

1 - Pre-Area Quantity Recognizer (PAQR)

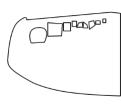
Shows little specific concept of area. Uses side matching strategies in comparing areas.



May draw approximation of circles or other figures in a rectangular tiling task.

2 - Area Quantity Recognizer (AQR)

Perceives space and objects within the space. Requests to fill the space may result in placing



objects or drawing short paths (open or closed) on and around it.

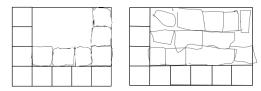
May compare areas using only one side of figures

5 - Area Unit Relater and Repeater (AURR)

- Children count individual units, guided by rows.
- Draws a complete covering based on an intuitive row or column structure. Attends to drawing equal sized units, one at a time. In comparing contexts, relates size and number of units.
- Recognizes that differently sized area units will result in different measures. Also recognizes that identical units should be used, at least intuitively.
- May compare areas by counting units.

3 - Physical Coverer and Counter (PCC)

Attends to some aspects of the structure and may cover it completely. Covers a rectangular space with physical tiles. However, cannot organize, coordinate, and structure 2D space without such perceptual support. In drawing (or imagining and pointing to count), can represent only certain aspects of that structure, such as approximately rectangular shapes next to one another.



6 - Initial Composite Structurer (ICS)

Identifies a square unit as both a unit and a component of a unit of units; needs figural support to structure the space.

Uses additive reasoning to compute area (e.g., skip counting).

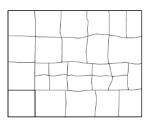
7 - Area Row and Column Structurer (ARCS)

Draws and counts rows and columns, drawing parallel lines in both dimensions.

Counts the number of squares by iterating the number in each row. Uses multiplicative reasoning.

4 - Complete Coverer and Counter (CCC)

Draws a complete covering of a specific region without gaps or overlaps and in approximations of rows. When provided with more than the total number of physical tiles needed, can build a region of specified area (e.g., build a rectangle with an area of 12 from a pile of 20 tiles).



8 - Array Structurer (AS)

With linear measures or other similar indications of the two dimensions, multiplicatively iterates squares in a row or column to determine the area. Has an abstract understanding of the rectangular area formula.

9 - Conceptual Area Measurer (CAM)

- Has an abstract and generalizable understanding of the rectangular area formula.
- Restructures regions to determine how to use known area measures to find the areas of triangles, kites, trapezoids, and parallelograms. Recognizes that these formulas for areas are related to the formula for the area of a rectangle.
- When comparing non-congruent regions with equal areas, integrates and operates on qualitative and quantitative aspects of the regions.

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