Session 6: Area Learning Trajectory – Instructional tasks and teaching strategies

Description of the session

In Session 6, participants will complete their work on the Learning Trajectory for area measurement by focusing on the third component of the Learning Trajectory: instructional tasks. At the beginning of the session, participants will review the levels of the developmental progression for area measurement by watching videos of students' performance on tasks and identifying the levels of performance shown. Then, they will engage in their second anecdotal notes workshop, which will provide them with an opportunity to examine their own students' developmental levels as well as their methods of taking anecdotal notes. After that, participants will analyze a variety of instructional tasks to identify the developmental level they target. They will practice modifying tasks from their own curriculum materials in order to target the levels of the students in their class.

*Activities and goals of the session**

Activities	Times	Corresponding parts of the session	Goals
I. Review, overview, and anecdotal notes workshop	35 minutes	Parts 1 & 2	• Participants will be oriented to the work of the session. • Participants will demonstrate understanding of the Learning Trajectory for area measurement. • Participants will use the Learning from Practice Protocol to describe and learn about students' thinking. • Participants will generate ways of improving their taking of anecdotal notes. • Participants will determine ways of improving their learning from the workshop.
II. Instructional tasks for area measurement	50 minutes	Parts 3, 4, & 5	• Participants will recognize the Learning Trajectory level an instructional activity is designed to target. • Participants will connect activities in their curriculum to the Learning Trajectory levels. • Participants will revise an activity to target the levels of the students in their classroom.
III. Wrap up	5 minutes	Part 6	• Participants will recall the work they have done in the three sessions that have focused on the Learning Trajectory for area measurement. • Participants will understand ways of connecting the session content to their classroom.

*Conversations about the CCAs from the last session are integrated into this session.

Classroom Connection Activities

Optional
Type of task: Anecdotal notes extension Description: Facilitate students' work on an area measurement activity and use the anecdotal notes form to record observations.
Type of task: Preparation for Sessions 7-9 Description: Start looking for an activity or assessment focused on the measurement of volume.

Preparing for the session

☐ Make copies as needed: Handout: Content cube – Area Learning Trajectory (Parts 1, 3, 4, & 5); Handout: Anecdotal notes workshop protocol (Part 2); Handout: Anecdotal notes form – Area Learning Trajectories (Part 2); Handout: Array and area challenge (Part 4)

☐ Assemble and prepare materials needed for Parts 3 and 4:

- A set of rectangular cards (for each participant) with the dimensions 1x12, 2x6, and 3x4;
- A set of rectangular cards (for each participant) with the dimensions 8 x 6 and 10 x 5
- Rulers (in the same units as the dimensions of the rectangles above)
- Scissors

☐ Customize and make copies of the Classroom Connection Activities

☐ Text technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.

5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• recognizing the mathematical goal as the first component of a complete Learning Trajectory • understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement) • understanding how measurement of length, area, and volume are represented and developed in the CCSS • understanding how measurement connects with the CCSS standards for mathematical practice • **understanding concepts and skills involved in measuring** length, **area**, and/or volume • understanding connections between length, area, and volume measurement and between metric measurements for each	• recognizing student development as the second component of a complete Learning Trajectory • understanding children's development of measurement through Learning Trajectories for length, area, and volume • recognizing principles of measurement in student work • interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement • **interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement** • interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement	• **recognizing instruction as the third component of a complete Learning Trajectory** • using anecdotal notes to document what students say and do when working on measurement tasks • **connecting measurement activities in curricula to measurement Learning Trajectory levels** • **modifying measurement tasks to target different and/or particular Learning Trajectory levels**	• **understanding the anecdotal notes workshop process** • **using the anecdotal notes workshop to improve the practice of note taking** • **using the anecdotal notes workshop to improve teaching**

Part 1: Test ourselves (~10 minutes)

Goals

- Participants will be oriented to the work of the session.
- Participants will demonstrate understanding of the Learning Trajectory for area measurement.

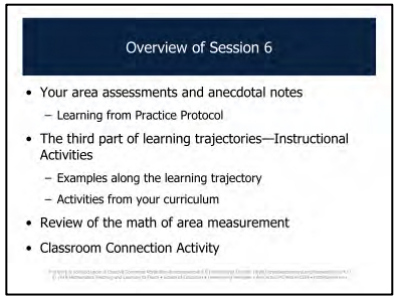
Instructional sequence

1. Introduce the session by watching Video A.
2. Introduce this part by watching Video B.
3. Have participants test their knowledge of the Learning Trajectory for area measurement by watching and discussing Videos C and D.
4. Watch and discuss Videos E and F to continue assessing participants' understanding of the Learning Trajectory for area measurement.

Resources

- Video A (01:04): Overview
- Video B (00:36): Test ourselves
- Video C (01:15): Test ourselves 1: Tiling an area 1
- Video D (02:22): Commentary on "Test ourselves 1"
- Video E (02:49): Test ourselves 2: Creating a rectangle
- Video F (01:14): Commentary on "Test ourselves 2"
- Handout: Content cube – Area Learning Trajectory

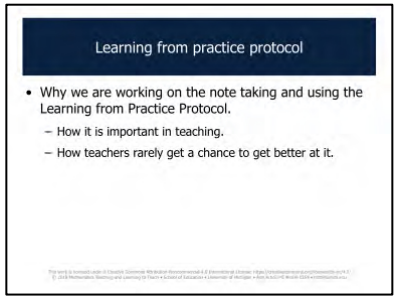
Detailed description of activity	Comments & other resources
1. Introduce the session by watching *Video A*: In Sessions 4 and 5, participants examined the mathematics of area measurement and a developmental progression of students' thinking about these ideas. The focus of this session is on instructional tasks that can be used to help advance students' thinking about area. In this session, participants will • Use the Learning from Practice Protocol to analyze their anecdotal notes about students' work on area assessments • Examine instructional activities for teaching area, including examples along the Learning Trajectory as well as activities from their own curricula • Review the mathematics of area measurement	*Advance preparation required for this session:* *In Parts 3 and 4 of this session, participants will try out some of the instructional activities they will discuss. The following materials are needed for these activities:* *"Covering space" activity:* • Rectangular cards (for each participant) that have the following dimensions: ○ *1 x 12* ○ *2 x 6* ○ *4 x 3* *Note: The same unit should be used to create each of these rectangular cards, but these square units should not be visually marked on the cards.* *"Visualizing arrays" activity:* • Rulers (for each participant) • Rectangular cards (for each participant) that have the following dimensions: ○ *8 x 6* ○ *10 x 5*

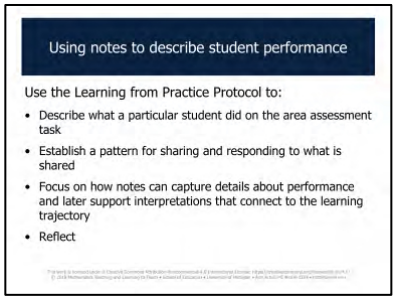
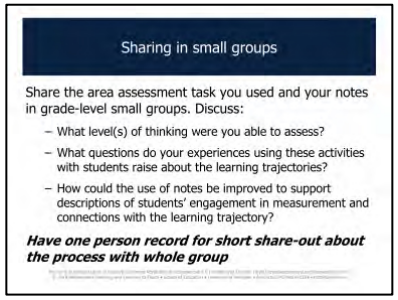


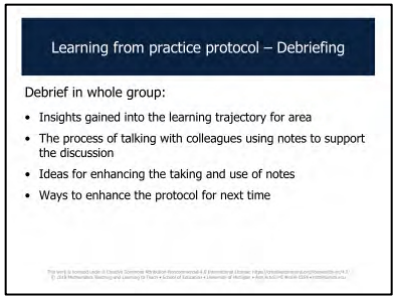
Detailed description of activity	Comments & other resources
2. Watch *Video B,* where Dr. Sarama and Dr. Clements set up the activity in this part. Distribute *Handout: Content cube – Area Learning Trajectory* for participants to use as a resource in this part and in subsequent parts in this session.	*This part is designed to provide a quick review of what participants learned about the developmental progression for area measurement before considering strategies for teaching area measurement.*
3. Have participants watch *Video B* , discuss their observations, and identify the level of thinking demonstrated in the video. Conclude this discussion by having participants watch *Video C*, where Dr. Clements and Dr. Sarama facilitate a discussion about why the student's work in this video is characterized by the Area Unit Relater and Repeater level.	*Video B: Test ourselves 1: Tiling an area* *The student in this video is asked to draw squares to cover a rectangle that has one square drawn and has marks on each side that indicate the dimensions. The student covers the rectangle by drawing equal-sized squares, one by one.* *Video C: Commentary on "Test ourselves 1"* *In this video, teachers discuss aspects of the student's work that reflect the Area Unit Relater and Repeater level; for instance, for the most part, the student drew one square at a time. She also covered the entire rectangle (although she was initially not sure whether the entire rectangle needed to be filled in).*
4. Watch *Video D* and have participants discuss their observations and identify the level of thinking demonstrated in the video. Conclude this discussion by having participants watch *Video E,* where teachers discuss why this student's work appears to reflect the Array Structurer level.	*Video D: Test ourselves 2: Creating a rectangle* *In this video, the student is asked to use a ruler to draw a rectangle that has an area of 24 centimeters. He draws a rectangle with side lengths of 6 cm and 4 cm. He is then asked to draw in the square units, which he does by marking each centimeter on the length and width of the rectangle and then using those marks to draw lines across the rectangle.* *Video E: Commentary on "Test ourselves 2"* *In this video, teachers wonder whether the student may be at the Conceptual Area Measurer level, but they acknowledge that they do not have evidence from his work on this task to determine this. Dr. Sarama points out that it is not possible to make definitive claims about students' levels of understanding from their work on a single task. She also acknowledges that students may work at different levels on different tasks, depending on the difficulty of the tasks and the amount of scaffolding they receive when working on them.*

Part 2: Anecdotal notes workshop (~25 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will use the Learning from Practice Protocol to describe and learn about students' thinking. • Participants will generate ways of improving their taking of anecdotal notes. • Participants will determine ways of improving their learning from the workshop.	1. Introduce the session by watching Video A. 2. Distribute handouts and introduce the anecdotal notes workshop protocol. 3. Have participants meet in small groups to engage in an anecdotal notes workshop. 4. Watch Video A and debrief the workshop; watch Video B during the discussion.	• Video A (03:01): Anecdotal notes workshop launch • Video B (00:55): Anecdotal notes workshop debrief • Video C (01:08): Multiple uses for anecdotal notes • Handout: Anecdotal notes workshop protocol • Handout: Anecdotal notes form – Area Learning Trajectories

Detailed description of activity	Comments & other resources
1. Introduce Part 2 by watching *Video A,* where Dr. Sarama introduces the anecdotal notes workshop and reminds teachers of the rationale for working on the practice of note taking during this professional development module. In this video, Dr. Sarama points out that, while the practice of note taking is important in teaching, teachers rarely have opportunities to get better at it.  	*The terms "learning from practice protocol" and "anecdotal notes workshop protocol" are used interchangeably in the module and refer to the same process.*

Detailed description of activity	Comments & other resources
2. Distribute *Handout: Anecdotal notes workshop protocol* and *Handout: Anecdotal notes form – Area Learning Trajectories.* Remind participants that the goal of this workshop is to use the anecdotal notes to support discussion about the mathematics that their students demonstrate and how students think about mathematics. A related goal is to discuss differences in students' thinking. As in Session 3, participants will use the Learning from Practice Protocol for anecdotal notes workshop to structure their discussion:  • The "presenter" will use his or her notes to support sharing information about the performance of a student on a particular task (and any key background information) (~ 3 minutes) • "Colleagues" will ask questions to better understand the task used, the student's performance on the task, the connections between the student's performance and the Learning Trajectories, and the presenter's method of taking anecdotal notes. (~3 minutes) • Participants switch roles until all have shared about a student's performance on a task that was used. • With whatever time remains, participants should reflect on what they learned, including discussing how the process of sharing with colleagues worked/didn't work for them. (~5 minutes)	*If possible, have the participants meet with the same group of grade-level-alike colleagues as the last time they shared their anecdotal notes. This will support the development of norm for sharing and discussing.* *Limit participants to working in groups of three, as this will ensure that all will be able to share an instance of student thinking from their classrooms. If it is not possible to group all participants in threes, use pairs for the remaining groups (instead of 4 per group).*
3. Have the groups of three begin their anecdotal notes workshop time. During this time, participants should use their anecdotael notes to share what they learned about students from the tasks. They should discuss the following questions: • What levels of thinking were made visible by the tasks? • What new insights or questions do you have about the Learning Trajectory levels? • How could you enhance your next use of anecdotal notes?  As each "presenter" shares, encourage participants to think about • What the presenter is able to say about his or her students based on the anecdotal notes taken • What rest of the group is able to say about the presenter's students based on the anecdotal notes taken	*The major focus of this workshop is on connecting the levels of the Learning Trajectory's developmental progression to instructional tasks (including analyzing the content of tasks and videos). Also integrated with this is a focus on taking notes.* *While participants meet with their small groups, circulate and record:* • *examples of student work that will be helpful in illustrating the different levels of the Learning Trajectory* • *ways that participants use the notetaking forms*

Detailed description of activity	Comments & other resources
4. After participants have had time for small group discussion, return to the whole group to debrief the workshop. Watch *Video B* to set up the debrief. In this video, Dr. Sarama introduces the focus the debrief. When debriefing, discuss • Insights gained into the Learning Trajectory for area • The process of talking with colleagues using notes to support the discussion • Ideas for enhancing the taking and use of notes • Ways to enhance the protocol for next time  As part of the debriefing conversation, watch *Video C: Multiple uses for anecdotal notes.* In this video, Dr. Sarama points out that anecdotal notes can be useful when communicating with parents about students' progress, working with grade-level colleagues, and communicating with colleagues across grade levels.	*It may help to project the anecdotal notes form as participants make comments about its use.*

Part 3: Connecting instructional tasks with early Learning Trajectory levels (~20 minutes)

Goals

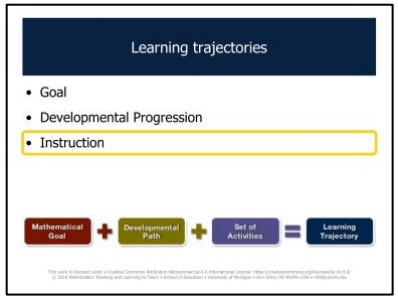
- Participants will recognize the Learning Trajectory level an instructional activity is designed to target.

Instructional sequence

1. Introduce Part 3 by watching Video A and discussing what makes a good instructional task.
2. Watch Video B, try out the two activities, discuss the activities, and watch Video C.
3. Watch and discuss Videos D and E.
4. Watch and discuss Videos F and G.

Resources

- Video A (02:14): Selecting instructional tasks
- Video B (01:04): Covering space tasks
- Video C (02:55): Commentary on covering spaces
- Video D (00:55): Tiling squares task
- Video E (01:56): Commentary on tiling squares
- Video F (00:43): Counting within an array task
- Video G (01:25): Commentary on counting within an array
- Handout: Content cube – Area Learning Trajectory

Detailed description of activity	Comments & other resources
1. Introduce Part 3: In this part, participants consider examples of instructional tasks that are useful for supporting the learning of students at the early levels of the Learning Trajectory for area measurement.  Watch *Video A,* where Dr. Sarama and Dr. Clements discuss the importance of the "instruction" component of the Learning Trajectory and introduce the activity of analyzing examples of instructional tasks designed to advance students' thinking about area measurement. Explain that, as participants examine different instructional tasks, they will seek to answer the question: "**What level is this developing?"** As they do so, encourage them to use *Handout: Content cube – Area Learning Trajectory* as a reference. Before showing the first example task, take a moment to have participants discuss the question: **"What makes a good instructional task?"** Encourage them to continue to consider this question as they examine particular examples of tasks, and ask them to think about how these examples are similar to and different from "typical" curriculum activities.	*In Video A, Dr. Sarama and Dr. Clements point out that instructional tasks are useful not only for helping students advance to higher levels of the Learning Trajectory, but they also provide ongoing opportunities for formative assessment of students' thinking.* *The hallmark of a "good" task is that it connects the student's current level of thinking (which can be determined by the assessments like the ones participants considered in Parts 1 and 2) with where the teacher wants the student to go mathematically. In a larger sense—especially in later grades—it also provides teachers with opportunities to help students who are at different places work productively as a class.* *A good instructional task:* *1. Engages children at different levels—children can solve with different-level strategies.* *2. Requires concepts, skills, and problem solving.* *3. Most important, aligns with the level just beyond the "mastered" level of majority of children.*

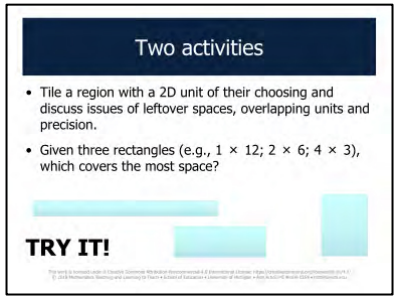
Detailed description of activity	Comments & other resources

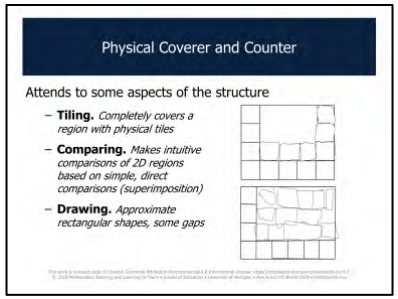
2. Have participants watch *Video B*, in which Dr. Clements introduces one activity that asks students to tile a region and discuss issues that come up and another activity in which students are asked which rectangle covers the most space.

   Before discussing the tasks, have participants try these tasks themselves (using the materials prepared in advance of the session), and ask them to think about the ways in which students would approach the tasks.

   After participants have had an opportunity to try the tasks, discuss the question: **"What level is this developing?"**

   Then watch *Video C,* where Dr. Sarama and Dr. Clements explain why these tasks help develop students' skills with physically cover and count in order to determine the area of rectangles.





Materials needed for this activity:

- *Rectangular cards (for each participant) that have the following dimensions:*
  - *1 x 12*
  - *2 x 6*
  - *4 x 3*

Point out that, in the activity, the dimensions of the rectangles (1 x 12; 2 x 6; and 4 x 3) would not be provided to students. They are listed on the slide to help teachers make sense of the activity. Participants might want to use scissors as they think through this task.

Video C: Commentary on covering spaces

In this video, Dr. Sarama points out that the discussions students might have with one another as they work on these tasks can provide opportunities for them to make progress with their thinking at this level.

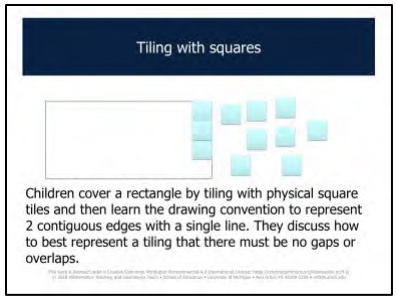
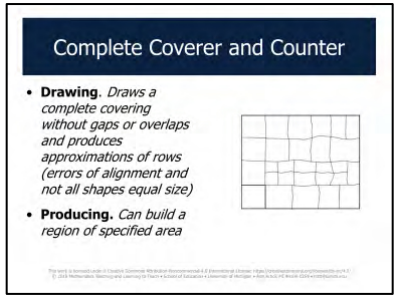
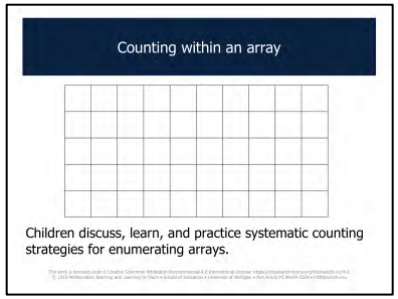
Task 1 (physically tiling a 2D region) focuses on the "Tiling" portion of the PCC level. At this level, children can completely tile the region without leaving gaps.

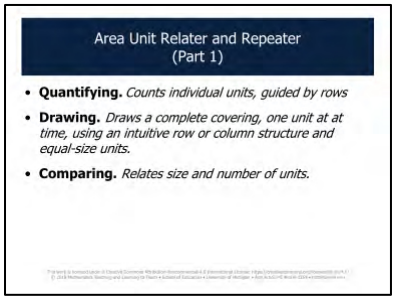
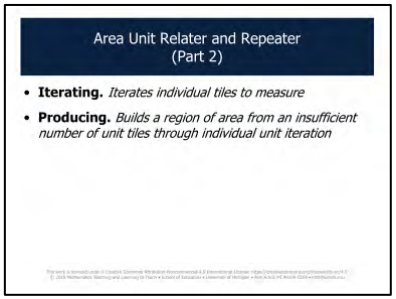
Task 2 (which rectangle covers the most space) focuses on the "Comparing" portion of the Physical Coverer and Counter level. At this level, children use position (i.e., placing one rectangle on top of another) to make intuitive comparisons.

Note that children at this level draw all four sides of the shapes they are drawing in, leaving gaps between shapes.

Dr. Clements and Dr. Sarama also describe possible student responses to the second task, including

- Comparing the rectangles by comparing one dimension only (a characteristic of the "Area Quantity Recognizer" level)
- Directly comparing the rectangles by placing them on top of one another, and sometimes by cutting one into pieces to make it fit (which is a technique that students should not be discouraged from doing to solve this task)

Detailed description of activity	Comments & other resources
3. Have participants read the "Tiling with squares" activity (described on the slide). Then watch *Video D*, which shows Dr. Clements and Dr. Sarama provide more information about the activity. After viewing the video, discuss the question: **"What level is this developing?"** Then show *Video E* where Dr. Sarama and Dr. Clements explain why this instructional task helps work towards the Complete Coverer and Counter level.  	*In this activity, students are given physical squares to cover the entire region. In some situations, the student may not need to completely cover the entire rectangle, but rather use some of the available squares to determine structure and patterns that could be used in drawings to determine the total number of squares needed to cover the region.* *It is important to ask children to draw a complete covering. By helping children understand that a single line segment can be used to represent 2 contiguous edges, this helps eliminate the gap between continuous drawn squares. The focus, then, is on elimination of the gaps/spaces.* *The resources in parts 3 and 4 of this session are sequenced to start with an introduction to each activity, followed by participants thinking about which portion of the learning trajectory would be served by the activity, and concludes with expert commentary from Dr. Clements and Dr. Sarama. If this structure becomes too repetitive, consider having participants watch the overview of the activity and follow that quickly with the video of expert commentary connecting the activity with the learning activity. Then ask participants to discuss their questions or ways of thinking about the connections.*
4. Next, watch *Video F,* where Dr. Sarama and Dr. Clements introduce the "Counting within an array" activity. Again, give participants time to discuss the question: **"What level is this developing?"** 	*As Dr. Sarama and Dr. Clements model in Video F, teachers may introduce this activity to students by asking them to* • Count the squares and write each number they counted on the square • Think about a faster way to count the squares *After giving students time to count, the teacher could then lead a class discussion about the different ways students counted. The idea that a person can skip count is based on the understanding that the same number of squares will be in each row. Thus, when drawing, they will make sure that there is the same number of squares in each row. This is an important transition for the child.*

Detailed description of activity	Comments & other resources
After this discussion, watch *Video G,* where Dr. Sarama and Dr. Clements lead a discussion about why this activity is meant to target the Area Unit Relater and Repeater level.   	*Video G: Commentary on counting with an array* *In this video, Dr. Sarama points out that this task may support students in noticing and using the row structure in order to count the squares, which begins to provide a foundation for the later levels of the Learning Trajectory for area measurement.*

Part 4: Connecting instructional tasks with later Learning Trajectory levels (~20 minutes)

Goals

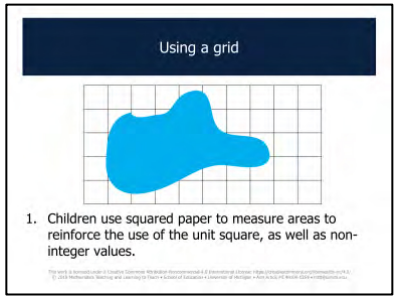
- Participants will recognize the Learning Trajectory level an instructional activity is designed to target.

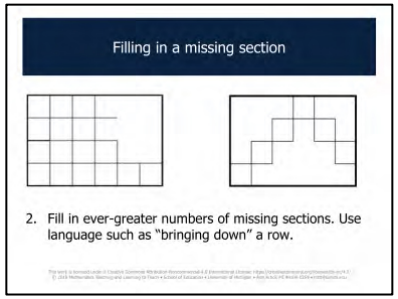
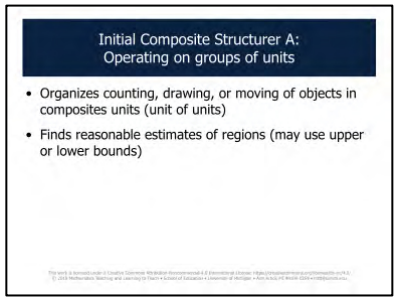
Instructional sequence

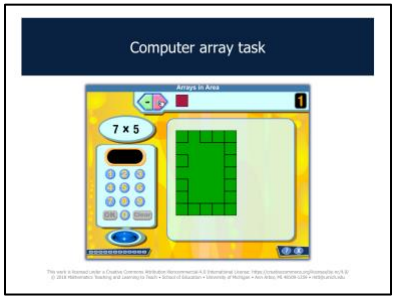
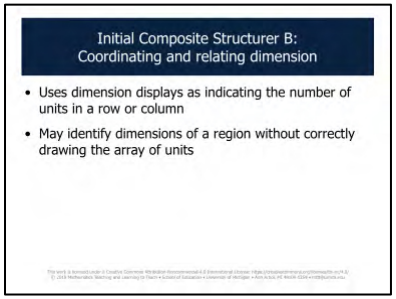
1. Introduce the part and watch and discuss Videos A-C.
2. Watch and discuss Videos D and E.
3. Watch and discuss Videos F and G.
4. Watch Video H, try out the Visualizing arrays task, discuss the task, and then watch Video I.
5. Watch and discuss videos J and K.

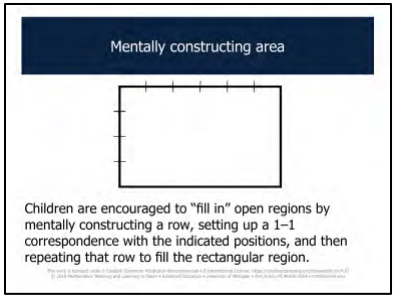
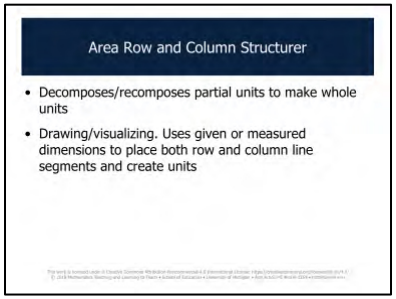
Resources

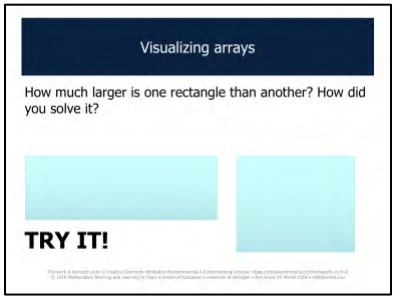
- Video A (00:21): Using a grid task
- Video B (02:40): Filling in a missing section task
- Video C (02:41): Commentary on using a grid and missing section task
- Video D (00:57): Computer array task
- Video E (00:25): Commentary on computer array task
- Video F (01:07): Mentally constructing area task
- Video G (01:47): Commentary on mentally constructing area task
- Video H (00:21): Visualizing arrays task
- Video I (00:44): Commentary on visualizing arrays task
- Video J (01:26): Array and area challenge tasks
- Video K (01:21): Commentary on array and area challenge tasks
- Handout: Content cube – Area Learning Trajectory
- Handout: Array and area challenge

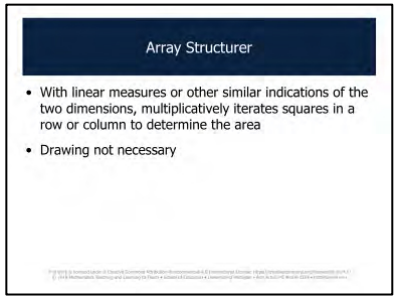
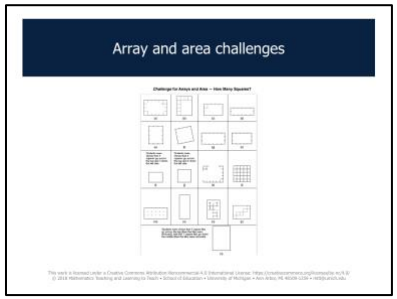
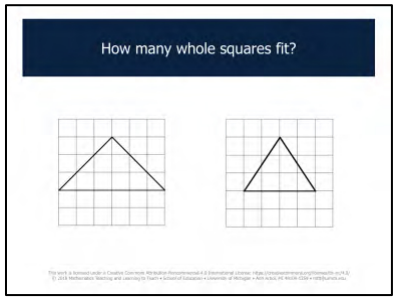
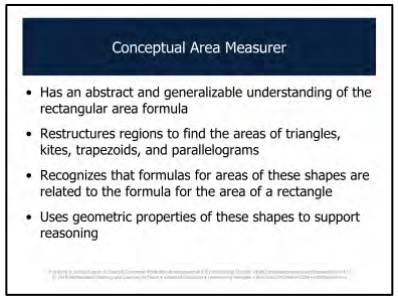
Detailed description of activity	*Comments & other resources*
1. Introduce Part 4: In this part, participants will discuss instructional tasks that are designed to target later levels of the Learning Trajectory for area measurement. Watch *Video A*, where Dr. Clements and Dr. Sarama introduce the "Using a grid" task and pose the question: "What level is this developing?"  	*Participants can continue to use Handout: Content cube – Area Learning Trajectory as a reference during this part.* *Grids can be used to help determine the areas of irregular shapes, like the blue region in the slide. However, the blue region in this example can also be viewed as presenting an obstacle to the goal of determining the number of squares in the shown rectangle. These are substantially different measurement goals that are accommodated by the same representation.*

Detailed description of activity	*Comments & other resources*
After giving participants time to discuss this question, show *Video B,* where Dr. Clements and Dr. Sarama present another task that is designed to target the same level and point out some of the similarities and differences between the two tasks. Give participants time to consider both of these tasks and identify the Learning Trajectory level they are designed to target. Show *Video C*, where teachers Dr. Clements and Dr. Sarama explain why these tasks would be useful for developing the Initial Composite Structerer A level.  	*There are many variations that can be made of this task. A representation of many variations will be seen in the slide "array and area challenges" later in this part of the module* *In Video C, there are initially different opinions among teachers about which level these tasks were designed to target. Dr. Clements and Dr. Sarama point out that one feature of "good" instructional tasks is that they can be approached by students who are at multiple levels of the Learning Trajectory. Teachers can also modify tasks to make them accessible and productive for students at various levels.*

Detailed description of activity	*Comments & other resources*
2. Have participants watch *Video D*, in which Dr. Clements introduces the Computer Array task and talks about how it was designed to help students see a row in a rectangular array and iterate. After watching the video, discuss the question: **"What level is this developing?"** After participants have had time to discuss, watch *Video E,* in which Dr. Sarama and Dr. Clements share that this activity targets the Initial Composite Structurer B level by providing a scaffolded opportunity for students to determine the number of units in a row, see that row as a composite unit, and to use skip counting to determine the area of the rectangle.  	*At this level, children are creating composite units (generally rows or columns) and using those composites to determine the area of a rectangular region. For example, a child might determine that there are 5 squares in the top row, then determine that three rows are needed to completely fill the rectangle. They would, then, find the area by using skip counting ("5, 10, 15… so the area is 35")* *It may help to remind participants that the difference between level B and level A:* *Level A- The student organizes counting, drawing, or moving of objects in composite units (units of units) and finds reasonable estimates of regions* *Level B- The student uses dimensions as indicating the number of units in a row or column and may identify dimensions of a region without drawing the array*

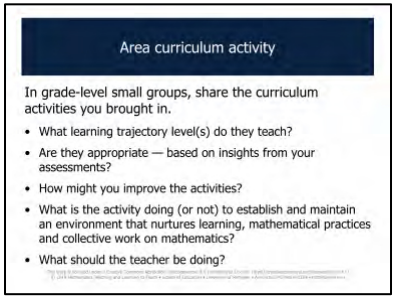
Detailed description of activity	*Comments & other resources*
3. Have participants watch *Video F,* in which Dr. Sarama and Dr. Clements present the "Mentally constructing area" task and model how teachers could set their students up to work on this task. Again, take time to discuss: **"What level is this developing?"** Once participants have had time to discuss this question, watch *Video G,* where Dr. Clements and Dr. Sarama explain why this task targets the Area Row and Column Structurer level.  	*Video G includes information about why it is important for students to have experiences that support them in understanding the spatial structuring of arrays (especially before learning the "length x width" formula for computing area).*

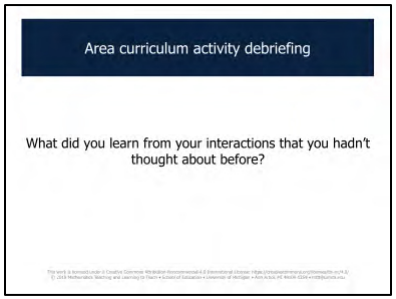
4. Watch *Video H*, in which Dr. Clements and Dr. Sarama introduce the "Visualizing arrays" activity. Have participants try this activity for themselves (using the materials prepared in advance of the session) before discussing which level it is designed to target. Participants can use rulers for this task, but encourage them to work on it first without using rulers.
 	
After participants have tried the activity, give them time to discuss: **"What level is this developing?"**
 	
 
	The following materials are needed for this activity:
	
	• *Rulers (for each participant)*
	• *Rectangular cards (for each participant) that have the following dimensions:*
	  ○ *8 x 6*
	  ○ *10 x 5*

Detailed description of activity	*Comments & other resources*
Watch *Video I,* in which Dr. Sarama explains why this task is useful for developing the Array Structurer level. 	*In this video, Dr. Sarama starts by talking about what the student is instructed to do when completing the task. She then points out the importance of understanding the connection between the linear measurements of the dimensions and the 2-D measurement of the area.*
5. Distribute *Handout: Array and area challenges*. Watch *Video J,* where Dr. Clements and Dr. Sarama introduce this task and the "How many whole squares fit?" task.   Again, give participants time to discuss: **"What level are these tasks developing?"** After participants have had time to discuss, watch *Video K,* where Dr. Clements reviews the characteristics of the Conceptual Area Measurer level and explains why these tasks are useful for developing this level. 	*Array and area challenges – This handout shows some examples of ways to set up a task involving finding the area of a rectangular region. Presenting multiple ways like this encourages children to build a more abstract and generalizable understanding of area. It's not just about knowing the length and the width, but looking at how we might determine those dimensions, as well as helping children better visualize that finding the area is really about determining the number of whole squares that fit inside.* *How many whole squares fit? – Presenting children with triangular regions (or kites or trapezoids or parallelograms, etc.) on a grid like this helps children think about how to restructure a non-rectangular region to find the area, such as by breaking off different pieces and moving them to fit together into a rectangle that aligns with the grid. It also encourages children to think about partial units, as well as how to use the geometric properties of the non-rectangular shapes to help determine the area.*

Part 5: Area measurement in the school curriculum (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• *Participants will connect activities in their curriculum to the Learning Trajectory levels.* • *Participants will revise an activity to target the levels of the students in their classroom.*	1. Introduce Part 5 by showing Video A; have participants discuss their curriculum activities in grade-level groups. 2. Share in whole group, watching Videos B and C as time and interest permit.	• Video A (02:02): Sharing curriculum activities • Video B (00:44): Analyzing the mathematics of curriculum activities • Video C (02:51): Modifying activities with students' knowledge and skill in mind • Handout: Content cube – Area Learning Trajectory

Detailed description of activity	Comments & other resources
1. Introduce Part 5: Show *Video A*, which explains that participants will discuss activities in their curriculum and the ways in which they connect with the levels of the Learning Trajectory for area measurement. Have participants discuss the activities they brought in grade-level small groups, focusing on the following questions: • What Learning Trajectory level(s) do they teach? • Are they appropriate for your students—based on insights from your assessments? • How might you improve the activities? • What is the activity doing (or not) to establish and maintain an environment that nurtures learning, mathematical practices, and collective work on mathematics? • What would a teacher need to do to focus the activity on a different and more appropriate Learning Trajectory level for his/her students? Give participants about five minutes to work in their grade-level groups.	 *In Video A, Dr. Sarama acknowledges that it may be difficult to make time for everyone to share and receive input on the activities they brought. She suggests several ways to address this, including* • *Starting with a group member who did not have an opportunity to share the last time the group met together* • *Choosing a time keeper who can keep the discussion moving* • *Sharing in partners first and then coming together as a larger group* *Participants can continue to use Handout: Content cube – Area Learning Trajectory as a reference during this part.* *If time is tight, it may help to have participants work with a grade level partner instead of small groups. Further it may help to focus the discussion on just the first three bullet points on the slide.*

Detailed description of activity	Comments & other resources
2. Invite participants to share what they discussed in their grade-level groups with the whole group. If it would be useful to support the discussion, show one or both of the following videos: • Video B: Analyzing the mathematics of curriculum activities • Video C: Modifying activities with students' knowledge and skill in mind  	*Video B: Analyzing the mathematics of curriculum activities* *In this video, Dr. Sarama and Dr. Clements discuss a task from curriculum materials that seemed to be trying to tie area to multiplication by asking the students to find the area of their hand and then multiply by 100 to find the area of their skin. Dr. Clements points out that the multiplication involved in this task does not help advance students' understanding of area.* *Video C: Modifying activities with students' knowledge and skill in mind* *In this video, Dr. Sarama and Dr. Clements point out that students in a class are never all "on grade level", so teachers must always work to modify "grade level" tasks in order to meet students where they are and to ensure that they have experiences that will be productive for pushing their thinking forward.*

Part 6: Wrap up (~5 minutes)

Goals

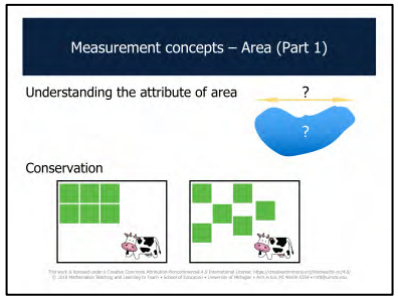
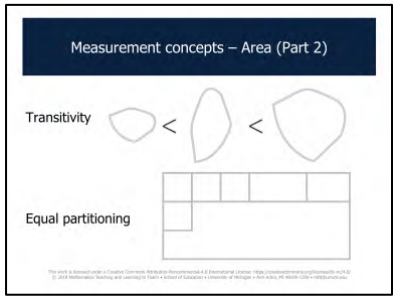
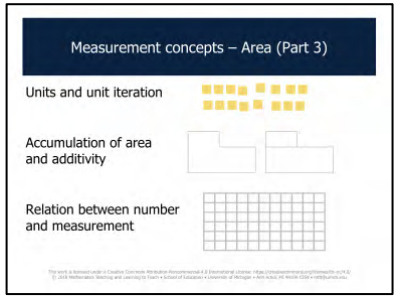
- Participants will recall the work they have done in the three sessions that have focused on the Learning Trajectory for area measurement.
- Participants will understand the ways of connecting session content to their classroom.

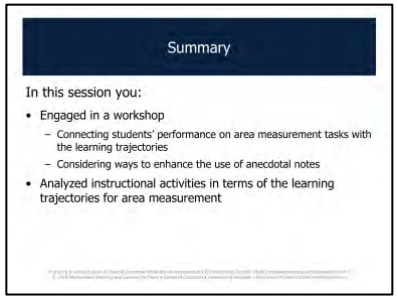
Instructional sequence

1. Watch Video A.
2. Summarize the work of the session and preview the focus of the upcoming sessions.
3. Explain and distribute the Classroom Connection Activities.

Resources

- Video A (03:30): Reconnecting with principles of measurement

Detailed description of activity	Comments & other resources
1. Watch *Video A*, where Dr. Clements and Dr. Sarama connect the work of the session to the principles of measurement introduced at the beginning of the module.   	*The eight key concepts discussed are foundational or all types of measurement. For example, regardless of the attribute being measured, all measurement involves selecting a unit and then iterating that unit (units and unit iteration). With length, the units are one dimensional, and they can be iterated by placing them end to end with no overlaps or gaps. With area, the units are two dimensional, but they can also be iterated with no overlaps or gaps to cover the entire surface of an object.* *After viewing the video, it may be useful to talk with participants about the ways in which these ideas apply to an example activity that was shared in parts 3-5. Alternatively, it may be useful to see if participants have questions about any of the concepts, how it applies to area, and/or how these ideas were embodied in measurements of length.*

Detailed description of activity	Comments & other resources
2. Summarize the session by emphasizing that participants: • Engaged in a workshop ○ Connecting students' performance on area measurement tasks with the Learning Trajectories ○ Considering ways to enhance the use of anecdotal notes • Analyzed instructional activities in terms of the Learning Trajectories for area measurement  Note that participants have now completed sessions focused on the three components of the area Learning Trajectory: the mathematics, the developmental progression of students' thinking, and instructional activities designed to advance students' thinking about area measurement. Explain that the focus of the next three sessions will be on volume measurement.	
3. Distribute the *handout* you customized with the Classroom Connection Activities, which could include the following: • Participants use an area measurement activity (or some portion of it) with their students ○ Could be something from their curriculum (perhaps the activity brought to this session or something completely different) ○ Use the anecdotal notes form • In preparation for our upcoming sessions, start looking for an activity or assessment focused on the measurement of volume.	