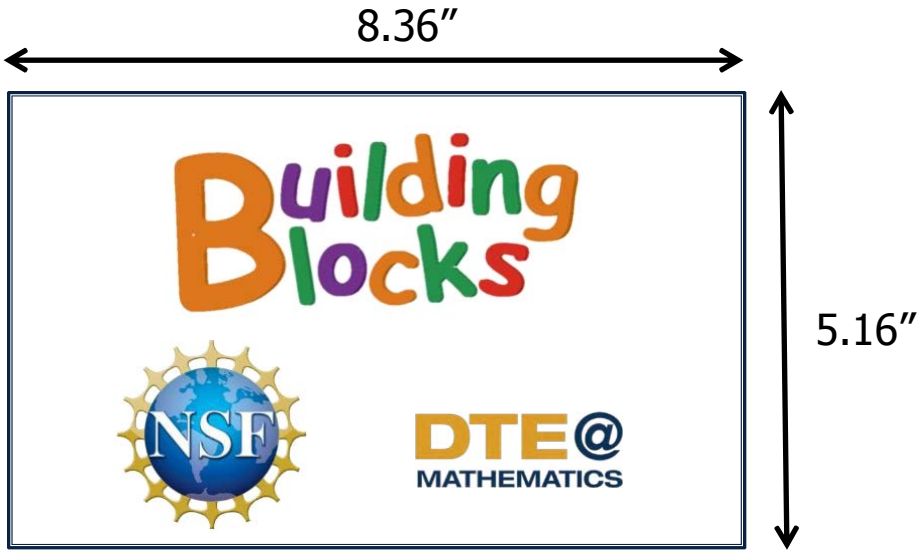
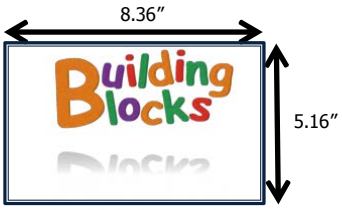
Session 5: Area Learning Trajectory – Developmental Progression



Overview of Session 5

- Discussing what you learned about student thinking from the CCA assessment task(s)
- Unpacking the Developmental Progression of the Learning Trajectory for area by watching students measure
- Classroom Connection Activity



CCA – Focal tasks from last time

- Piagetian conservation tasks
- Arrays and area
  - Copy an array
  - Fill in an incomplete array (what processes?)

CCAs – What did you find?

In groups of 2-4, discuss your students' responses to the two area tasks.

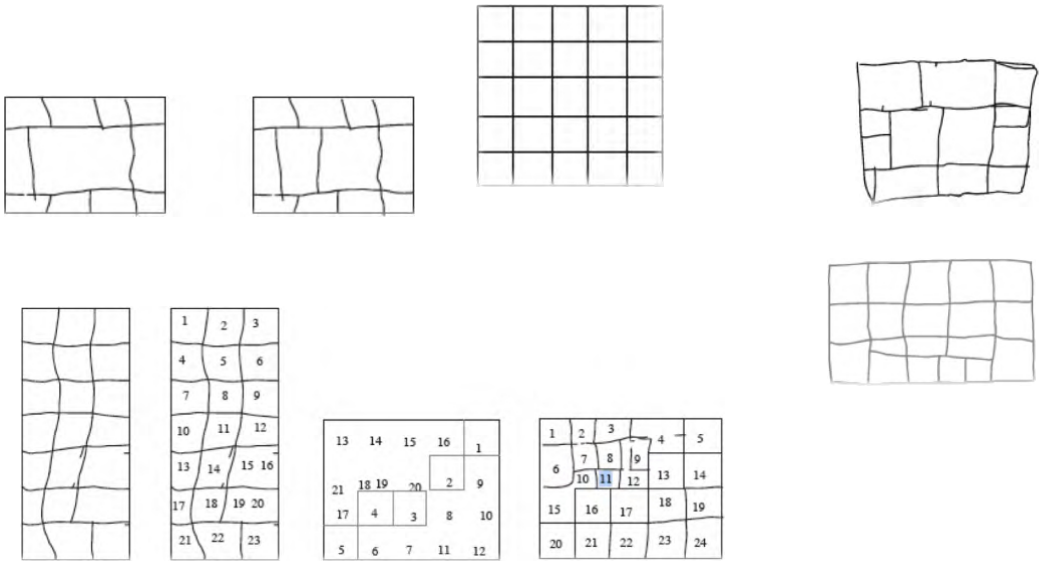
Think about:

- What mathematics do they know?
- How do they think about the math?
- What differences did you notice?

Students' thinking

What started our investigations?

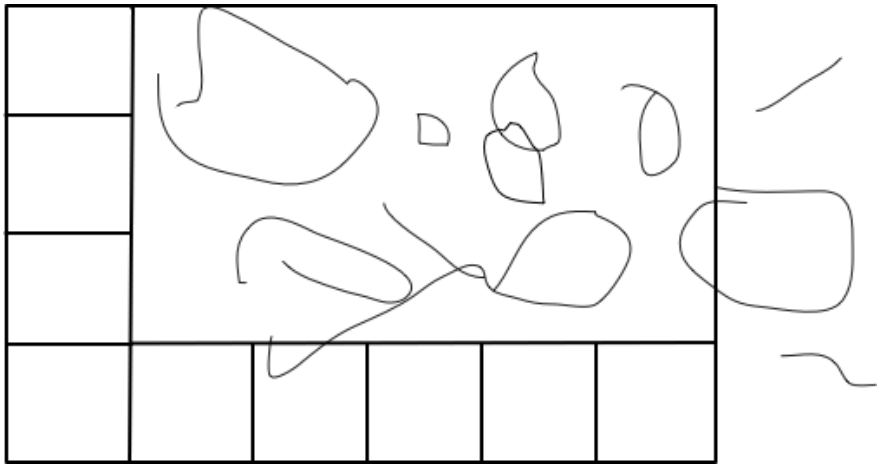
Arrays and area



Learning trajectories approach

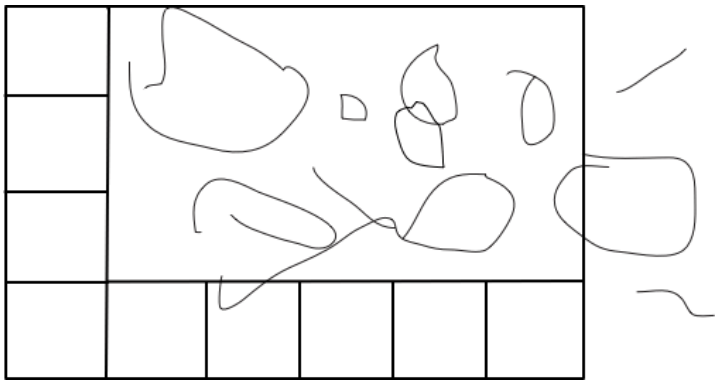
- Goal
- Developmental Progression
- Instruction

Student work sample



Before the first level…
Pre-Area Quantity Recognizer (PAQR)

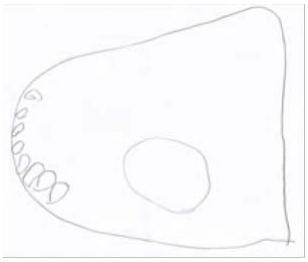
- Little specific concept of area
- Draws in and out of boundary



What characteristics?

Many children at this level…

- Draw arrays like this:

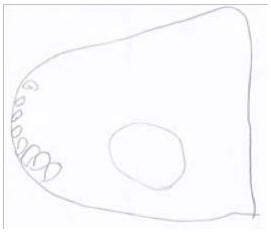


- Use side-matching strategies to compare

Area Quantity Recognizer (AQR)

- Little specific concept of area
- Uses side matching strategies in comparing areas
  (Silverman, York, & Zuidema, 1984)

- May draw approximation of circles or other figures in a rectangular tiling task

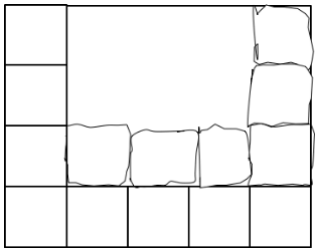


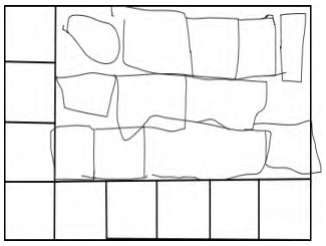
How about this level?

Physical Coverer and Counter (PCC)

Attends to some aspects of the structure

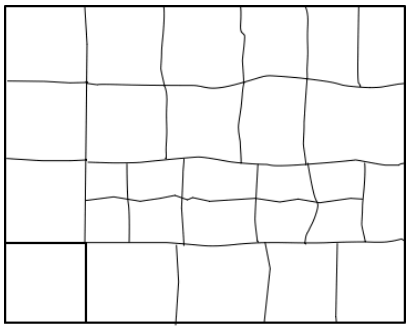
- **Tiling**. Completely covers a region with physical tiles
- **Comparing**. Makes intuitive comparisons of 2D regions based on simple, direct comparisons (superimposition)
- **Drawing**. Approximate rectangular shapes, some gaps





Complete Coverer and Counter (CCC)

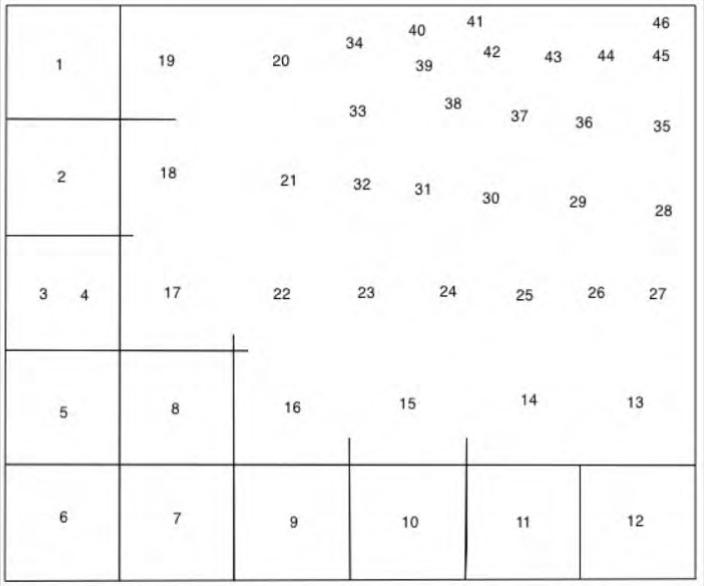
- Drawing. Draws a complete covering without gaps or overlaps and in approximations of rows (errors of alignment and not all shapes equal size)



- Producing. Can build a region of specified area

Example: Grade 2

He was unsystematic in his counting of individual shapes, yet he demonstrated an explicit understanding that the entire region needed to be covered.



Learning trajectory levels - Area

- Pre-Area Quantity Recognizer
- Area Quantity Recognizer
- Physical Coverer and Counter
- Complete Coverer and Counter

What about more advanced levels?

Let's watch a new example…

Area Unit Relater and Repeater (AURR) (Part 1)

- **Quantifying**. Counts individual units, guided by rows
- **Drawing**. Draws a complete covering, one unit at at time, using an intuitive row or column structure and equal-size units
- **Comparing**. Relates size and number of units

Area Unit Relater and Repeater (AURR) (Part 2)

- **Iterating**. Iterates individual tiles to measure
- **Producing**. Builds a region of area from an insufficient number of unit tiles through individual unit iteration

What characterizes this level?

Initial Composite Structurer (ICS)

- Identifies a square unit as both a unit and a component of a unit of units (a row, column, or group)
- Two sub-levels…this video represents the "A" sub-level

Initial Composite Structurer (ICS) A: Operating on groups of units

- Organizes counting, drawing, or moving of objects in composite units (unit of units)
- Finds reasonable estimates of regions (may use upper or lower bounds)

Initial Composite Structurer (ICS) B: What's different?

Initial Composite Structurer (ICS) B: Coordinating and relating dimension

- Uses dimension displays as indicating the number of units in a row or column
- May identify dimensions of a region without correctly drawing the array of units

How about this level?

Area Row and Column Structurer (RCS)

- Decomposes/recomposes partial units to make whole units
- Drawing/visualizing. Uses given or measured dimensions to place both row and column line segments and create units

What characterizes this level?

Array Structurer (AS)

- With linear measures or other similar indications of the two dimensions, multiplicatively iterates squares in a row or column to determine the area
- Drawing not necessary

Test ourselves!

To develop understanding of the developmental progression, we will:

- Watch video examples
- Take notes in a form similar to the one we used last time, recording
  - what the student does
  - the level of thinking from the learning trajectory
- Think-Pair-Share
- Check!

Note taking

1 - Pre-Area Quantity Recognizer (PAQR)

Shows little specific concept of area. Uses side matching strategies in comparing areas.

May draw approximation of circles or other figures in a rectangular tiling task.

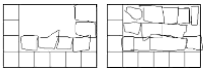
2 - Area Quantity Recognizer (AQR)

Perceives space and objects within the space. Requests to fill the space may result in placing objects or drawing short paths (open or closed) on and around it.

May compare areas using only one side of figures

3 - Physical Coverer and Counter (PCC)

Attends to some aspects of the structure and may cover it completely. Covers a rectangular space with physical tiles. However, cannot organize, coordinate, and structure 2D space without such perceptual support. In drawing (or imagining and pointing to count), can represent only certain aspects of that structure, such as approximately rectangular shapes next to one another.



4 - Complete Coverer and Counter (CCC)

Draws a complete covering of a specific region without gaps or overlaps and in approximations of rows. When provided with more than the total number of physical tiles needed, can build a region of specified area (e.g., build a rectangle with an area of 12 from a pile of 20 tiles).

5 - Area Unit Relater and Repeater (AURR)

- Children count individual units, guided by rows.
- Draws a complete covering based on an intuitive row or column structure. Attends to drawing equal sized units, one at a time. In comparing contexts, relates size and number of units.
- Recognizes that differently sized area units will result in different measures. Also recognizes that identical units should be used, at least intuitively.
- May compare areas by counting units.

6 - Initial Composite Structurer (ICS)

Identifies a square unit as both a unit and a component of a unit of units; needs figural support to structure the space.

Uses additive reasoning to compute area (e.g., skip counting).

7 - Area Row and Column Structurer (ARCS)

Draws and counts rows and columns, drawing parallel lines in both dimensions.

Counts the number of squares by iterating the number in each row. Uses multiplicative reasoning.

8 - Array Structurer (AS)

With linear measures or other similar indications of the two dimensions, multiplicatively iterates squares in a row or column to determine the area. Has an abstract understanding of the rectangular area formula.

9 - Conceptual Area Measurer (CAM

- Has an abstract and generalizable understanding of the rectangular area formula.
- Restructures regions to determine how to use known area measures to find the areas of triangles, kites, trapezoids, and parallelograms. Recognizes that these formulas for areas are related to the formula for the area of a rectangle.
- When comparing non-congruent regions with equal areas, integrates and operates on qualitative and quantitative aspects of the regions.

LT code	Evidence (Notes/Images)

Taking notes to support identifying a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Physical Coverer and Counter (PCC)

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Area Unit Relater and Repeater (AURR)

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Area Quantity Recognizer (AQR)

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Physical Coverer and Counter (PCC)

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Initial Composite Structurer (ICS) B: Coordinating and Relating Dimension

Taking notes to support identifying a learning trajectory level

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In pairs

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Session 5 CCAs

- Select assessment tasks to use with your students and take notes on what they do
- Bring in a curriculum lesson or activity on area

Summary

In this session you:

- Analyzed examples of student engagement in measurement in terms of the learning trajectory for area measurement
- Used note taking to describe student work and student thinking before trying to interpret what the student was doing