## *Description of the Session*

Session 8 extends the work from Session 7 that focused on studying the mathematics of volume measurement. This session focuses on the second component of the Learning Trajectory for volume measurement: the developmental progression of learning to measure volume. First, participants will interpret their own students' performance on volume tasks using what they already know about volume measurement. Then, they will be introduced to the levels of the developmental progression in the volume Learning Trajectory. Next, they will use what they have learned about the Learning Trajectory levels to predict how students at certain levels might approach different tasks and to discuss ways to modify tasks to make them more accessible to students at different levels. After that, they will practice using the anecdotal notes form to observe and describe students' performance on measurement tasks and identify the level at which they are performing. At the end of the session, participants will be introduced to the Classroom Connection Activities they will complete prior to Session 9.

## *Activities and goals of the session*

| Activities | Times | Corresponding parts of the session | Goals |
|---|---|---|---|
| I. Overview of session and discussion of CCAs | 10 minutes | Parts 1 & 2 | • Participants will be oriented to the work of the session.<br>• Participants will describe their students' thinking on the volume tasks. |
| II. Studying student thinking: Introduction to the developmental progression for volume | 60 minutes | Parts 3, 4, & 5 | • Participants will recognize and understand the early levels of the Learning Trajectory for volume measurement.<br>• Participants will recognize and understand the later levels of the Learning Trajectory for volume measurement.<br>• Participants will use knowledge of the Learning Trajectory levels to anticipate student responses to volume tasks.<br>• Participants will consider ways to modify tasks to make them more accessible to students. |
| III. Taking anecdotal notes to identify students' Learning Trajectory levels | 15 minutes | Part 6 | • Participants will demonstrate understanding of the Learning Trajectory for volume measurement.<br>• Participants will be able to take descriptive notes while watching a student measure and then use those notes to later interpret the student's thinking. |
| IV. Wrap up | 5 minutes | Part 7 | • Participants will understand ways of connecting the session content to their classroom. |

## *Classroom Connection Activities*

| **Required** |
|---|
| Type of task: Assessment task/ Collecting records of practice<br>Description: Complete volume tasks with 3-4 students of different (hypothesized) achievement levels. Ask the students to write down or draw how they measured. Use the anecdotal notes form to record how students engage in the task and the Learning Trajectory level that is associated with the students' performances. Respond to the reflection questions listed on the CCA handout. |
| Type of task: Analysis of curriculum materials<br>Description: Select a lesson or a short sequence of instructional activities from your curriculum that could be used to support learning about volume measurement. Bring these materials with you to the next session. |

## *Preparing for the session*

- ☐ Make copies as needed:
  - Resources: Handout: Comment Cubes – Common Core State Standards Volume (Part 2); Handout: Content cube – Volume Learning Trajectory (Parts 3 & 4); Handout: Anecdotal notes form – Volume Learning Trajectories (copies for Part 6 & for the CCA)
- ☐ Customize and make copies of the Classroom Connection Activities
- ☐ Test technical setups: Internet connection, speakers, projector

## *Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)*

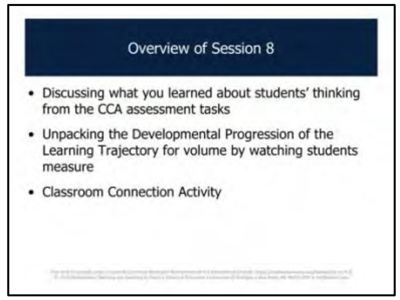
1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

***Scope of the module*** *(focal content of this session in* ***bold****)*

| Mathematics | Student thinking | Teaching practice | Learning from practice |
|---|---|---|---|
| • recognizing the mathematical goal as the first component of a complete Learning Trajectory<br>• understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement)<br>• understanding how measurement of length, area, and volume are represented and developed in the CCSS<br>• understanding how measurement connects with the CCSS standards for mathematical practice<br>• understanding concepts and skills involved in measuring length, area, and/or volume<br>• understanding connections between length, area, and volume measurement and between metric measurements for each | • **recognizing student development as the second component of a complete Learning Trajectory**<br>• **understanding children's development of measurement through Learning Trajectories for length, area, and volume**<br>• recognizing principles of measurement in student work<br>• interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement<br>• interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement<br>• **interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement** | • recognizing instruction as the third component of a complete Learning Trajectory<br>• **using anecdotal notes to document what students say and do when working on measurement tasks**<br>• connecting measurement activities in curricula to measurement Learning Trajectory levels<br>• modifying measurement tasks to target different and/or particular Learning Trajectory levels | • understanding the anecdotal notes workshop process<br>• using the anecdotal notes workshop to improve the practice of note taking<br>• using the anecdotal notes workshop to improve teaching |

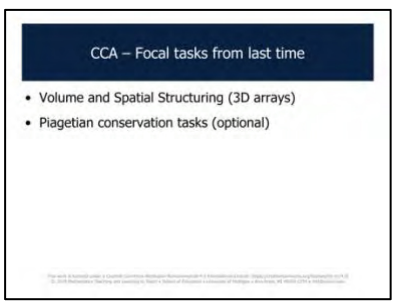
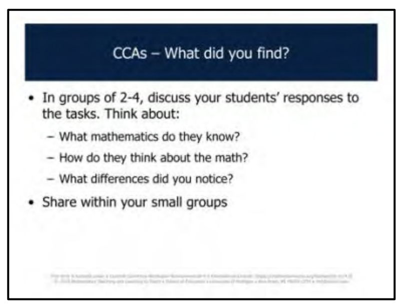
## *Part 1: Overview (~5 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will be oriented to the work of the session. | 1. Introduce the session and watch *Video A.* | |

| Detailed description of activity | | Comments & other resources |
|---|---|---|
| 1. Introduce the session by watching Video A. This session continues work on volume measurement by focusing on the developmental progression portion of the Learning Trajectory for volume measurement. In this session, participants will<br>• Discuss what they learned about student thinking from the Classroom Connection Activity assessment tasks<br>• Unpack the developmental progression of the Learning Trajectory for volume by watching students measure<br>• Learn about the Classroom Connection Activity that they will complete before the next session |  | |

## *Part 2: Sharing experiences from the Classroom Connection Activity (~5 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will describe their students' thinking on the volume tasks. | 1. Watch Video A and have participants share their students' responses on the volume tasks in small groups and then with the whole group.<br>2. Conclude this part by watching Video B. | • Video A (01:07): Discussing the mathematics revealed through the CCA<br>• Video B (00:57): Focusing on what students are thinking<br><br>***Supplements***<br><br>• Handout: Content cubes – Common Core State Standards volume |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 2 showing Video A, in which Dr. Sarama reviews the tasks included in the Classroom Connection Activity from the previous session and launches a discussion of students' work on these tasks.<br><br>Allow participants a few minutes to discuss their students' thinking in small groups, focusing on the following questions:<br>• What mathematics do students know?<br>• How do students think about the math?<br>• What differences do you notice (across tasks, students, and grade levels)?<br><br><br><br>After a few minutes, have participants briefly share their observations with the whole group. | *This is meant to be a quick launch activity for work in this session to understand the developmental progression in volume, so do not take time for an in-depth discussion. The main goal is for participants to consider their own students' thinking about mathematical ideas related to volume measurement.*<br><br>*Handout: Content cubes – Common Core State Standards volume, which was introduced in the previous session, may serve as a helpful reference during this discussion.* |

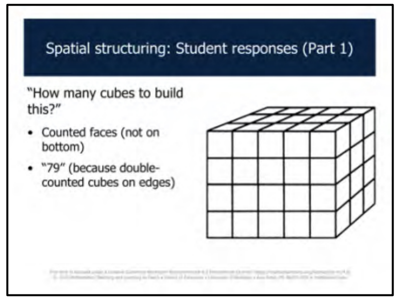
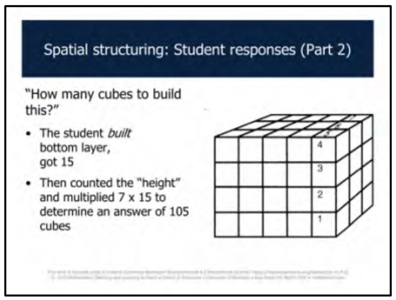
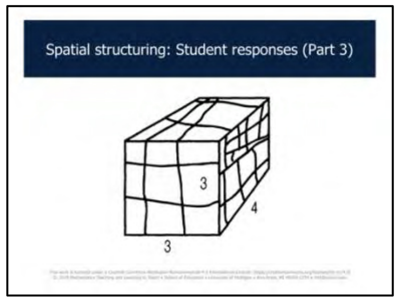
| Detailed description of activity | Comments & other resources |
|---|---|
| 2. Conclude this part by watching Video B, in which Dr. Sarama and Dr. Clements encourage | *In Video B, Dr. Sarama points out that interviewing* |

| Detailed description of activity | Comments & other resources |
|---|---|
| participants to think about their own students as they begin to examine the levels of the developmental progression. | *students about a sequence of tasks (which participants did as part of the CCA) not only provides information to the teacher about students' thinking but it can also serve as a learning experience for the students.*<br>*Probing students' thinking about a sequence of strategically designed tasks can help push their thinking forward.* |

## Part 3: Learning Trajectories – early levels (~25 minutes)

| Goals | Instructional sequence | Resources |
|---|---|---|
| • Participants will recognize and understand the early levels of the Learning Trajectory for volume measurement. | 1. Introduce Part 3 and watch Video A.<br>2. Distribute the Handout; watch and discuss Videos B and C to introduce the developmental progression for volume and the Volume Quantity Recognizer level.<br>3. Introduce the Volume Filler level by watching and discussing Videos D and E.<br>4. Introduce the Volume Filler level by watching and discussing Videos F and G.<br>5. Introduce the Volume Unit Relater and Repeater level by watching and discussing Videos H and I. | • Video A (03:33): Students' thinking about volume and spatial structure<br>• Video B (01:57): Overview of developmental levels for volume<br>• Video C (00:50): Commentary on "comparing the volume of liquids"<br>• Video D (02:15): Placing cubes in an open box<br>• Video E (02:36): Commentary on "placing cubes in an open box"<br>• Video F (01:02): Building a 3D structure<br>• Video G (02:55): Commentary on "building a 3D structure"<br>• Video H (00:56): Scoops of rice in a container<br>• Video I (01:31): Commentary on "scoops of rice in a container"<br>• Handout: Content cube – Volume Learning Trajectory |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 3: In this part, the early levels of the Learning Trajectory for volume will be introduced and illustrated using video examples of students who are working on various measurement tasks.<br><br>Remind participants that, in the previous session, they discussed the "goal" of the Learning Trajectory for volume measurement (i.e., the mathematical ideas involved in volume measurement) and they started considering examples of student thinking about volume. Now, they will focus on the developmental progression piece of the Learning Trajectory.<br><br>Have participants watch Video A, where Dr. Clements explains why he and his colleagues began to investigate the learning progression for volume measurement—because they were interested in understanding how students develop the capacity for spatial structuring (i.e., seeing things in 3-D space as being "organize-able" into rows, columns, and layers). In this video, Dr. Clements presents student responses to volume tasks and discusses how responses like these prompted him and his colleagues to investigate how students developed the spatial structuring ability needed for determining volume. | *In Video A, Dr. Clements presents students' responses to two volume tasks. One task involves figuring out how many cubes are in a 5 x 3 x 4 rectangular prism. (Note: a 2-D image of this structure is included on the slides, but children whose responses are discussed were asked to work with a 3-D prism.) The second task involves drawing lines on a 2-D figure of a rectangular prism (labeled with measures of length, width, and height) to show its structure.* |

| Detailed description of activity | Comments & other resources |
|---|---|
|     | *Slide: Spatial Structuring: Student responses (Part 1) In Video A, Dr. Clements explains that some students figured out "how many cubes" were in the rectangular prism by counting all of the visible faces of the cubes.*<br><br>*Children who used this strategy did not seem to attend to the cubes (or faces) that were not visible, nor did they seem to recognize that they were double (or triple) counting cubes with two (or three) visible faces.* |
|     | *Slide: Spatial Structuring: Student responses (Part 2) When a student was asked to build the bottom layer of the prism as a way to help solve this task, he correctly built a 5 x 3 array of cubes and figured out that there were 15 cubes in that layer. However, to figure out how many layers were in the prism, he counted not only a column of 4 cubes on the side of the prism, but also a "column" of 3 cubes across the top of the prism (see slide). Thus, he calculated the volume by multiplying 15 by 7 (rather than by multiplying 15 by 4).*<br><br>*Slide: Spatial Structuring: Student responses (Part 3) When a child was asked to add lines to a figure of a 3 x 4 x 3 rectangular prism to show the units that comprised it, she did not seem to recognize the row or column structure of this prism.*<br><br>*All three of these student responses reflect a lack of spatial structuring.* |

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<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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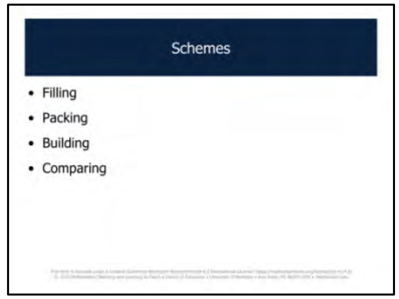
2. Distribute Handout: Content cube – Volume Learning Trajectory, and introduce the research- based, developmental progression as a part of the Learning Trajectory for volume measurement by watching Video B. In this video, Dr. Clements first explains that children have multiple schemes for thinking about volume, including filling, packing, building, and comparing.

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*Direct participants to the Content cube handout to support their recall and later use of the ideas that are discussed.*

*Schemes for thinking about volume, include*

- *A filling scheme (e.g., pouring into a container)*
- *A packing scheme (e.g., fitting as many cubes as possible into a box)*
- *A building scheme (e.g., building structures with cubes without the constraints of a box)*
- *A comparing scheme (e.g., comparing the volumes of two containers or structures)*

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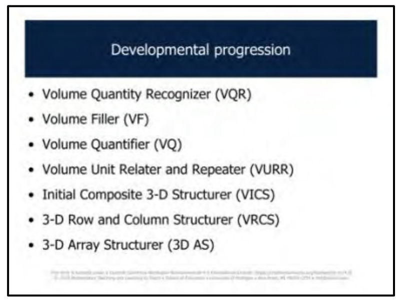
Dr. Clements then gives an overview of the levels of the developmental progression for volume:

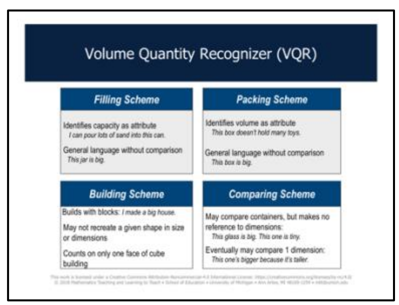
- Volume Quantity Recognizer (VQR)
- Volume Filler (VF)
- Volume Quantifier (VQ)
- Volume Unit Relater and Repeater (VURR)
- Initial Composite 3-D Structurer (VICS)
- 3-D Row and Column Structurer (VRCS)
- 3-D Array Structurer (3DAS)




The end of Video B shows an example of a student that is starting to get to the Volume Quantity Recognizer (VQR) level.

Following the video, have participants discuss the question: **What characterizes the student's thinking?**

After that, watch Video C, where Dr. Clements describes the Volume Quantity Recognizer level.

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*Part 3 introduces the early levels of the developmental progression for volume measurement:*

- *Volume Quantity Recognizer (VQR)*
- *Volume Filler (VF)*
- *Volume Quantifier (VQ)*
- *Volume Unit Relater and Repeater (VURR) The later levels will be discussed in Part 4.*

*When using the names of these levels, continue to keep in mind that the **student** is not the level; instead, the student's **behavior in this case** corresponds with a particular level.*

*In this example, a student is asked which jar might hold more water. He points to the bigger jar and justifies his response by saying that this jar is "big small" and "short big."*

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<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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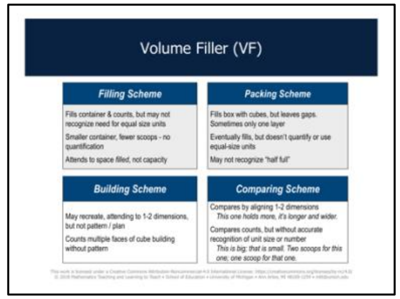
3. Have participants watch Video D, which shows an example of the Volume Filler (VF) level. Then discuss: **Why is this child an example of a Volume Filler?**




After participants have responded to this question, watch Video E, where Dr. Clements and teachers discuss characteristics of the Volume Filler level. Dr. Clements explains that, though these characteristics are not visible in this particular example, Volume Fillers do not attend to equal-sized units when determining volume, and they tend to focus only on two dimensions of objects when attempting to compare their volumes.

Allow time for any follow-up questions/discussion as time and need permit.

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*In Video D, the student is asked to fill an open box with cubes. She packs the box without seeming to attend to the gaps between cubes. When the interviewer prompts her to consider whether there are any spaces between the cubes, she adds a couple more cubes to fill in the gaps in the top layer of the box.*

*In Video E, a teacher asks a question about the part of the description for the Volume Filler level that says that students "eventually" pack an entire box leaving no gaps. Dr. Clements explains that students at this level are generally only able to do this with support or prompting. They may also begin to do this as they are transitioning to the next level. However, most students at this level leave gaps when packing boxes. Some may even consider a box to be filled after packing only one layer.*

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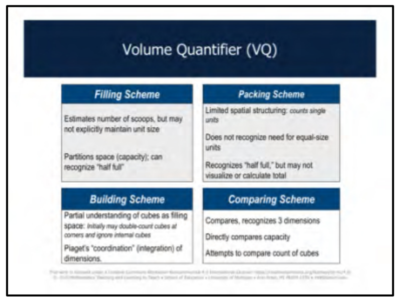
4. Next, watch and discuss Video F, an example of the Volume Quantifier (VQ) level. Ask: **What characterizes this student's thinking?**




Then, have participants watch Video G, where Dr. Clements discusses the video of the child and talks about the description of the Volume Quantifier level.

Allow time for any follow-up questions/discussion as time and need permit.

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*In Video F, the child is asked to use cubes to build a structure that matches a 2-D picture of a rectangular prism. She builds the "front" and the "side" of the prism that are visible in the picture, without recognizing that one column of cubes is part of both the front and the side of the prism. As a result, she builds an extra column of cubes.*

*In Video G, Dr. Clements explains that this student is having difficulty with what Piaget calls "coordination of dimensions." When translating the 2-D image to a 3-D structure, she is not able to see that one column of cubes in one dimension is also part of another dimension.*

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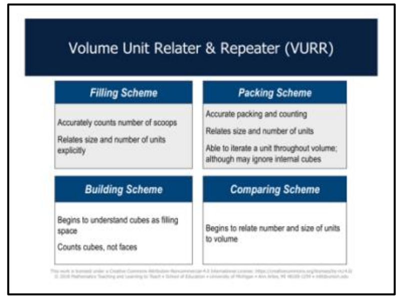
| Detailed description of activity | Comments & other resources |
|---|---|
| 5. Conclude this part by watching Video H, an example of the Volume Unit Relater and Repeater level. Again, ask participants to discuss: **What characterizes this student's thinking?**<br><br>After participants have had time to discuss, watch Video I, where Dr. Clements describes the characteristics of the Volume Unit Relater and Repeater level. In this video, he also points out that students do not naturally see the relationship between "filling" and "packing", and it is important for teachers to design experiences that will help them make this connection.<br><br> | *In Video H, a student is asked to predict how many scoops of rice it will take to fill a container. She has previously filled the container with one large scoop, and now the interviewer has asked her to predict how many scoops it will take when she uses a scoop that is half the size of the first scoop. She is able to correctly predict how many scoops it will take to fill the container by comparing the size of the smaller scoop to the size of the larger scoop.*<br><br>*In Video I, Dr. Clements explains that the student is at the Volume Unit Relater and Repeater level because she is able to relate the size and the number of the scoops in order to anticipate (and then count) the number of scoops it will take to fill a container (i.e., iterating the scoops to find the volume).* |

## Part 4: Learning Trajectories – later levels (~20 minutes)

| Goals | Instructional sequence | Resources |
|---|---|---|
| • Participants will recognize and understand the later levels of the Learning Trajectory for volume measurement. | 1. Introduce Part 4 and watch and discuss Videos A and B to introduce the Initial Composite 3-D Structurer level.<br>2. Watch and discuss Videos C and D to introduce the 3-D Row and Column Structurer level.<br>3. Watch and discuss Videos E and F to introduce the 3-D Array Structurer level<br>4. Discuss how these levels fit with what participants have seen with their students. | • Video A (00:41): Using a small set of cubes to find volume<br>• Video B (01:22): Commentary on "using a small set of cubes to find volume"<br>• Video C (02:04): Using one cube to find volume<br>• Video D (01:52): Commentary on "using one cube to find volume"<br>• Video E (03:39): Outlined containers<br>• Video F (01:12): Commentary on "outlined containers"<br>• Handout: Content cube – Volume Learning Trajectory |

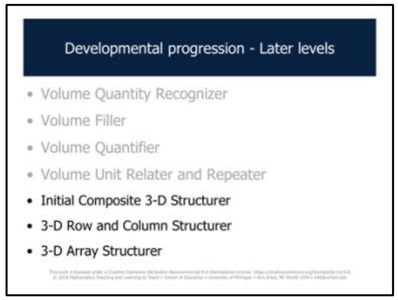
**Detailed description of activity**

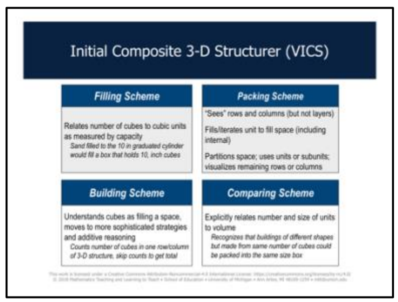
1. Introduce Part 4: In this part, participants will be introduced to the later levels of the Learning Trajectory for volume measurement.

   Have participants watch Video A, which illustrates the Initial Composite 3-D Structurer level. Discuss**: Why is this child an example of an Initial Composite 3-D Structurer?**

   **What characterizes the student's thinking?**

   Then have participants watch Video B, where Dr. Clements describes behaviors that characterize the Initial Composite 3-D Structurer (VICS) level. Following the video, allow time for follow-up questions or discussion as time and need permit.

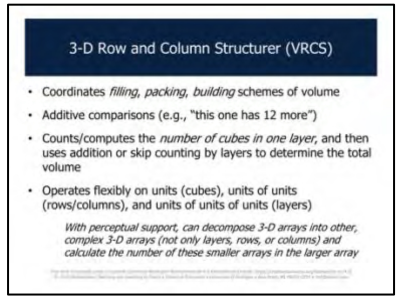


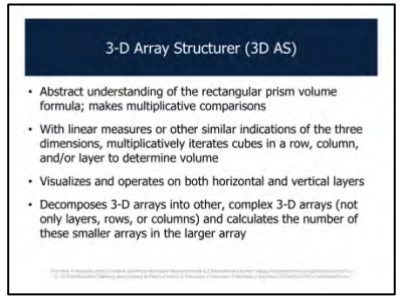




**Comments & other resources**

*In Video A, the interviewer shows a small group of students an open box that has one row of five cubes, and he asks them if they could use those five cubes to figure out how many cubes would be needed to fill a complete layer of the box. The students figure this out by counting by fives as they estimate how many rows of five it would take to fill the entire layer. Dr. Clements points out that these students have started to use composites, or "units of units."*

*In Video B, Dr. Clements points out that these students were counting a composite unit rather than counting each cube individually. However, they do not yet realize that the number of cubes along the other dimension of the bottom layer would be an indicator of how many of those composite units would be needed to fill the entire layer. Because they are skip counting rather than multiplying, they are still using additive reasoning to determine how many cubes would be needed to fill the bottom layer of the box.*

| Detailed description of activity | Comments & other resources |
| --- | --- |
| 2. Next, view Video C, which illustrates an example of the 3-D Row and Column Structurer (VRCS) level. Discuss: **Why is this child an example of a 3-D Row and Column Structurer? What characterizes the student's thinking?**<br><br>Then, watch Video D, where Dr. Clements describes the distinguishing characteristics of the 3-D Row and Column Structurer level.<br><br> | *In Video C, a student is given a figure of an empty box that has grids on the two sides to show the area of those sides (4 x 3 units and 6 x 3 units, respectively). She is asked "How many cubes would it take to fill this box?" To solve this problem, she counts the number of square units on one visible side of the box (12 square units) and then counts the length of the other visible side of the box (6 units). She then multiplies 12 x 6 to get an answer of 72 cubes.*<br><br>*In Video D, a teacher asks whether or not the three parts of the description of the 3-D Row and Column Structurer (on the content cube handout) are distinct sub-levels of this level. Dr. Clements explains that these parts of the description are not separate—students who are at the 3-D Row and Column Structurer level will demonstrate all three of these characteristics. However, Dr. Clements points out that students may develop these skills at different times as they are progressing toward this level.*<br><br>*Dr. Clements also points out that, while the student in Video C is using multiplication to solve this problem, she is not yet using multiplicative thinking to figure out the volume of the cube (which would be a characteristic of the 3-D Array Structurer Level). Instead, she is conceptualizing each layer of 12 cubes and then multiplying by the number of layers that would be in the box. This is slightly different from being able to compute the volume by multiplying length times width times height.* |

| Detailed description of activity | Comments & other resources |
|---|---|
| 3. Have participants watch Video E, an example of a student who is on the verge of the 3-D Array Structurer (3D AS) level. Discuss: **What characterizes this student's thinking?**<br><br>Then, view Video F, where Dr. Clements talks about the 3-D Array Structurer level and describes evidence from the video that the student is progressing toward that level. He also points out that students are commonly asked to use the formula for volume even though many have not yet reached this level of understanding. He argues that in order for students to actually understand the formula for volume, they need to go through these levels. | *Video E shows a student who is progressing toward the 3-D Array Structurer level. The student is presented with an outline of a container and cubes that that partially fill it in. In the partial structure, there is one row of 10 cubes to show the length of the complete container, one row of 8 cubes to show the width, and one tower of 5 cubes to show the height. The bottom layer is partially filled in with a 5 x 4 array of cubes (including the cubes along the edges of the layer). To figure out how many cubes would fill the entire container, the student first determines the volume of the portion of the structure that would have the 5 x 4 array as its base, seeing 20 cubes in that layer and iterating that layer up the height, counting by 20s. (he calculates 100 cubes in that section). Then, with prompting from the interviewer, he determines that 4 of those smaller structures would comprise the entire structure, so he multiplies 100 by 4 to get his final answer of 400 cubes.*<br>*In Video F, Dr. Clements explains that, while this student is not yet at an abstract level of understanding, he is able to imagine the 3-D arrays that would compose the entire container. If he had been able to visualize the entire container, determine the number of cubes along each edge of the container, and multiply these together to find the volume, he would have demonstrated a more abstract understanding of the formula for volume and could have been classified as a 3-D Array Structurer.* |
| 4. Conclude this part by having participants look at all of the levels in the volume developmental progression. Discuss: **How do the levels make sense in view of your assessments and our work here?**<br>Encourage participants to think about how these levels relate to the measurement skills of the students in their classrooms. | |

## *Part 5: Using Learning Trajectory levels to predict students' approaches to tasks (~15 minutes)*

***Goals***

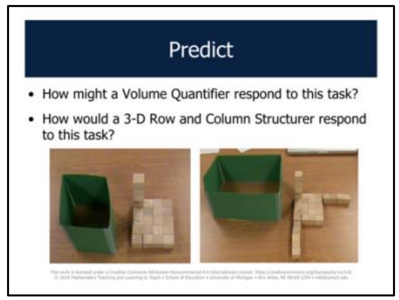
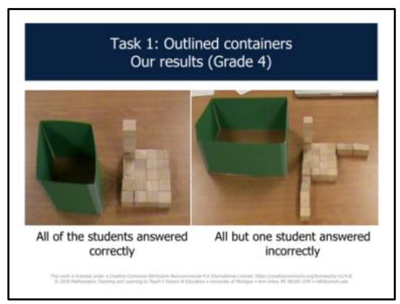
- Participants will use knowledge of the Learning Trajectory levels to anticipate student responses to volume tasks.
- Participants will consider ways to modify tasks to make them more accessible to students.

***Instructional sequence***

1. Introduce Part 5 and watch and discuss Videos A and B.
2. Discuss possible modifications to the outlined container task and watch Video C.
3. Watch Video D and discuss the volume of the solid task; compare this to the previous task.
4. *Optional*: If time permits, watch Videos E and F and discuss the drawing vs. building task.

***Resources***

- Video A (00:35): Predicting student responses to the outlined container
- Video B (02:11): Outlined container 4th grade results
- Video C (01:02): Modifying the outlined containers task
- Video D (00:54): Student responses to the volume of the solid task
- Video E (00:38): Drawing vs. building task
- Video F (02:08): Commentary on "drawing vs. building"

| Detailed description of activity | Comments & other resources |
| --- | --- |
| 1. Introduce Part 5: In this part, participants will consider how students at different levels of the developmental progression for volume measurement might respond to particular volume tasks.   Watch Video A, where Dr. Clements asks how a Volume Quantifier or a 3-D Row and Column Structurer would respond to the two versions of an "outlined container" task that are displayed on the slides.<br><br>Give participants time to discuss this question and to think about whether students would respond differently to the different versions of this task.<br><br>Then, watch Video B, where Dr. Clements discusses this question with teachers and then summarizes how the fourth graders he studied responded to these tasks. He explained that all fourth graders were able (after struggling) to solve the first task correctly but only one student in the group was able to solve the second task correctly.  <br><br>Have participants discuss the question Dr. Clements posed to teachers in the professional development (at the end of Video B): **how might this task be modified to make it more accessible to the fourth graders?** | *In Video B, a teacher predicts that students at the Volume Quantifier level would try to solve the problem by filling the entire container with cubes. Another teacher predicts that students at the 3-D Row and Column Structurer level might want to manipulate the cubes when trying to solve the problem.*<br><br>*Dr. Clements predicts that, if students at the Volume Quantifier level did not have enough cubes to fill the container, they would likely just make a guess about how many cubes there were.*<br><br>*He then explains that, for the group of eight, fourth graders he interviewed, the second version of the task was much more difficult than the first version. He points out that what might seem (to adults) like minor task modifications can make a huge difference to how children respond to these tasks.* |

<table>
<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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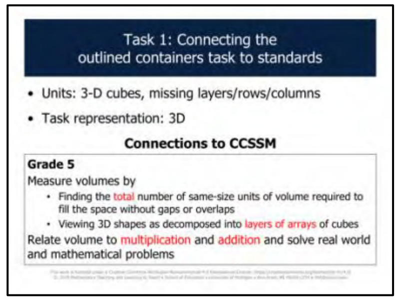
2. Then, watch Video C, where Dr. Clements explains that he and his colleagues found that this task became more accessible to the students once the interviewers removed the cubes that were not at the edges of the container.

   He then points out how these tasks relate to the Grade 5 Common Core State Standard that asks students to "relate volume to multiplication and addition and solve real world and mathematical problems."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*In Video C, a teacher asks if the students were allowed to move the "extra" cubes when they originally approached the task. Dr. Clements explains that, while they would have been allowed to do this, they usually did not attempt to do this. He points out that in order to recognize that the potential benefit of removing those cubes, students would need to understand the nature of their difficulty and how to address it (which would draw on higher levels of understanding of volume than the students had at that time).*

*Using the outlined container task to support students' learning of the Common Core State Standards:*

*The Common Core expects students at Grade 5 to "Measure volumes by viewing 3-D shapes as decomposed into layers of arrays of cubes." The first version of the outlined container task, which provides students with a complete bottom layer as well as cubes that indicate the height of the container, can support them in visualizing the layers of arrays of cubes that would comprise the entire volume of the container. Thinking about these layers provides a conceptual basis for relating volume to repeated addition and multiplication.*

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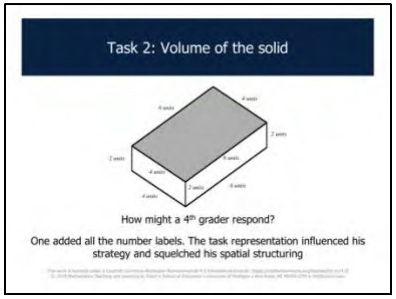
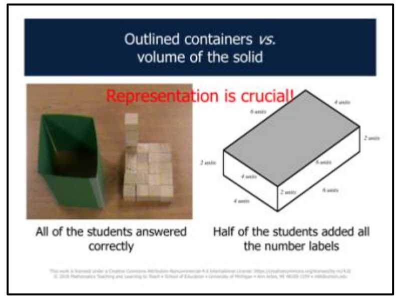
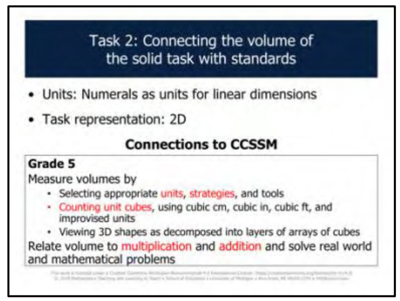
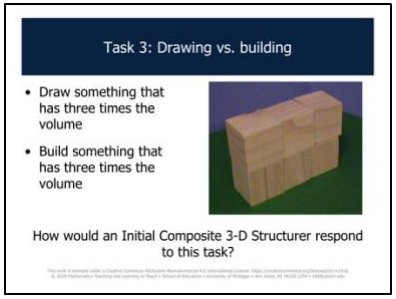
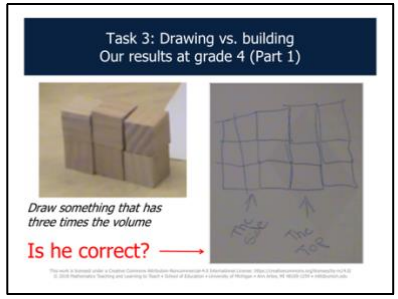
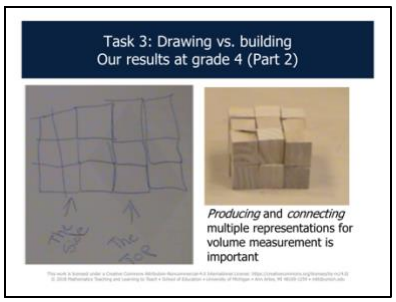
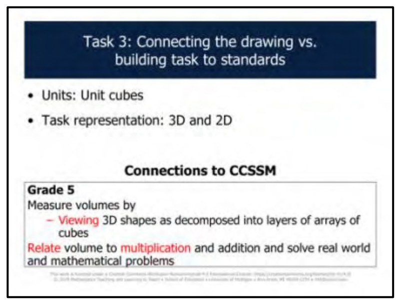
| Detailed description of activity | Comments & other resources |
| --- | --- |
| 3. Show Video D, where Dr. Clements presents a second task he and his colleagues gave to the group of fourth graders and talks about responses to this task. He makes the point that the representation of the task has a huge influence on how students respond.<br><br><br><br><br>After watching Video D, discuss the implications of Dr. Clements's comments for supporting students in meeting the Common Core volume standards in Grade 5.<br><br> | *In the task presented in Video D, students are asked to find the volume of a solid rectangular prism using the provided edge lengths. Dr. Clements explains that one student strategy for solving this problem was to add all of the number labels together. He points out that students approach this kind of task much differently than they approach the first task he shared, and he argues that students need to be exposed to the more "concrete" volume tasks (like Task 1) before they move to more abstract tasks (like Task 2).*<br><br>*Otherwise, students tend to solve the abstract tasks by manipulating numbers rather than by thinking conceptually about volume.*<br><br>*Dr. Clements and colleagues found that, when they provided a 3-D space for children to represent their thinking, children as early as Grade 4 demonstrated that they were able to think about volume in terms of layers to solve the tasks (which draws on a conceptual understanding of volume).*<br><br>*Participants will likely point out that it is important for teachers (in Grade 5 and before) to give students experiences with concrete volume tasks (involving packing or building with cubes, as well as reasoning about partially packed containers or partially built structures) and to use those kinds of experiences to help build their conceptual understanding of volume. As students demonstrate this conceptual understanding, teachers can then begin to help them apply it to the more abstract tasks.* |

| Detailed description of activity | Comments & other resources |
|---|---|
| 4. *Optional*: If time permits, watch Video E, where Dr. Clements introduces the "drawing vs. building" task. Ask participants to predict how an Initial Composite 3-D Structurer might respond to this task. Then, show Video F, where Dr. Clements and Dr. Sarama discuss one student's response to the task and talk about how this task connects to the Common Core State Standards for Grade 5. | *Video F: Commentary on "drawing v. building"* |




*In this video, Dr. Clements and Dr. Sarama point out that the student's work does not have a 3-D perspective; however, his drawing does have labels indicating which part of the drawing reflects each perspective (i.e., a view of the side and a view of the top). Even though his drawing does not reflect a 3-D perspective, he was able to use this drawing to correctly determine the number of cubes in the structure. They point out that, while his representation might be identified as incorrect (because it lacks the 3-D perspective), it does reflect an understanding of volume as iterating layers.*

*In Video F, Dr. Sarama points out that this task connects nicely to the Common Core State Standards. At Grade 5, the CCSS focuses on "viewing 3-D shapes as decomposed into layers of arrays of cubes" as well as relating volume to multiplication. By providing students with a starting "layer" and asking them about a shape with 3 times the volume, students need to first be able to visualize a shape that has three times of the volume of the given shape. This involves visually iterating the layer 3 times. They, then, need to determine the number of cubes in that shape, which requires multiplicative thinking, thus helping build the connection between volume and multiplication (or repeated addition).*

## *Part 6: Test ourselves: Connecting students' thinking with Learning Trajectories (~15 minutes)*

***Goals***

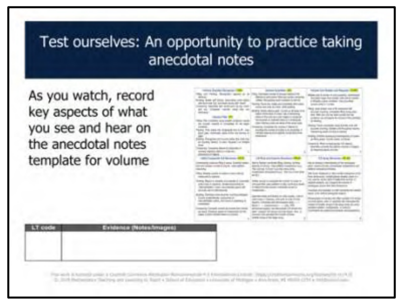
- Participants will demonstrate understanding of the Learning Trajectory for volume measurement.
- Participants will be able to take descriptive notes while watching a student measure and then use hose notes to later interpret the student's thinking.

***Instructional sequence***

1. Introduce Part 6 by watching Video A.
2. Think-pair-share about Video B (**Answer: 3-D Row and Column Structurer);** watch Video C during the whole-group discussion.
3. Think-pair-share-about Video D (**Answer: Volume Quantifier);** watch Video E during the whole-group discussion.
4. Think-pair-share about Video F **(Answer: Initial Composite 3-D Structurer);** watch Video G during the whole-group discussion.

***Resources***

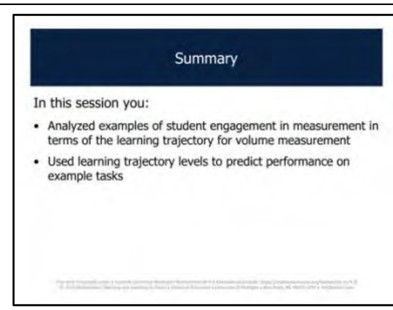
- Video A (00:16): Introducing "test ourselves"
- Video B (00:31): Test ourselves 1: Drawing on a blank cube
- Video C (00:34): Commentary on "Test ourselves 1"
- Video D (00:09): Test ourselves 2: Number of cubes in a shape
- Video E (00:13): Commentary on "Test ourselves 2"
- Video F (00:54): Test ourselves 3: Volume of a cube in a diagram
- Video G (00:31): Commentary on "Test ourselves 3"
- Handout: Anecdotal notes form – Volume Learning Trajectories

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 6 by watching Video A, in which Dr. Clements explains that participants will practice using the anecdotal notes form as they "test" their understanding of the levels of the developmental progression for volume measurement. Participants will also practice using these forms as part of their CCA for this session.<br><br>Distribute Handout: Anecdotal notes form – Volume Learning Trajectories and review this form with participants, zooming in on using the table to record notes and make connections with Learning Trajectory levels.<br><br><br><br>After each video, participants will use these notes to identify the level of the volume Learning Trajectory that was demonstrated in the video. They will then talk with a partner about what they noticed and whether/how the notes they took helped them identify the appropriate level.<br><br>Explain that, as participants watch the videos, they should focus on two questions:<br>- How are students reasoning about measuring?<br>- How are students making sense of the volume? | *During this part, encourage participants to use specific evidence from the video to support their claims.*<br><br>*As participants analyze students' responses according to the levels of the developmental progression, keep in mind that they should also be developing an orientation that is focused towards teaching. Allow them opportunities to think about what "next steps" they might want to take with the student based on what the student is currently doing.*<br><br>*For the purpose of this activity, participants only need to record the student name (or, in this case, video number), LT code, and evidence. They do not need to record anything about the task or focal Learning Trajectory levels.* |
| 2. Have participants watch Video B, using the anecdotal notes form to record what they are noticing. Afterwards, give participants time with their partners to identify the Learning Trajectory level | *In Video B, a student is asked to use a ruler to find the volume of a box. He determines that one side* |

| Detailed description of activity | Comments & other resources |
|---|---|
| represented in the video.<br>After participants have had time to talk with their partners, return to the whole group watch Video C, where teachers in the professional development determine that the student is at the 3-D Row and Column Structurer level because he is skip counting to determine the volume of the box. | *of the box is about 9 inches, and he then skip counts by nine (seemingly estimating how many layers of nine there would be in the box) to determine his answer. This performance reflects the 3-D Row and Column Structurer level.* |
| 3. Repeat the process of taking notes about a student's performance on a volume task by watching Video D.<br>After participants have had time to talk with their partners about what they noticed, return to the whole group watch Video E. In this video, Dr. Clements explains that, because this student is counting the cubes he sees (without structuring), this performance reflects the Volume Quantifier level. | *In Video D, a student determines the volume of a 2 x 2 x 2 structure by counting each cube individually.*<br>*This performance reflects the Volume Quantifier level.* |
| 4. Repeat the process of watching and taking notes on a video by watching Video F.<br>After participants have had time to talk with their partners, return to the whole group and watch Video G. Dr. Clements explains that, while he characterizes this performance as reflecting the Initial Composite 3-D Structurer level, he would want to see how this student does with other tasks in order to gain a fuller understanding of this student's thinking. | *In Video F, a student is presented with a 2-D representation of a 3 x 3 x 2 prism. After he determines that there are 9 cubes in the top layer, the interviewer asks him how many cubes would be in the bottom layer. He first guesses that there might be 5, but then the interviewer asks him to think about the fact that there are 9 cubes in the top layer. With this prompt, he realizes that there would also be 9 in the bottom layer. This performance is an example of the Initial Composite 3-D Structurer level.* |

### *Part 7: Wrap up (~5 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will understand ways of connecting the session to their classroom. | 1. Watch Video A and explain and distribute the Classroom Connection Activities.<br>2. Summarize the work of the session. | • Video A (03:36): Previewing the next session and the CCA<br>• Handout: Classroom Connection Activity 2 – Volume |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Preview the next session by having participants watch Video A, where Dr. Clements previews the next session (which will be focused on instruction) and introduces the Classroom Connection Activities.<br><br>After watching the video, distribute the handout you customized with selected Classroom Connection Activities and accompanying documents described below.<br><br>Required:<br><br>• Volume Assessment Tasks: Select volume tasks to complete with 3-4 students of different (hypothesized) achievement levels. Ask the students to write down and/or draw how they measured. Use the anecdotal notes form to record how students engage in the task and the Learning Trajectory level that is associated with the students' performances. Respond to the reflection questions listed on the CCA handout.<br><br>• Select a lesson or a short sequence of instructional activities from your curriculum that could be used to support learning about volume measurement. Bring these materials with you to the next session. | *Explain the Classroom Connection Activity and the materials participants need to bring to the next session.*<br><br>*Make sure to explain that it is important to use the anecdotal note taking form so that the group can work together on understanding its affordances and limitations.*<br><br>*If you would like to have participants submit the notes, question responses, etc. prior to the next session, then make sure they understand the method by which they should submit those.* |
| 2. Summarize this session by explaining that participants:<br><br>• Analyzed examples of student engagement in measurement in terms of Learning Trajectory for volume measurement<br><br>• Used Learning Trajectory levels to predict performance on example tasks<br><br><br> | |