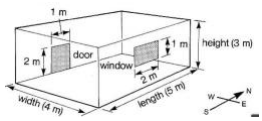
Session 7: Volume Learning Trajectory – Mathematical goals



Discussing the CCA from the previous session

- What did you do?
- What mathematical practices did you see?
- If you engaged students with area…
  - What levels of the learning trajectory did you see?
  - Did the activity promote new levels of thinking?
- Did you use the note taking form? How did you record on the form (text, pictures, …)?

Overview of Session 7

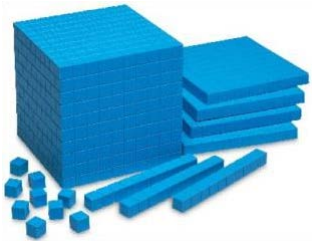
- Measuring the room in a new way
- What is volume and how do students think about volume?
- Analyzing the ways that volume appears in standards for student learning
- Unpacking the mathematics involved in measuring volume

Learning trajectories approach

- Goal
- Developmental Progression
- Instruction

Measuring volume: Cubic units

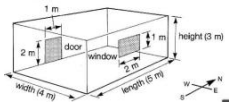
- Here's a..
  - Cubic centimeter
  - Cubic decimeter



Measuring the volume of the room

- Here's a cubic decimeter
- How many cubic decimeters does this room hold?

  Consider the room as if it were a rectangular prism



- Here's a cubic meter to help you…
- Work individually for 10 minutes
- Discuss your results with a partner for 5 minutes
- Share methods and results—whole group

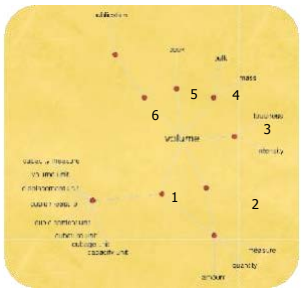
Discussion

- How did different selections and methods affect the resulting measurements?
- How did you deal with partial units?
- What about fluid capacity?

Reflection

What did you learn
or what are you now thinking about volume?

What is volume?



1. The amount of 3-D space occupied by an object
2. A relative amount
3. The magnitude of sound (usually in a specified direction)
4. The property of something that is great in magnitude
5. Physical objects consisting of number of pages bound together
6. A publication that is one of a set of several similar publications

http://www.visualthesaurus.com

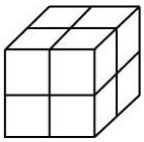
What is volume? Students' ideas (Part 1)

- The number of cubes in a shape
- How big it is
- How "loud" or "soft" sounds/music are
- How many things that would fit inside of it
- Height
- Like how much, how many there are
- It's like levels of some sort

What is volume? Students' ideas (Part 2)

- Size or shape or how many square units it is
- The number of shapes that you can put in a shape
- To find the volume you would measure the height
- It is the mass

1996 NAEP assessment: Grade 4



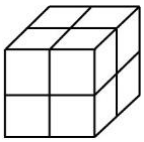
What is the correct answer?

How many 4th graders answered it correctly in 1996?

In this figure, how many small cubes were put together to form the large cube?

A. 7
B. 8
C. 12
D. 24

1996 NAEP assessment: Grade 4 result



What is the correct answer?

33% of our nation's Grade 4 students answered this item correctly in 1996

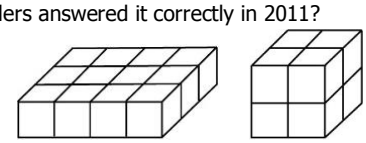
In this figure, how many small cubes were put together to form the large cube?

A. 7
B. 8
C. 12
D. 24

2011 NAEP assessment: Grade 4

What is the correct answer?

How many 4th graders answered it correctly in 2011?



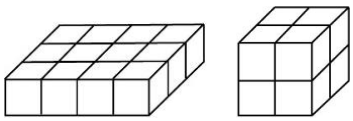
How many more small cubes were used to make Solid *A* than Solid *B*?

A. 2
B. 4
C. 6
D. 7

2011 NAEP assessment: Grade 4 result

What is the correct answer?

54% of our nation's Grade 4 students answered this item correctly in 2011



How many more small cubes were used to make Solid *A* than Solid *B*?

A. 2
B. 4
C. 6
D. 7

Volume measurement

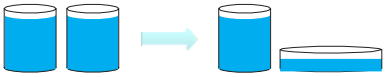
- Volume measurement is difficult for students.
- Students cannot solve volume measurement tasks correctly
  - Applying the volume formula incorrectly
  - Counting the number of cubes in 3-D arrays incorrectly
  - Confounding volume and surface area measurement
  - Little understanding of a unit of volume

(Battista & Clements, 1996; Ben-Haim, Lappan, & Houang, 1985; Enochs & Gabel, 1984)

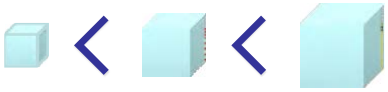
Measurement Concepts – Volume (Part 1)

Understanding of the attribute of volume

Conservation

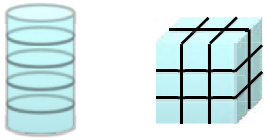


Transitivity



Measurement Concepts – Volume (Part 2)

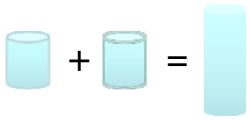
Equal partitioning



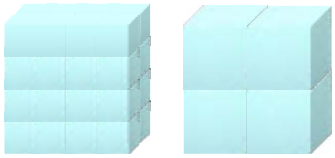
Units and unit iteration

Measurement Concepts – Volume (Part 3)

Additivity



Relation between (discrete) number and measurement



How do we ask about volume?

- Are we asking about how much the container holds?
  - What about filling vs. packing?
- Are we asking about how much actual space the materials making up the container occupy?
- Are we asking about the largest exterior dimensions of the object and how much space it appears to occupy?

The mathematics of volume measurement

- Measuring consists of two aspects:
  - Identifying a unit of measure and subdividing (mentally and physically) the object by that unit, and
  - Iterating, filling the object being measured without gaps
- Compare with area measurement….

Volume measurement task design variations

- Units
  - Cubic units (e.g. cm cube, inch cube)
  - Linear dimensions (label side lengths or provide a ruler)
  - Grid forms on prisms
  - Missing or all given units
- Task Representation
  - 2D (on paper)
  - 3D (real objects)

Volume in the Common Core

- Where is volume in the Common Core?
- What is the mathematical progression?
- What "connective tissue" may be appropriate in grades in between the explicit standards?
  - What should be done at each grade so volume understandings develop?
  - e.g., Confrey's "bridging standards"

Common Core State Standards grade 5

- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
  - A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
  - A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
  - Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
  - Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumesof right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
  - Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Volume in the Common Core: Considering the mathematical practices

What mathematical practices?

- Reason abstractly and quantitatively (MP #2)
- Model with mathematics (MP #4)
- Use appropriate tools strategically (MP #5)
- Attend to precision (MP #6)
- Look for and make use of structure (MP #7)

Volume in the Common Core breakout questions

- Where is volume in the Common Core?
- What is the mathematical progression?
- What "connective tissue" may be appropriate in grades in between the explicit standards?
- What mathematical practices could be worked on through a focus on volume?

CCSSM 4.MD

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

CCSSM 5.MD.5

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
   a) Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
   b) Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

Summary

In this session you:

- Determined and compared different measures of volume
- Unpacked the mathematics involved in measuring volume
- Analyzed the ways that volume appears in standards for student learning