Session 7: Volume Learning Trajectory – Mathematical goals

Description of the session

In Session 6, participants concluded their work on the Learning Trajectory for area measurement. In this session, participants begin a sequence of work on volume measurement, focusing specifically on the mathematics of volume measurement. They start by measuring the volume of the room and discussing the issues that arise. After that, participants explore the concept of volume and examples of how students think about volume. Participants then explore how volume measurement is addressed in the Common Core State Standards and identify mathematical practices that are involved with measuring volume. The session closes with an overview of the Classroom Connection Activities that will be completed prior to the next session.

*Activities and goals of the session**

Activities	Times	Corresponding parts of the session	Goals
I. Overview	10 minutes	Part 1	• Participants will reflect on their students' thinking and on their experience using the note taking form when facilitating a measurement activity in their classrooms. • Participants will be oriented to the work of the session.
II. Studying the math of volume measure	40 minutes	Parts 2 & 3	• Participants will begin to recognize and understand the concepts and skills involved in measuring volume. • Participants will recognize the challenges of measuring volume. • Participants will be able to describe their approaches to measuring volume.
III. Studying mathematical goals for students' learning about volume measurement	35 minutes	Parts 4 & 5	• Participants will recognize foundational mathematical ideas of volume measurement. • Participants will begin to gain insight into students' thinking about volume. • Participants will recognize and identify volume measurement within the Common Core State Standards. • Participants will understand connections between the CCSS standards for volume measurement across the grade levels. • Participants will recognize opportunities to work on the mathematical practices within the context of measuring volume.
IV. Wrap up	5 minutes	Part 6	• Participants will understand ways of connecting the session content to their classroom.

*A conversation about a CCA from the last session is integrated into this session (in Part 1).

Classroom Connection Activities

Required
Type of task: Practice and extension of in-class work Description: Complete two volume assessment tasks with 2-3 students of different (hypothesized) achievement levels, and ask students to record how they solved the tasks. Video record your interactions with students around these tasks.

Preparing for the session

- ☐ Gather materials: base ten blocks (including little cubes, longs, and big cubes), meter sticks, tape
- ☐ Construct a cubic meter using 12 meter sticks and tape (or a way to connect the meter sticks to form a cube)
- ☐ Make copies as needed: Handout: Measurement concepts – Volume (Part 4); Handout: Content cubes – Common Core State Standards volume (Part 5); Handout: Learning Trajectory display of measurement standards (Part 5)
- ☐ Customize and make copies of the Classroom Connection Activities
- ☐ Test technical setups: Internet connection, speakers, projector, document camera

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Developing an understanding of the principles of measurement

Foundational concepts of measurement include: understanding of the attribute, conservation, transitivity, equal partitioning, iteration of a standard unit, accumulation, origin, and relation between measurement and number.

- **Attribute-** understanding what is being measured
  - *Key question: What is being measured?*
- **Conservation-** understanding that an attribute being measured does not change when moved
  - *Key question: Does the measurement change if I move what is being measured?*
- **Transitivity-** understanding that a third object can be used to compare the measures of two other objects
  - *Key question: How could I know how the measurements of these objects relate without directly comparing them?*
- **Equal partitioning-** understanding that an attribute to be measured can be partitioned into the same-sized units
  - *Key question: How can we partition this into equal sized parts?*
- **Units and unit iteration**- understanding that an attribute can be measured with a smaller unit without gaps or overlaps
  - *Key question: How can this small unit be used to measure something so large?*
- **Accumulation-** understanding that as you iterate a unit the count represents the total of all units used
  - *Key question: How many copies of this unit were used to measure this attribute?*
- **Origin** is the notion that any point on a ratio scale can be used as the origin. Young children who lack this understanding often begin a measurement with "1" instead of zero.
  - *Key question: Where could I start the process of measuring?*
- **Relation between number and measurement**- Understanding that there is an inverse relation between the size of the unit and the number of those units in a given measure.
  - *Key question: How does the number of units change when I use a larger unit of measure?*

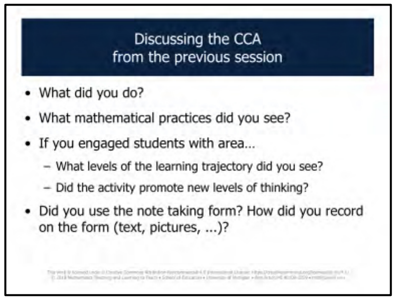
Scope of the module *(focal content of this session in **bold**)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **recognizing the mathematical goal as the first component of a complete Learning Trajectory** • **understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement)** • **understanding how measurement of length, area, and volume are represented and developed in the CCSS** • **understanding how measurement connects with the CCSS standards for mathematical practice** • **understanding concepts and skills involved in measuring** length, area, and/or **volume** • understanding connections between length, area, and volume measurement and between metric measurements for each	• recognizing student development as the second component of a complete Learning Trajectory • understanding children's development of measurement through Learning Trajectories for length, area, and volume • **recognizing principles of measurement in student work** • interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement • interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement • interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement	• recognizing instruction as the third component of a complete Learning Trajectory • using anecdotal notes to document what students say and do when working on measurement tasks • connecting measurement activities in curricula to measurement Learning Trajectory levels • modifying measurement tasks to target different and/or particular Learning Trajectory levels	• understanding the anecdotal notes workshop process • using the anecdotal notes workshop to improve the practice of note taking • using the anecdotal notes workshop to improve teaching

Part 1: Overview (~10 minutes)

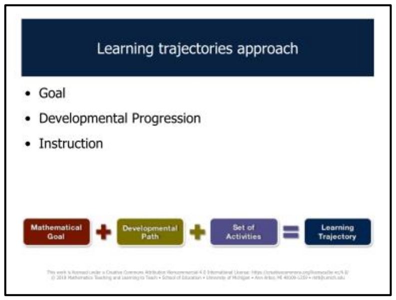
Goals	***Instructional sequence***	***Resources***
• Participants will reflect on their students' thinking and on their experience using the note taking form when facilitating a measurement activity in their classrooms. • Participants will be oriented to the work of the session.	1. Discuss the CCA from the previous session. 2. Introduce the sequence of sessions focused on the Learning Trajectory for volume measurement. 3. Introduce the session by watching Video A.	

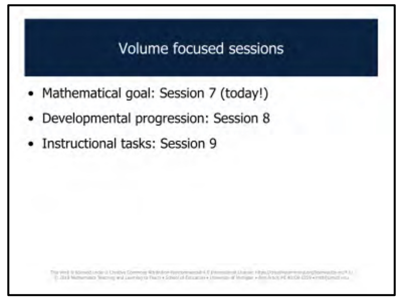
Detailed description of activity	Comments & other resources
1. Direct the participants to have a conversation about the CCA from the previous session. After they have had a chance to talk with a partner, ask a few participants who tried a whole-class measurement activity to share their experiences. Use the following questions to guide the discussion: • What did you do? • What mathematical practices did you see? • If you engaged students with area, what levels of the Learning Trajectory did you see? Did the activity promote new levels of thinking? • Did you use the note taking form? How did you record on the form (text, pictures, …)?	*This portion of the session is meant to provide participants with a chance to check in with colleagues about their CCA and to discuss how it worked to use the note taking form when teaching at a whole class level.* *There will be another opportunity to discuss and develop these ideas through the CCA in Session 9.*



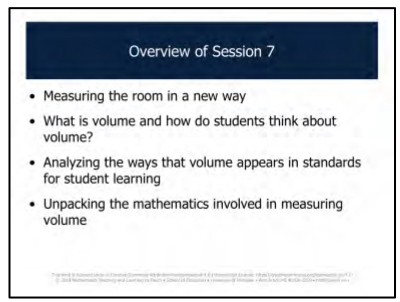
Detailed description of activity	Comments & other resources

2. Introduce the sequence of sessions focused on the Learning Trajectory for volume measurement. These sessions will parallel the three components of the Learning Trajectories:
   - Mathematical goal (Session 7)
   - Developmental progression (Session 8)
   - Instructional tasks (Session 9)





3. Watch *Video A,* in which Dr. Sarama and Dr. Clements provide overview of this session. In this session, participants will
   - Measure the room in a new way
   - Explore the concept of volume and how students think about volume
   - Analyze the ways that volume appears in the standards for student learning
   - Unpack the mathematics involved in measuring volume



Part 2: Measuring the volume of the room – set up (~20 minutes)

Goals

- Participants will begin to recognize and understand the concepts and skills involved in measuring volume.
- Participants will recognize the challenges of measuring volume.

Instructional sequence

1. Watch Video A and have participants work independently to measure the volume of the room.
2. Have participants discuss their results with a partner.
3. *Optional*: Have participants solve an extension problem.

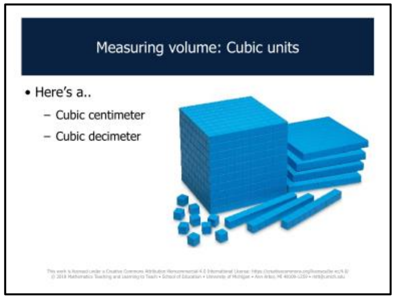
Resources

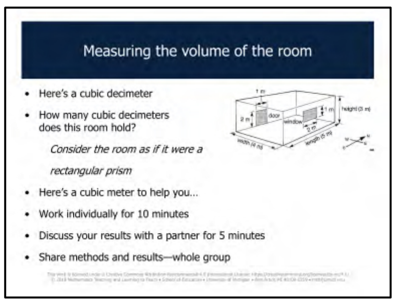
- Video A (02:18): Measuring the volume of the room

Supplements

- Video (01:01): Extension problem using traditional units

Detailed description of activity	Comments & other resources
1. Introduce Part 2 by watching *Video A,* where Dr. Clements sets up the task of using cubic decimeters to measure the volume of the room. Note: For the purpose of this task, participants should consider the room to be one large rectangular prism—even if it actually more complex than this. Make sure participants understand this before they begin to work on the task. Distribute base ten blocks and meter sticks and give participants about 10 minutes to work on this task individually. After about 5 minutes, stop to discuss the questions: • What are you doing to solve this problem? • How have you made progress?	*Base ten blocks (including little cubes and big cubes), meter sticks, and a cubic meter formed with 12 meter sticks are needed for this activity.*





Detailed description of activity	Comments & other resources
2. Once they have completed their measurements individually, have participants discuss their results with a partner for a few minutes.	*Encourage participants to compare and contrast their approaches. What are the benefits, drawbacks, limitations and/or challenges they encountered?* *As they talk with partners, be listening for the ways in which participants describe their approaches and the challenges/benefits of those approaches. You will be able to draw on what you hear as you lead the discussion in Part 3.* *Use the question about fluid capacity to encourage participants to think about the challenges of measuring fluid capacity and connecting the volume with cubed units.*

3. Optional: Have participants solve an extension problem that asks them to figure out how many 1 ft. x 1 ft. x 2 ft. boxes could fit in the room. This problem is introduced in *Video: Extension problem using traditional units*, which is included as a supplement.	*The main purpose for presenting a problem like this is to challenge participants to think about volume when the given unit is not a cube.*
	• *How do you handle the different dimensions?*
	• *Did you work numerically or more visually?*
	  ○ *Most people will attempt to visualize lining up the boxes along one dimension (and then the others) rather than turn to a mathematical formula.*
	• *Did you at any point rotate the boxes to make them fit?*
	  ○ *Most people will line up the boxes along the 2-ft dimension so that 12 boxes will with along a 24-foot wall. What happens, though, if the wall is 25 feet long? Do you turn the last box to make it fit? Do you line up the boxes along the 1-ft dimension so 25 of them fit?*

Part 3: Measuring the volume of the room – discussion and reflection (~20 minutes)

Goals

- Participants will begin to recognize and understand the concepts and skills involved in measuring volume.
- Participants will be able to describe their approaches to measuring volume.
- Participants will recognize the challenges of measuring volume

Instructional sequence

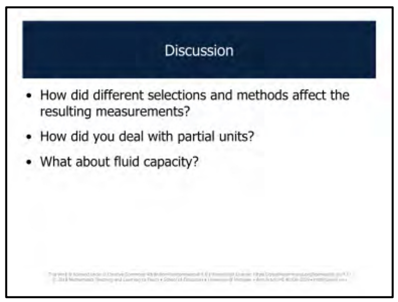
1. Watch Video A and discuss methods participants used to solve the problem; watch Videos E and/or F as needed.
2. Watch Video B and discuss participants' measurements of the room; watch Video C and discuss discrepant measurements, and watch Video G as needed.
3. Watch Video D; watch Video H as time and interest permit.
4. Have participants reflect on their work on this problem, watching Videos H and I as time and need permit.

Resources

- Video A (01:01): Launching a discussion of methods
- Video B (01:50): Comparing volume measurements
- Video C (02:19): Making sense of discrepant measurements
- Video D (00:36): The challenge of thinking across dimensions
- Video E (00:42): Method 1: Measuring with a meter stick
- Video F (00:50): Method 2: Using wall panels
- Video G (01:25): Discrepant measurements – dimension and units
- Video H (00:39): Teaching volume as layers or liquids
- Video I (01:06): Reflection: Thinking with a larger unit

Detailed description of activity

1. Introduce Part 3 by watching *Video A*, in which Dr. Sarama explains the most common method teachers in the professional development used to solve this problem.

   Ask participants if anyone solved the problem differently, using this question to launch a discussion of the <u>methods</u> participants used to solve this problem.

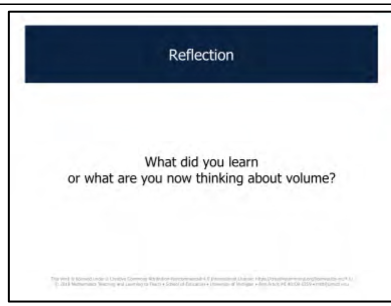
   As time and interest permit, consider watching *Videos E* and/or *F*, which illustrate methods that teachers used to determine the volume of a room.

Comments & other resources

The following themes may emerge as participants discuss and reflect on their work during Part 3:

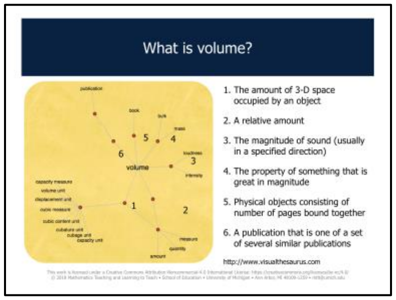
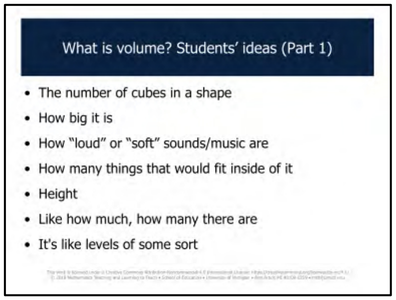
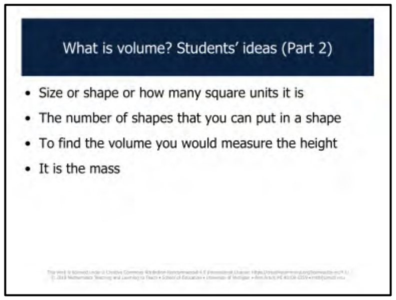
- *<u>Connecting area and volume:</u> One way to think about volume is through thinking about translating an area across a third dimension. For example, with a cylinder, you find the area of one face and then multiply that by (or translate that area along) the height as a way of extending the face into 3D space.*
- *<u>How the linear measure of a prism relates to the number of cubes that fill up that prism:</u> The linear dimension tells how many cubes of that particular unit will fit along that particular dimension. For example, if the room is 25 feet long, then participants should picture 25 cubes, each 1 ft x 1 ft x 1 ft along that dimension.*
- *<u>Building up formulas and connecting them to volume formulas for prisms and cylinders:</u> Connect with the first two points above. Knowing that the linear dimension tells you how many cubes fit along that dimension helps make sense of the l x w x h formula for finding the volume of a prism. Also, understanding that volume is translating an area along a third dimension helps make sense of the formulas for the volume of prisms and cylinders.*

Detailed description of activity	Comments & other resources
	• *<u>Why participants got different answers, and what differences are and are not acceptable:</u> Errors in measurement can happen in many ways – imprecise use of a tool, rounding errors, ignoring partial units, converting using non-exact ratios, etc.* • *<u>Dealing with partial units</u>: Often, partial units are translated into decimals in calculations. For example, if the dimension is 5 and a half feet, we just use 5.5 in our calculations. However, in the "box problem" from Part 2, participants had to visualize filling the room with boxes. In this case, 0.5 of a box doesn't make sense.* • *<u>The features of measurement (e.g., continuous versus discrete quantity; measurement errors):</u> The box problem also highlights the different between continuous and discrete units. Filling the room with sand would be different than filling it with boxes.* *Video E: Method 1: Measuring with a meter stick* *In this video, teachers use a meter stick to measure the height of the room. They estimate that the room is "almost 3" meters high. Among other things, this conversation illustrates the measurement error that occurs when measuring a single dimension of the room. These measurement errors will multiply when the volume of the room is calculated.* *Video F: Method 2: Using wall panels* *A group of teachers discusses how to use their measures of 14 wall panels (which they measured as being 1.2 meters each) to estimate the width of the room. Different teachers suggest different estimates of the width of the room (i.e., 17, 16.5, and 16.48 meters). This work on conversation illustrates the how errors might arise when converting between standard units (meters) to non-standard units (panels) and back again.*

Detailed description of activity	Comments & other resources
2. Watch *Video B,* in which teachers in the professional development compare the measurements they calculated. Use this to launch a discussion of the participants' <u>measurements</u> of the room. After discrepant measurements have emerged, watch *Video C,* where Dr. Sarama and Dr. Clements discuss what might account for the variation that exists among the measurements. Allow participants to add to this discussion, commenting on the methods they used to measure the room and the reasons their answers might vary. As time and interest permit, consider watching *Video G*, which illustrates an example of an approach that could lead to discrepant measures.	*Video G: Discrepant measurements – dimensions and units* *In this video, teachers discuss their method for measuring the volume of the room, which involved: measuring the dimensions of the room in yards, converting them to meters, multiplying the dimensions by each other, and then multiplying by 1000 to convert from cubic meters to cubic decimeters. Dr. Sarama wonders if they made an error when converting from feet to yards. She points out that is a lot of potential for error when converting between different units.*
3. Then, watch *Video D,* in which Dr. Sarama talks about the challenges of thinking in three dimensions and figuring out what answers would be reasonable. As time and interest permit, consider watching *Video H*, which illustrates another challenge of measurement – the challenge of connecting volume measures of solid and liquids.	*Video H: Teaching volume as layers or liquids* *A teacher comments that volume is often taught as "layers" and she is not sure how that fits in with liquid capacity. Dr. Sarama points out that there is often a disconnect in math instruction between measuring solid volume and liquid capacity.*
4. Conclude the discussion of the problem by asking participants to reflect on what they learned or what they are now thinking about volume. As time and need permit, watch *Video I* to support this discussion.  	*The slide with the reflection question is available in the resources section. It is also available in the right-hand side of the viewer at the end of Video D and in Videos H and I. Video I provides an example of what teachers may take out of the activity and might be interesting for participants to hear.* *Video I: Reflection: Thinking with a larger unit* *In this video, teachers explain that they are still trying to make sense of their answer, and they describe their attempt to visualize (and estimate) how many cubic **meters** could fit in the room.*

Part 4: Volume and the way that students think about it (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will recognize foundational mathematical ideas of volume measurement. • Participants will begin to gain insight into students' thinking about volume.	1. Introduce this part by watching Video A. 2. Watch and discuss Video B. 3. Watch and discuss Video C. 4. Watch and discuss Video D. 5. Watch and discuss Video E. 6. Watch Video F.	• Video A (01:24): Meanings of volume • Video B (01:31): What do children think? • Video C (00:32): 1996 NAEP result and 2011 NAEP item • Video D (02:13): 2011 NAEP result • Video E (01:12): Students' proficiency with volume measurement • Video F (01:55): Volume concepts • Handout: Measurement concepts – Volume

Detailed description of activity	Comments & other resources
1. Introduce Part 4 by watching *Video A,* where Dr. Clements discusses different meanings of the word "volume." He points out that, when teachers introduce "volume" in the context of mathematics, they must be aware that students might be thinking about other meanings of this term rather than the mathematical meaning.  After viewing, engage participants in a brief discussion of the challenges posed by the multitude of meanings of the word volume.	*Often, mathematical terms are introduced that have multiple everyday life meanings. This can lead to a situation where children may not understand exactly what is being discussed. This term "volume" is an example of a term that can be challenging. Children's exposure to the term "volume" often involves the context of loud and soft sounds. So it may not be clear when a teacher holds up an object and ask children about its volume. It is often necessary to clarify the idea by noting that volume in this context is focused on one of the following: how much material there is; how much can fit inside; or how much space the object takes up?*
2. Watch *Video B,* where Dr. Clements presents and comments on different students' ideas about what volume is.   	*In Video B, Dr. Clements points out that there is a spatial-visualization component to a true understanding of volume that goes beyond the definition or formulas.*

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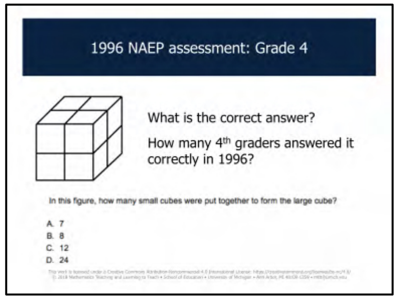
He then shows a question from the 1996 NAEP assessment and asks teachers to predict how many fourth graders answered this question correctly.

Give participants a moment to figure out the answer to the assessment question and to predict how many fourth graders answered it correctly in 1996.



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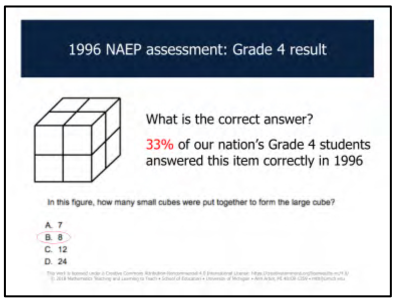
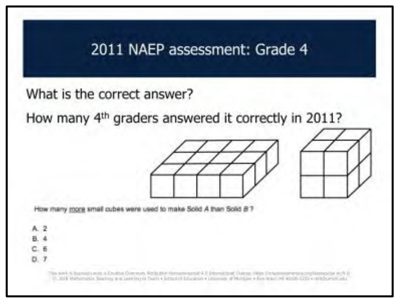
3. Watch Video C, where Dr. Clements reveals that 33% of fourth graders answered the question correctly. He then poses a problem from the 2011 NAEP assessment an again asks teachers to predict how many fourth graders answered it correctly. Give participants a few moments to do this.





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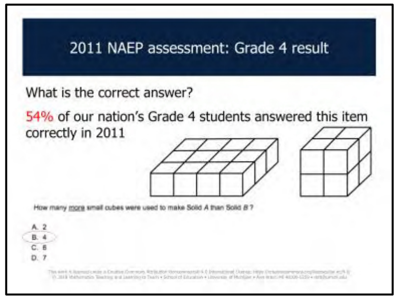
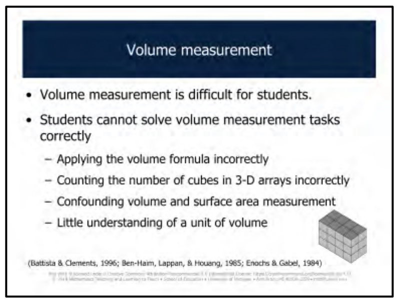
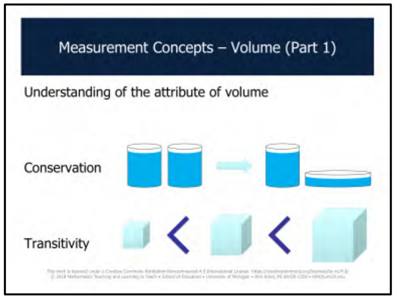
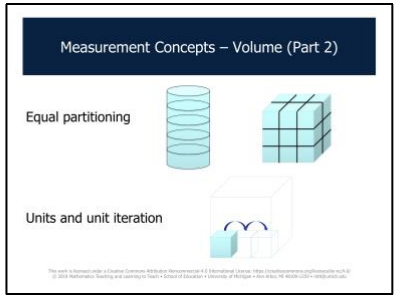
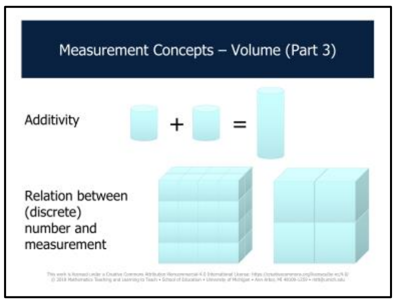
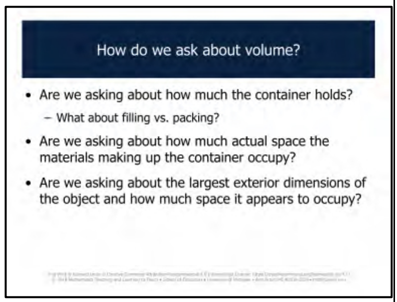
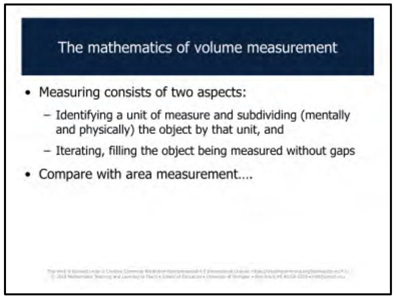
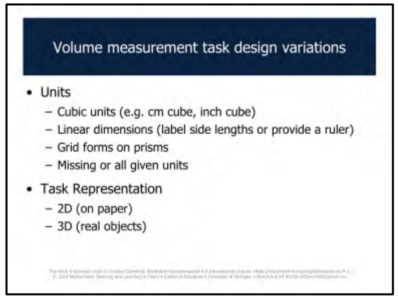
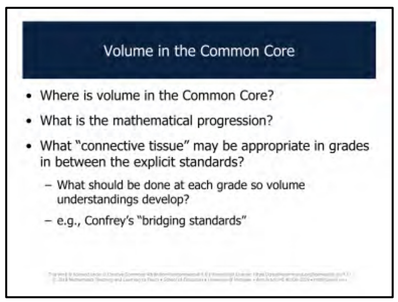
Detailed description of activity	Comments & other resources
4. Watch *Video D,* where teachers in the professional development predict whether students would find the NAEP task from 2011 to be easier than the task from 1996. Dr. Clements reveals that 54% of students answered this task correctly, and teachers think about reasons this task might be easier for students than the previous task. Dr. Sarama discusses why the percentages on both of these tasks might be so low.  Ask participants to consider for themselves why students might have scored so much higher on this item than the previous item, even though this item involves a comparison.	*In Video D, a teacher wonders whether students performed better on this item than they did on the previous one because instruction related to volume had improved from 1996 to 2011. Dr. Sarama explains that she doubts that this is the case, pointing out that measurement is typically not a high priority for instruction in the elementary grades, and it is the area in which students consistently score the lowest.*
5. Next, watch *Video E,* where Dr. Clements discusses how students often think about volume tasks when they do not yet have a well-established 3-D spatial ability.  After viewing, ask participants which of these challenges seems the most crucial to address. If time allows, see if participants can generate a few ways that teachers across the k-5 grade span might provide experiences that would address these challenges.	*When students' 3-D spatial ability is not well developed, they often confound volume and surface area measurement.* *This last discussion point foreshadows work that will happen in session 9 to connect learning experiences with the volume learning needs of their students.*

Detailed description of activity	Comments & other resources
6. Conclude this part by watching *Video F,* where Dr. Clements applies the measurement concepts to volume measurement.   	*<u>Conservation</u>: Piaget's classic conservation tasks involved, first, establishing that two volumes were equal, then transforming one of the two and asking children whether they were still equal. For example, the image on the slide shows two identical containers filled with the same amount of water. Children should be able to recognize that the two containers hold the same amount of water. The water from the second container, then, is poured into a short, wide container. Children who conserve volume will recognize that the amount of water has not changed, even though it looks different.* *<u>Transitivity</u>: As with length and area, transitivity involves a recognition that if the volume of one object is smaller than the volume of a second object and, in turn, the volume of the second object is smaller than the volume of the third object, then the volume of the first object is smaller than the volume of the third object. Note: This is also true if the adjective is "larger" rather than "smaller".* *<u>Equal partitioning:</u> To measure the volume of an object or container, it is important to understand that the object or container must be partitioned into equal-size units for the measurement to make sense.* *<u>Units and unit iteration:</u> To measure volume (or capacity), the units must completely fill the container; there can be no gaps between units.* *<u>Additivity:</u> Additivity is the understanding that a volume can be broken into pieces, the volume of each piece can be found, and those volumes added together to equal the volume of the whole object.* *<u>Relation between number and measurement:</u> The relationship between volume and multiplication is valuable. Volume is a good way to understand multiplication of three numbers, but multiplication is also a good way to understand volume. The second piece is often overlooked. Additionally, understanding the relationship between the linear dimensions and the volume is an important concept; for example, if the linear dimensions of an object are doubled, the volume of the object is multiplied by 8 or 2^3.*

Part 5: Analyzing volume in standards for student learning (~20 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will recognize and identify volume measurement within the Common Core State Standards (CCSS). • Participants will understand connections between the CCSS standards for volume measurement across the grade levels. • Participants will recognize opportunities to work on the mathematical practices within the context of measuring volume.	1. Introduce the part and watch Video A. 2. Watch Video B. 3. Distribute the handouts, watch Videos C and D, and have participants explore how Common Core State Standards. address volume measurement. 4. Discuss in whole group, using Video D as well as Videos E-H as needed.	• Video A (01:10): How do we ask about volume • Video B (02:10): The math of volume measurement • Video C (01:37): The measurement of volume in the Common Core • Video D (03:17): The mathematical practices in volume standards • Video E (01:28): Volume as an application of number and operations • Video F (00:28): Grade level placement of standards • Video G (00:37): Sequencing understanding and the use of formulas • Video H (02:56): Foundations for volume in early grades • Handout: Content cubes – Common Core State Standards volume • Handout: Learning Trajectory display of measurement standards

Detailed description of activity		Comments & other resources
1. Introduce Part 5: In this part, participants explore the development of volume measurement in the Common Core State Standards. Watch *Video A*, where Dr. Clements poses some of the questions teachers must consider when deciding how to help students work on volume measurement.		*In Video A, Dr. Clements raises questions about how volume should be taught. For instance, should teachers talk about volume filling (i.e., liquid capacity) or packing (i.e., figuring out how many of something a container holds)? He points out that measuring liquid capacity tends to be easier for children because, when measuring with a graduated cylinder, they only need to consider one dimension (height) to figure out the volume. He points out that, when volume is taught in schools, "filling" is rarely related to liquid capacity.*

Detailed description of activity	Comments & other resources
2. Watch *Video B,* in which Dr. Clements reviews the two aspects of measurement and explores some of the decisions that must be made when designing volume measurement tasks.   	*In Video B, Dr. Clements reminds teachers that measurement of all kinds consists of (1) identifying a unit of measure and subdividing an object by that unit, and (2) iterating the object without gaps. He then discusses some of the things teachers must consider when designing volume tasks, including:* • *What unit should students be asked to use?* • *Should the measures of the edges of an object be labeled? Or will that lead students to apply a formula without conceptualizing the volume?* • *Should objects have grids on them?* • *Should volume tasks be represented on paper or should 3-D objects be used?* *All of these decisions have implications for the nature of the work students will have to do on these tasks and their opportunities to learn from them.* *In Video B, Dr. Clements makes a reference to the optional extension problem included in Part 2, which involved figuring out how many 1 ft. x 1 ft. x 2 ft. boxes could fit into the room.*
3. Distribute *Handout: Learning Trajectory display of measurement standards* and *Handout: Content cubes – Common Core State Standards volume.* Watch *Video C,* in which Dr. Clements introduces the activity of investigating how volume measurement is addressed in the Common Core State Standards. In this video, Dr. Clements poses questions participants should consider as they examine these documents, including  • Where is volume in the Common Core? • What is the mathematical progression? • What "connective tissue" may be appropriate in the grades in between the explicit standards?	*The Handout: Learning Trajectory display of measurement standards was also used in Session 1, so participants may already have a copy of this handout.* *In Video C, Dr. Clements points out that, because the Common Core State Standards are not a curriculum, teachers must figure out what experiences students need that will prepare them to meet the standards at the different grade levels.*

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<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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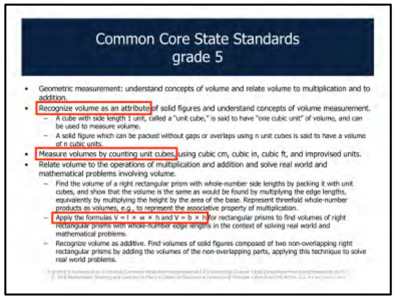
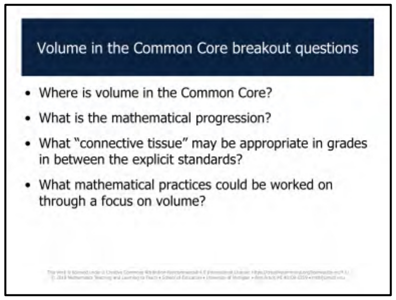
In this video, Dr. Clements shows the Grade 5 standards related to volume and asks participants to consider what kinds of experiences students should have before Grade 5 in order to be prepared to meet these standards.

At the end of the video, Dr. Sarama also asks participants to consider how work on volume measurement provides a context in which students can work on the Common Core Standards for Mathematical Practice. She poses a fourth question for consideration during this activity: **What mathematical practices could be worked on through a focus on volume?**

After viewing the video, give participants time to explore how the Common Core State Standards address volume measurement.





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Dr. Sarama uses the term "processes" in this video to refer to. the mathematical practices. In many cases the phrase "mathematical processes" is interchangeable with the phrase "mathematical practices".

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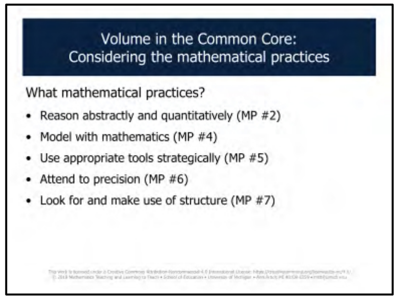
Detailed description of activity	Comments & other resources

Detailed description of activity

4. After participants have had time to review the handouts and record their ideas, encourage them to share what they noticed in the Common Core State Standards.

   Towards the end of the discussion, show *Video D,* where Dr. Sarama and the teachers in the professional development discuss how work on the standards for volume measurement might involve the use of mathematical practices.

   Continue this discussion, asking participants to identify how the following mathematical practices could be worked on in the context of volume measurement:

   - Reason abstractly and quantitatively (MP #2)
   - Model with mathematics (MP #4)
   - Use appropriate tools strategically (MP #5)
   - Attend to precision (MP #6)
   - Look for and make use of structure (MP #7)

   If applicable at any point during the discussion, show one or more of the following videos (described in the right-hand column of this guide) to support the discussion:

   - *Video E: Volume as an application of number and operations*
   - *Video F: Grade level placement of standards*
   - *Video G: Sequencing understanding and use of formulas*
   - *Video H: Foundations for volume in early grades*

Comments & other resources

One of the purposes of this discussion is to help participants recognize how work on volume measurement (and measurement more generally) can provide rich opportunities to focus on several of the mathematical practices. Working with students on measurement is not only important because it corresponds with particular content standards, but it also provides an important context for work on the standards of mathematical practice.

Some of the ways in which work on volume measurement might involve the use of specific mathematical practices are:

- *Reason abstractly and quantitatively (MP #2) is involved when students develop a formula for measuring volume*
- *Modeling with mathematics (MP #4) is involved when students try to represent situations involving volume.*
- *Use appropriate tools strategically (MP #5) is involved when students are able to select and use a tool (e.g., a beaker with a measurement scale, a ruler) to measure volume*
- *Attend to precision (MP #6) is involved when students choose units decide to use whole and/or portions of units*

Looking for and making use of structure (MP #7) is involved when students are able to conceptualize the spatial structure of rows, columns, and layers of units Video E: Volume as an application of number and operations

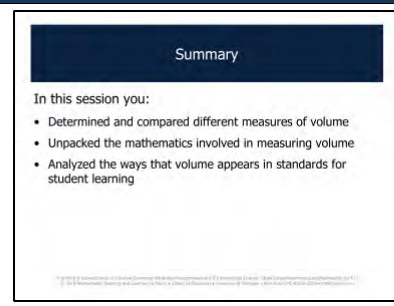
In this video, a teacher points out that, in fourth grade, volume seems to be an application of work on number and operations rather than a concept to be developed itself.

Note: While this video is playing, the right-hand side of the viewer displays slides that show the standards being discussed. Once the video is finished playing, the arrows at the bottom of the viewer can be used to return to slides that were shown during the video.

Detailed description of activity	Comments & other resources
	Video F: Grade level placement of standards *In this video, Dr. Sarama and teachers discuss their surprise that the standard involving "packing a right rectangular prism with unit cubes" was not in an earlier grade level.* *Video G: Sequencing understanding and the use of formulas* *In this video, Dr. Clements refers to research that suggests that it is more difficult for students to figure out the meaning of formulas they have already learned by rote than it is for students to develop understanding of how to solve a type of problem and then transition to using a formula.* *Video H: Foundations for volume in early grades* *In this video, Dr. Sarama acknowledge that there is not much explicit work on volume in the early grades. A teacher points out, though, that the Common Core does provide a foundation for work on volume by expecting students to develop a deep understanding of length measurement and area measurement in the younger grades.*

Part 6: Wrap Up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom.	1. Summarize the work of the session. 2. Explain and distribute the Classroom Connection Activities.	• Handout: Classroom Connection Activity 1 - Volume

Detailed description of activity	Comments & other resources
1. Summarize the session by emphasizing that participants • Determined and compared different measures of volume • Unpacked the mathematics involved in measuring volume • Analyzed the ways that volume appears in standards for student learning 	
2. Distribute the *handout* you customized with the Classroom Connection Activities. Introduce the two volume assessment tasks that are included in the handout: • Making a block that has the same volume as a 2 x 3 x 2 prism • Identifying the box that has the same volume as the liquid in a graduated cylinder Also explain that participants may want to start looking for a lesson in their curriculum materials that is relevant to volume. They will need this for Session 9. Give instructions about what participants should complete before the next session. <u>Required</u>: • Complete the two provided tasks with 2-3 students of different (hypothesized) achievement levels • Ask the students to write down and/or draw how they measured • Video record this to compare the video with what students drew and wrote	