

Session 5: Area Learning Trajectory –
Developmental Progression

8.36"

5.16"

Overview of Session 5

- Discussing what you learned about student thinking from the CCA assessment task(s)
- Unpacking the Developmental Progression of the Learning Trajectory for area by watching students measure
- Classroom Connection Activity

CCA – Focal tasks from last time

- Piagetian conservation tasks
- Arrays and area
 - Copy an array
 - Fill in an incomplete array (what processes?)

CCAs – What did you find?

In groups of 2-4, discuss your students' responses to the two area tasks.

Think about:

- What mathematics do they know?
- How do they think about the math?
- What differences did you notice?

Students' thinking

What started our investigations?

Arrays and area

The diagrams show several rectangular shapes and grids. Some are simple rectangles, while others are more complex, with internal lines forming smaller shapes. Some of the diagrams include numbers, possibly representing area calculations or patterns.

Learning trajectories approach

- Goal
- Developmental Progression
- Instruction

Mathematical Goal

+

Developmental Path

+

Set of Activities

=

Learning Trajectory



Student work sample

**Before the first level...
Pre-Area Quantity Recognizer (PAQR)**



- Little specific concept of area
- Draws in and out of boundary

What characteristics?

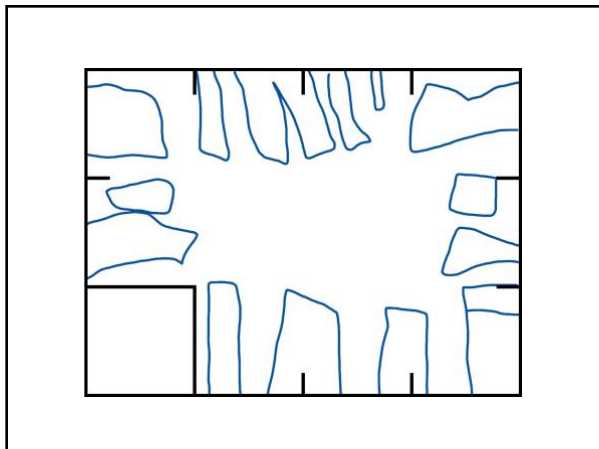
Many children at this level...

- Draw arrays like this: 
- Use side-matching strategies to compare 

Area Quantity Recognizer (AQR)

- Little specific concept of area
- Uses side matching strategies in comparing areas
(Silverman, York, & Zuidema, 1984) 
- May draw approximation of circles or other figures in a rectangular tiling task 

How about this level?



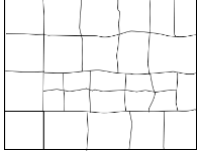
Physical Coverer and Counter (PCC)

Attends to some aspects of the structure

- **Tiling.** Completely covers a region with physical tiles
- **Comparing.** Makes intuitive comparisons of 2D regions based on simple, direct comparisons (superimposition)
- **Drawing.** Approximate rectangular shapes, some gaps

Complete Coverer and Counter (CCC)

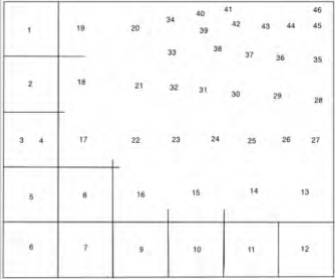
- Drawing. Draws a complete covering without gaps or overlaps and in approximations of rows (errors of alignment and not all shapes equal size)



- Producing. Can build a region of specified area

Example: Grade 2

He was unsystematic in his counting of individual shapes, yet he demonstrated an explicit understanding that the entire region needed to be covered.



Learning trajectory levels - Area

- Pre-Area Quantity Recognizer
- Area Quantity Recognizer
- Physical Coverer and Counter
- Complete Coverer and Counter

What about more advanced levels?
Let's watch a new example...

Area Unit Relater and Repeater (AURR)
(Part 1)

- **Quantifying.** Counts individual units, guided by rows
- **Drawing.** Draws a complete covering, one unit at a time, using an intuitive row or column structure and equal-size units
- **Comparing.** Relates size and number of units

Area Unit Relater and Repeater (AURR)
(Part 2)

- **Iterating.** Iterates individual tiles to measure
- **Producing.** Builds a region of area from an insufficient number of unit tiles through individual unit iteration

What characterizes this level?

Initial Composite Structurer (ICS)

- Identifies a square unit as both a unit and a component of a unit of units (a row, column, or group)
- Two sub-levels...this video represents the "A" sub-level

**Initial Composite Structurer (ICS) A:
Operating on groups of units**

- Organizes counting, drawing, or moving of objects in composite units (unit of units)
- Finds reasonable estimates of regions (may use upper or lower bounds)

**Initial Composite Structurer (ICS) B:
What's different?**

**Initial Composite Structurer (ICS) B:
Coordinating and relating dimension**

- Uses dimension displays as indicating the number of units in a row or column
- May identify dimensions of a region without correctly drawing the array of units

How about this level?

Area Row and Column Structurer (RCS)

- Decomposes/recomposes partial units to make whole units
- Drawing/visualizing. Uses given or measured dimensions to place both row and column line segments and create units

What characterizes this level?

Array Structurer (AS)

- With linear measures or other similar indications of the two dimensions, multiplicatively iterates squares in a row or column to determine the area
- Drawing not necessary

Test ourselves!

To develop understanding of the developmental progression, we will:

- Watch video examples
- Take notes in a form similar to the one we used last time, recording
 - what the student does
 - the level of thinking from the learning trajectory
- Think-Pair-Share
- Check!

Note taking

1- Area-Quantity Recognition (AQR)
Shows the specific content of the structure and they use the unit. They are not counting, they are recognizing the structure. They are recognizing the structure and they are recognizing the structure. They are recognizing the structure and they are recognizing the structure.

2- Physical Coverer and Counter (PCC)
Requires to use objects of the structure and they use it completely. Covers it completely and they use it completely. Covers it completely and they use it completely. Covers it completely and they use it completely.

3- Complete Coverer and Counter (CC)
Shows a comprehensive of the structure and they use it completely. Covers it completely and they use it completely. Covers it completely and they use it completely. Covers it completely and they use it completely.

4- Area Unit Estimator and Reporter (AUE)
Chosen mathematical, probably area. Shows a complete counting based on an inherent unit or column structure. Shows a complete counting based on an inherent unit or column structure. Shows a complete counting based on an inherent unit or column structure.

5- Inherently Composite Structures (ICS)
Identifies square and other unit area. Identifies square and other unit area. Identifies square and other unit area. Identifies square and other unit area.

6- Array Structure (AS)
Shows that structure is also under condition of the two dimensions. Independent from square in a two-dimensional space. Shows that structure is also under condition of the two dimensions. Independent from square in a two-dimensional space.

7- Area Row and Column Structures (ARCS)
Shows and counts rows and columns. Shows and counts rows and columns. Shows and counts rows and columns. Shows and counts rows and columns.

8- Composite Measure (CM)
Shows that structure is also under condition of the two dimensions. Independent from square in a two-dimensional space. Shows that structure is also under condition of the two dimensions. Independent from square in a two-dimensional space.

LT code	Evidence (Notes/Images)

Taking notes to support identifying a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Physical Coverer and Counter (PCC)

Taking notes to support identifying
a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Area Unit Relater and Repeater (AURR)

Taking notes to support identifying
a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Area Quantity Recognizer (AQR)

Taking notes to support identifying a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Physical Coverer and Counter (PCC)

Taking notes to support identifying
a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Initial Composite Structurer (ICS) B:
Coordinating and Relating Dimension

Taking notes to support identifying
a learning trajectory level

Individually

- Describe what the child did in your notes
- Identify the level of the area learning trajectory that fits the description

In pairs

- Discuss what you noticed and the level you selected
- How did the notes help identify levels?

Session 5 CCAs

- Select assessment tasks to use with your students and take notes on what they do
- Bring in a curriculum lesson or activity on area

Summary

In this session you:

- Analyzed examples of student engagement in measurement in terms of the learning trajectory for area measurement
- Used note taking to describe student work and student thinking before trying to interpret what the student was doing
