Description of the session

In Session 9, participants will complete their work on the Learning Trajectory for volume measurement by focusing on the third component of the Learning Trajectory: instructional tasks. At the beginning of the session, participants will review the levels of the developmental progression for volume measurement by watching videos of a student's performance on a task at three different time points and identifying the levels of development demonstrated. Then, they will engage in their third, and final, anecdotal notes workshop, which will provide them with an opportunity to examine their own students' behavior and understanding, as well as their own methods of taking anecdotal notes. After that, participants will analyze a variety of instructional tasks to identify the Learning Trajectory level each targets and discuss ways these activities could be modified to target different levels. Finally, participants will practice modifying tasks from their own curriculum materials in order to target the levels of the students in their class.

*Activities and goals of the session**

Activities	Times	Corresponding parts of the session	Goals
I. Review, overview, and anecdotal notes workshop	35 minutes	Parts 1 & 2	• Participants will be oriented to the work of the session. • Participants will demonstrate understanding of the developmental progression in the Learning Trajectory for volume measurement. • Participants will use the Learning from Practice Protocol to describe and learn about students' thinking • Participants will generate ways of improving their taking of anecdotal notes, • Participants will determine ways of improving their learning from the workshop.
II. Instructional Tasks for volume measurement	50 minutes	Parts 3, 4, & 5	• Participants will recognize the Learning Trajectory level an instructional activity is designed to target. • Participants will understand how to modify/differentiate an instructional activity to target a different level of the Learning Trajectory for volume measurement. • Participants will connect activities in their curriculum to the Learning Trajectory levels. • Participants will revise an activity to target the levels of the students in their classroom.
IV. Wrap up	5 minutes	Part 6	• Participants will recall the work they have done in the three sessions that have focused on the Learning Trajectory for volume measurement • Participants will understand ways of connecting the session content to their classroom.

*Conversations about the CCAs from the last session are integrated into this session.

Classroom Connection Activities

Optional
Type of task: Video recording a measurement activity Description: Facilitate students' work on a measurement activity and use the anecdotal notes form to record observations.
Type of task: Preparation for Session 10 Description: Reflect on the module using the questions provided and record ideas (for use during Session 10).

Preparing for the session

- ☐ Make copies as needed: Handout: Content cube – Volume Learning Trajectory (Parts 1, 3, 4, & 5); Handout: Anecdotal notes workshop protocol (Part 2); Handout: Anecdotal notes form – Volume Learning Trajectories (Part 2 & CCA); Anecdotal notes form (from session 6) – Area Learning Trajectories (CCA); Anecdotal notes form from session 3 – Length Learning Trajectories (CCA)
- ☐ Customize and make copies of the Classroom Connection Activities
- ☐ Text technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• recognizing the mathematical goal as the first component of a complete Learning Trajectory • understanding principles of measurement (e.g., attribute, conservation, transitivity, equal partitioning, units and unit iteration, accumulation, origin, and relation between number and measurement) • understanding how measurement of length, area, and volume are represented and developed in the CCSS • understanding how measurement connects with the CCSS standards for mathematical practice • **understanding concepts and skills involved in measuring** length, area, and/or **volume** • understanding connections between length, area, and volume measurement and between metric measurements for each	• recognizing student development as the second component of a complete Learning Trajectory • understanding children's development of measurement through Learning Trajectories for length, area, and volume • recognizing principles of measurement in student work • interpreting student work on measurement tasks using the levels of the Learning Trajectory for length measurement • interpreting student work on measurement tasks using the levels of the Learning Trajectory for area measurement • **interpreting student work on measurement tasks using the levels of the Learning Trajectory for volume measurement**	• **recognizing instruction as the third component of a complete Learning Trajectory** • using anecdotal notes to document what students say and do when working on measurement tasks • **connecting measurement activities in curricula to measurement Learning Trajectory levels** • **modifying measurement tasks to target different and/or particular Learning Trajectory levels**	• **understanding the anecdotal notes workshop process** • **using the anecdotal notes workshop to improve the practice of note taking** • **using the anecdotal notes workshop to improve teaching**

Part 1: Module preview (10 minutes)

Goals

- Participants will be oriented to the work of the session.
- Participants will demonstrate understanding of the developmental progression in the Learning Trajectory for volume measurement.

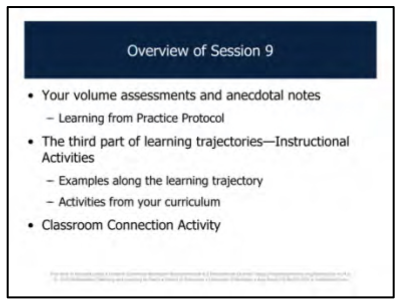
Instructional sequence

1. Introduce the session by watching *Video A.*
2. Watch Video B to introduce the "Test ourselves" activity.
3. Have participants test their knowledge of the developmental progression in the Learning Trajectory for volume measurement by watching and discussing Videos C (**Answer: (Pre?) Volume Quantity Recognizer)** and D.
4. Watch and discuss Videos E (**Answer: Volume Quantifier)** and F to continue assessing participants' understanding of the Learning Trajectory for volume measurement.
5. Watch and discuss Videos G (**Answer: Initial Composite 3-D Structurer)** and H.

Resources

- Video A (00:59): Overview
- Video B (00:47): "test ourselves"
- Video C (00:19): Test ourselves 1: Number of blocks in a larger cube
- Video D (00:35): Commentary on "Test ourselves 1"
- Video E (00:37): Test ourselves 2: Number of blocks in a larger cube 2
- Video F (04:00): Commentary on "Test ourselves 2"
- Video G (00:20): Test ourselves 3: Number of blocks in a larger cube 3
- Handout: Content cube – Volume Learning Trajectory

Detailed description of activity	Comments & other resources
1. Introduce the session by watching Video A: In Sessions 7 and 8, participants examined the mathematics of volume measurement and a developmental progression of students' thinking about these ideas. The focus of this session is on instructional tasks that can be used to help advance students' thinking about volume. In this session, participants will discuss • Their experiences with the volume assessment and anecdotal notes that made use of the Learning from Practice Protocol • The third part of the Learning Trajectory for volume measurement—Instructional Activities (including examples of tasks along the Learning Trajectory and activities from their curricula) • The Classroom Connection Activity they will complete prior to the final session.	*This part is designed to provide a quick review of what participants learned about the developmental progression for volume measurement before considering strategies for teaching volume measurement.*



Detailed description of activity	Comments & other resources
2. Show Video B, where Dr. Clements and Dr. Sarama introduce the "test ourselves" activity participants will do to review the levels of the developmental progression in the Learning Trajectory for volume measurement. In this part, they will watch three videos of the same student (at pre-K, kindergarten, and grade 1) and attempt to identify the Learning Trajectory levels that are represented in each video.	
3. Have participants watch Video C, discuss their observations, and identify the level of thinking demonstrated in the video. Conclude this discussion by having participants watch Video D, where Dr. Clements and Dr. Sarama facilitate a discussion about why the student's work in this video is characterized by the Volume Quantity Recognizer Level (or possibly the Pre-Volume Quantity Recognizer Level).	*Video C: Test ourselves 1: Number of blocks in a larger cube* *In this video, a student (who is in pre-K) is asked to determine "How many blocks are in this group of blocks?" First, the assessor gives him a set of cubes arranged in a 2 x 3 x 2 rectangular prism. Then, she asks the same question again while showing a picture of a 2 x 3 x 2 prism. In both cases, the student counts the 6 squares on the top of the prism and says that there are 6 blocks all together.* *Video D: Commentary on "Test ourselves 1"* *In this video, Dr. Clements explains that, while the student may be at the Volume Quantity Recognizer level, it is unclear from this video whether he even has a concept of volume at this point since he is only counting one dimension (and may either be thinking about cubes or just counting the squares he sees without thinking about volume at all).*

Detailed description of activity	Comments & other resources
4. Watch Video E and have participants discuss their observations and identify the level of thinking demonstrated in the video.	*Video E: Test ourselves 2: Number of blocks in a larger cube 2* *In this video, the same student (who is now in kindergarten) is presented with the same task as in Video B—determining how many blocks are in a 2 x 3 x 2 prism. He first counts squares on multiple faces of the rectangular prism and says that the answer is 9, but then he recounts 6 on the top layer and 6 on the bottom layer and determines that there are 12 blocks.*
After participants have shared their ideas, watch Video F, where teachers make sense of this student's response and discuss follow-up questions that could be asked to learn more about how the student is thinking. Dr. Clements indicates that his research team classified this student's performance at the "Volume Quantifier Level."	*Video F: Commentary on "Test ourselves 2"* *In this video, teachers discuss the differences between the student's first and second responses, and suggest follow-up questions the teacher might have asked to gain more information about whether the student was at the Volume Quantifier or Volume Filler level. When discussing potential follow-up questions (e.g., "Are you sure?"), Dr. Sarama points out the importance of establishing an environment that supports students in responding to these kinds of probing questions (rather than, for example, changing their answers because they think that the question implies that they were incorrect the first time.)* *The reason this video was classified as representing the Volume Quantifier Level is that it appears, by the end of the video, that the student has realized that he can figure out how many blocks there are altogether by counting the blocks in the top and bottom layer of the rectangular prism. Because he does not count multiple faces of the same cube, it appears he may be beginning to recognize that the squares on adjacent faces of a rectangular prism sharing a side are faces of the same cube.*

Detailed description of activity	Comments & other resources
5. Watch Video G and have participants discuss their observations and identify the level of thinking demonstrated in the video. After participants have shared their ideas, watch Video H, in which Dr. Sarama indicates that this video represents the Initial Composite Structurer level and asks teachers to explain what evidence from the video supports this claim.	*Video G: Test ourselves 3: Number of blocks in a larger cube 3* *In this video, the student (who is now in first grade) is presented with the same task as in Videos B and D. Without touching the prism, he answers that there are 12 cubes in the 2 x 3 x 2 prism, and when asked how he knows, he says, "Because there are six on the top and six on the bottom."* *Video H: Commentary on "Test ourselves 3"* *In this video, a teacher explains that he thinks that the student in this video reflects the Initial Composite Structurer level because the student was able to figure out the answer without picking up the prism. This suggests that the student was able to account for the cubes that were hidden from his view. Dr. Sarama and Dr. Clements point out that it is impossible to characterize this student's developmental level from his performance on this task alone—to learn more about his thinking, it would be necessary to examine how he performs on multiple tasks and to ask follow-up questions to learn more about his thinking.*

Part 2: Anecdotal notes workshop (25 minutes)

Goals

- Participants will use the Learning from Practice Protocol to describe and learn about students' thinking.
- Participants will generate ways of improving their taking of anecdotal notes.
- Participants will determine ways of improving their learning from the workshop.

Instructional sequence

1. Introduce the session by watching Video A.
2. Distribute and discuss the anecdotal notes workshop protocol and anecdotal notes form for volume
3. Have participants meet in small groups to engage in an anecdotal notes workshop
4. Watch Video B and debrief the workshop; watch Videos C-E (and the supplemental videos as time and need permit).

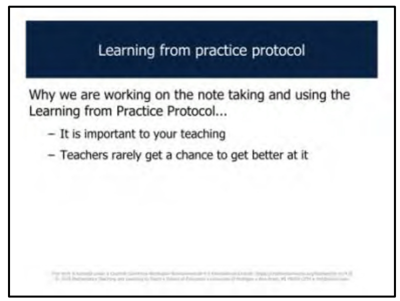
Resources

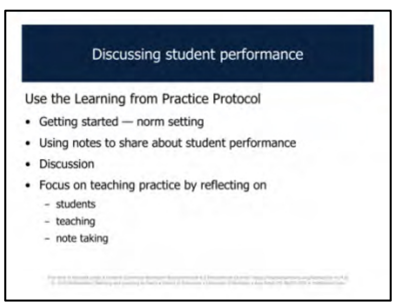
- Video A (02:56): Anecdotal notes workshop launch
- Video B (01:17): Debriefing anecdotal notes workshop
- Video C (0:18): Structured notes support later use
- Video D (01:14): Using assessment tasks as instruction
- Video E (01:03): The importance of formative assessment
- Handout: Anecdotal notes workshop protocol
- Handout: Anecdotal notes form – Volume Learning Trajectories

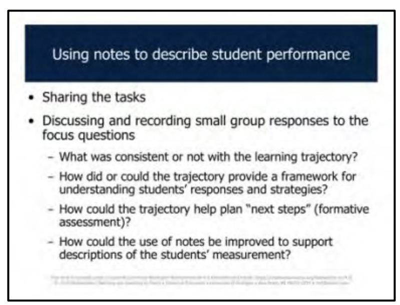
Supplements

- Video (01:54): How a student responds across tasks
- Video (00:43): How students reasoned about the volume of liquids

Detailed description of activity	Comments & other resources
1. Introduce Part 2 by watching Video A, where Dr. Sarama introduces the anecdotal notes workshop and reminds teachers of the rationale for working on the practice of note taking during this professional development module. In this video, Dr. Sarama explains that the goal is for teachers to have an opportunity to work on using formative assessment in the form of taking notes that they will actually be able to use in their practice. She also reviews the routine and norms for the anecdotal notes workshop.	







Detailed description of activity	Comments & other resources
2. Distribute Handout: Anecdotal notes workshop protocol and Handout: Anecdotal notes form – Volume Learning Trajectories. Remind participants that the goal of this workshop is to use the anecdotal notes to support discussion about the mathematics their students demonstrate and how their students think about the mathematics. A related goal is to discuss differences in students' thinking. As in Sessions 3 and 6, participants should use the anecdotal notes workshop protocol to structure their discussion: • The "presenter" will use his or her notes to support sharing information about the performance of a <u>student</u> on a particular task (and any key background information). (3 minutes) • "Colleagues" will ask questions to better understand the task used, the student's performance on the task, the connections between the student's performance and the Learning Trajectories, and the presenter's method of taking anecdotal notes. (3 minutes) • Participants switch roles until all have shared about a student's performance on a task that was used. • With whatever time remains, participants should reflect on what they learned, including discussing how the process of sharing with colleagues worked/didn't work for them. (5 minutes)	*If possible, have the participants meet with the same group of grade-level-alike colleagues as the last time they shared their anecdotal notes. This will support the development of norms for sharing and discussing.* *Try to limit participants to working in groups of three, as this will ensure that all participants will be able to share an instance of student thinking from their classrooms. If it is not possible to group all participants in threes, use pairs for the remaining groups (instead of 4 per group).*
2. Have the groups of three begin their anecdotal notes workshop time. During this time, participants should use their anecdotal notes to share what they learned about students from the tasks. They should discuss the following questions (listed on Slide: Using notes to record student performance) • What was consistent or not with the Learning Trajectory? • How did or could the trajectory provide a framework for understanding students' responses and strategies? • How could the trajectory help plan "next steps" (formative assessment)? • How could the use of notes be improved to support descriptions of the students' measurement? Participants should also record their groups' responses to these questions. As each "presenter" shares, encourage participants to think about • What the presenter is able to say about his or her students based on the anecdotal notes taken • What rest of the group is able to say about the presenter's students based on the anecdotal notes taken	*The major focus of this workshop is on connecting the levels of the Learning Trajectory's developmental progression to instructional tasks (including analyzing the content of tasks and videos). Also integrated with this is a focus on taking notes.* *While participants meet with their small groups, circulate and record:* • *examples of student work that will be helpful in illustrating the different levels of the Learning Trajectory* • *ways that participants use the notetaking forms*

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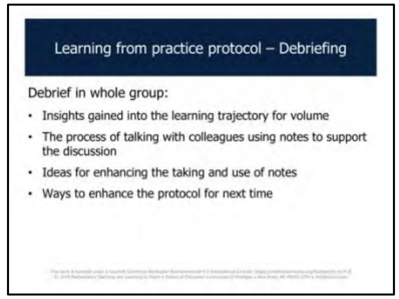
3. After participants have had time for small group discussion, return to the whole group and watch Video B. Then debrief the workshop by discussing

    - Insights gained into the Learning Trajectory for area
    - The process of talking with colleagues using notes to support the discussion
    - Ideas for enhancing the taking and use of notes
    - Ways to enhance the protocol for next time



As part of the debriefing conversation, watch Video C: Structured notes to support later use, which includes a discussion of how taking notes in a structured format can make it easier to interpret and use them later.

As part of the debriefing discussion, also include some attention to discussing the ways in which these assessment tasks (and the anecdotal notes form) could be used in instruction with the class. Watch Video D: Using assessment tasks as instruction, in which Dr. Sarama and Dr. Clements offer some suggestions for how these tasks could be used with the whole group.

Also, during the debriefing discussion, watch Video E: The importance of formative assessment. In this video, Dr. Clements explains that the use of formative assessment to guide instruction has been shown to be more effective than other efforts to improve student learning (e.g., a new curriculum, professional development) but it is only effective if teachers have guidance about how to get formative assessment information and what to do with it.

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It may help to project the anecdotal notes form as participants make comments about its use.

If time and interest permit, consider showing one or both of the following videos (available as Supplements on CTools):

- *Video: How a student responds across tasks*

  In this video, a teacher describes how a student demonstrated thinking at different levels of the LT for volume measurement on two different tasks; Dr. Sarama explains why this might be

- *Video: How students reasoned about the volume of liquids*

  In this video, a teacher shares how her students reasoned about which container would hold more liquid. Though they all got the correct answer, their reasons for their answer ranged from "this one you can't poke holes in" to "this one is the same size all the way down, and this one gets smaller at the bottom."

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Part 3: Connecting instructional tasks with early Learning Trajectory levels (20 minutes)

Goals

- Participants will recognize the Learning Trajectory level an instructional activity is designed to target.
- Participants will understand how to modify/differentiate an instructional activity to target a different level of the Learning Trajectory for volume measurement.

Instructional sequence

1. Introduce Part 3 by watching Video A.
2. Watch and discuss Videos B and C.
3. Watch and discuss Videos D and E.
4. Watch and discuss Videos F and G.

Resources

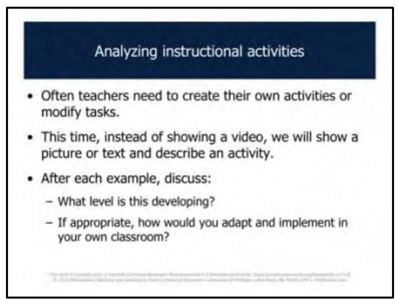
- Video A (01:49): Creating and modifying volume tasks
- Video B (00:39): Filling containers
- Video C (01:28): Commentary on "filling containers"
  Video D (00:11): Comparing volumes
- Video E (00:48): Commentary on "comparing volumes"
  Video F (00:17): Using cubes to fill a small box
- Video G (02:03): Commentary on "using cubes to fill a small box"
- Handout: Content cube – Volume Learning Trajectory

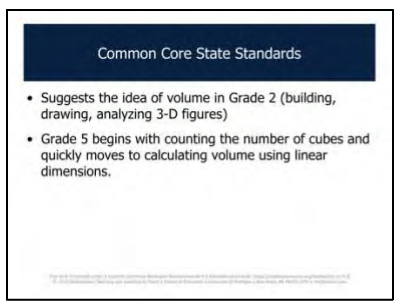
Detailed description of activity

1. Introduce Part 3: In this part, participants consider examples of instructional tasks that are useful for supporting the learning of students at the early levels of the Learning Trajectory for volume measurement. They also consider ways they could adapt these tasks to implement them in their classrooms.

   Watch Video A, where Dr. Clements encourages teachers to incorporate work on length, area, and volume in all grade levels—even if it isn't explicitly addressed in the Common Core at that grade level. In order to do this, teachers will often need to create their own activities or modify tasks. He explains that, in this part, participants will examine different instructional tasks for volume measurement and answer the following questions:

   - What level is this developing?
   - How would you adapt and implement this activity in your own classroom?





As they answer these questions, encourage participants to use Handout: Content cube – Volume Learning Trajectory as a reference.

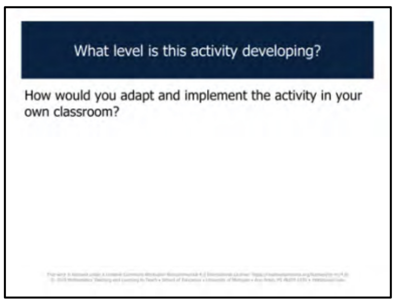
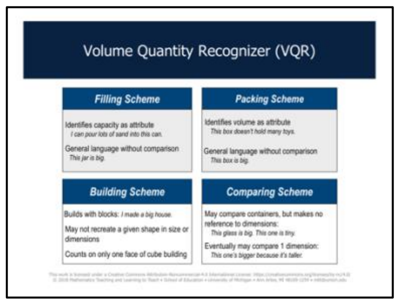
Comments & other resources

In Video A, Dr. Clements points out that, in the Common Core State Standards, some preparatory work on volume is done in Grade 2, and volume measurement is directly addressed in Grade 5, where students are expected to calculate volume using linear dimensions (after having experiences counting cubes to determine volume).

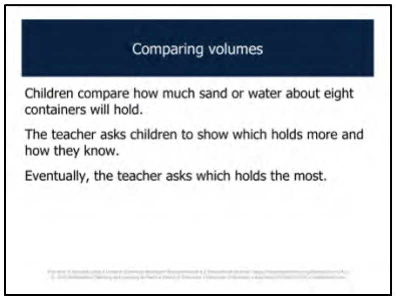
The hallmark of a "good" task is that it connects the student's current level of thinking (which can be determined by the assessments like the ones participants considered in Parts 1 and 2) with where the teacher wants the student to go mathematically. In a larger sense—especially in later grades—it also provides teachers with opportunities to help students who are at different places work productively as a class.

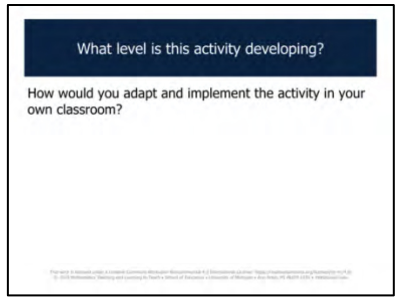
A good instructional task:

- *Engages children at different levels—children can solve with different-level strategies.*
- *Requires concepts, skills, and problem solving.*
- *Most importantly, aligns with the level just beyond the "mastered" level of the majority of children.*

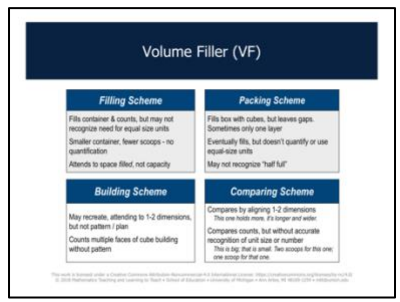
Detailed description of activity	Comments & other resources
2. Watch Video B: Filling containers, in which Dr. Clements introduces one activity in which students work together with a variety of different sized and shaped containers to figure out which container holds the most.  Have participants discuss • What level is this developing? • How would you adapt or implement this activity in your own classroom? After participants have had time to discuss, display Slide: Volume Quantity Recognizer and discuss why this activity might be useful for helping students begin to recognize volume as an attribute. Then watch Video C, where Dr. Sarama and Dr. Clements explain the importance of these kinds of experiences for supporting students' understanding of volume.   	*The Volume Quantity Recognizer level is the level at which children have begun to recognize volume as an attribute that can be measured. By allowing children to experiment with filling different containers with sand, and by asking them which container holds the most, teachers can focus children's attention on volume as an attribute that can vary from container to container.* *Video C: Commentary on "filling containers"* *In this video, Dr. Sarama explains that she and her colleagues did this activity with second graders because they realized the need for these students to have informal experiences with volume. She and teachers talk about how these kinds of informal experiences are often neglected in elementary classrooms because there is pressure to focus on the formal academic material that will appear on standardized tests. However, Dr. Clements points out that these informal, play-based experiences do not <u>compete</u> with experiences that will prepare students for assessments; rather, they provide important foundations that will ultimately <u>help</u> prepare students for these assessments.* *Differentiation ideas:* • *Encourage students to focus on dimensions by choosing containers where one dimension is identical and encouraging discussion about why one holds more than the other (Volume Filler)* • *Ask children to figure out how much bigger one container is than another (Volume Quantifier) and encourage thinking about units and relating the number of units (Volume Unit Relater and Repeater)*

Detailed description of activity	Comments & other resources
3. Watch Video D: Comparing volumes. In this video, Dr. Clements presents an activity where children compare how much sand or water about eight containers will hold. The teacher asks children to show which one holds more and how they know. Eventually, the teacher asks which container will hold the most. Have participants discuss • What level is this developing? • How would you adapt or implement this activity in your own classroom?	





Detailed description of activity	Comments & other resources
Then show Video E where Dr. Sarama and Dr. Clements explain why this instructional task helps work towards the Volume Filler level and how teachers might adapt this task for students at different levels.	*Video E: Commentary on "Comparing volumes"* *In this video, Dr. Sarama points out that this activity pushes students to go beyond just comparing containers by noticing which one "looks bigger" to comparing by actually determining which one holds more. Dr. Clements also acknowledges that teachers could extend this task for students working toward the Volume Quantifier level by asking students "how <u>much</u> more" one container holds than another. By asking different kinds of follow-up questions to different students during this activity, teachers can support the learning of students who are at various levels within the context of the same activity.*

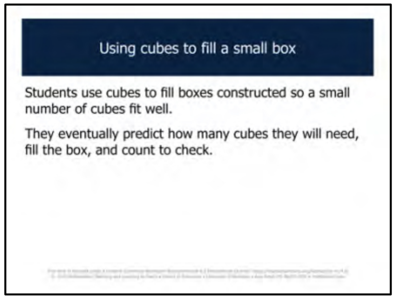


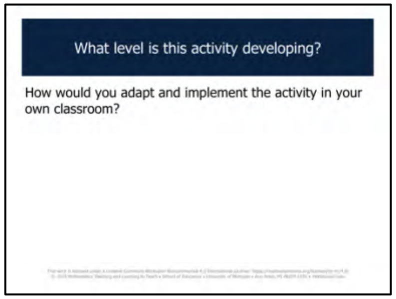
Detailed description of activity	Comments & other resources

4. Next, watch Video F: Using cubes to fill a small box. In this video, Dr. Clements describes an activity where students use cubes to fill boxes that have been constructed such that a small number of cubes fit well. Students eventually predict how many cubes they will need, fill the box, and count to check.

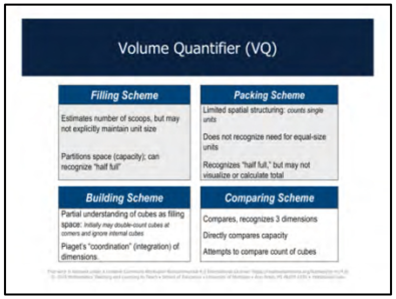
   Again, give participants time to discuss

   - What level is this developing?
   - How would you adapt or implement this activity in your own classroom?





After this discussion, watch Video G, where teachers discuss their thoughts about which level this task is developing and where Dr. Sarama and Dr. Clements explain why it targets the Volume Quantifier level (and begins to target the Volume Unit Relater and Repeater level as well).



Video G: Commentary on "using cubes to fill a small box."

In this video, a teacher explains that he sees this activity as including two different tasks that target two different levels. He explains that the "packing" part of the activity seems to target the Volume Quantifier level, because it supports students in working on packing a box without gaps. The part of the activity that involves predicting how many cubes will fill a box, he argues, seems to target the Volume Unit Relater and Repeater level because it asks students to "anticipate the volume of the container." Dr. Sarama explains that the purpose of asking students to predict how many cubes will fit into a container is to help them begin to develop a spatial numerical scheme rather than just counting the cubes they use as they fill the box.

Part 4: Connecting instructional tasks with later Learning Trajectory levels (20 minutes)

Goals

- Participants will recognize the Learning Trajectory level an instructional activity is designed to target.
- Participants will understand how to modify/differentiate an instructional activity to target a different level of the Learning Trajectory for volume measurement.

Instructional sequence

1. Introduce the part and watch and discuss Videos A and B.
2. Watch and discuss Videos C and D.
3. Watch and discuss Videos E and F.
4. Watch and discuss Videos G and H.

Resources

- Video A (00:25): Predict and test
- Video B (00:31): Commentary on "predict and test"
- Video C (00:33): Rectangular prism "nets"
- Video D (00:23): Commentary on "rectangular prism 'nets'"
- Video E (00:34): Units relating task
- Video F (02:21): Commentary on "units relating task"
- Video G (00:25): Calculating volume using linear dimensions
- Video H (00:45): Commentary on "calculating volume using linear dimensions"
- Handout: Content cube – Volume Learning Trajectory

Detailed description of activity

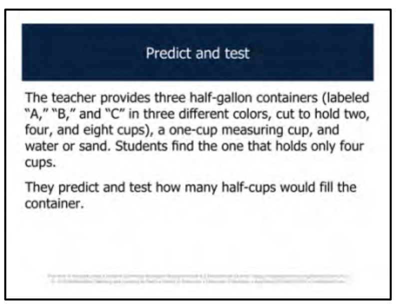
1. Introduce Part 4: In this part, participants will discuss instructional tasks that are designed to target later levels of the Learning Trajectory for volume measurement.

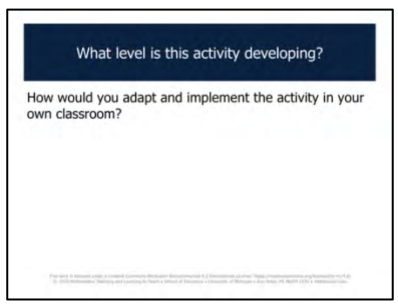
   Watch Video A: Predict and Test.

   In this video, Dr. Clements introduces an activity where the teacher provides three half-gallon containers (labeled "A", "B", and "C" in three different colors, cut to hold two, four, and eight cups), a one-cup measuring cup, and water or sand. Students are to (1) find the container that holds only four cups, and (2) predict and test how many half-cups would fill the container.

   Give participants time to discuss

   - What level is this developing?
   - How would you adapt or implement this activity in your own classroom?

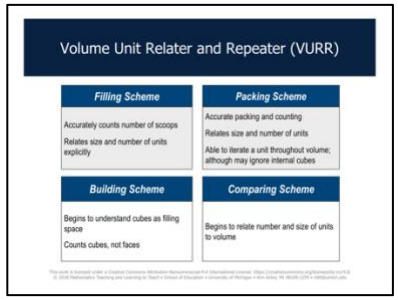




Comments & other resources

Participants can continue to use Handout: Content cube – Volume Learning Trajectory as a reference during this part.

<table>
<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
<tr><td>
After giving participants time to discuss, show Video B, Dr. Sarama and teachers discuss why this activity targets the Volume Unit Relater and Repeater level.



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Video B: Commentary on "predict and test" Dr. Sarama points out that by first having students determine the volume of a container with a one-cup measuring cup and then asking them to predict how many half-cups would fill the same container, this activity encourages students to think about and relate the size and number of units.

Differentiation ideas

- *This task can be modified to target higher levels by using a container and unit that are rectangular prisms and asking children to think about the dimensions, as well as the volume. For example, use a 2x4x4 container and a 2x2x2 unit, initially, then provide a 1x1x2 unit.*
- *As mentioned previously, by asking different kinds of follow-up questions to different students during this activity, teachers can support the learning of students who are at various levels within the context of the same activity.*
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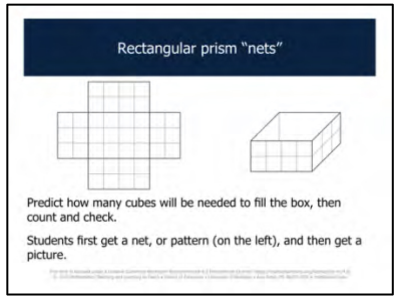
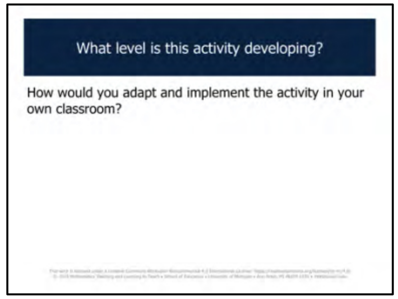
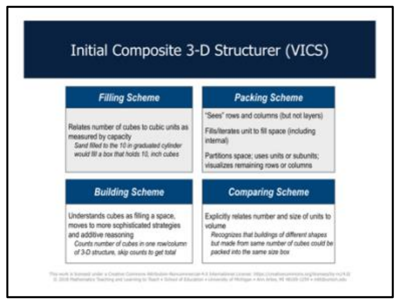
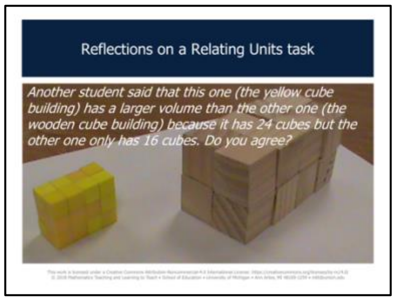
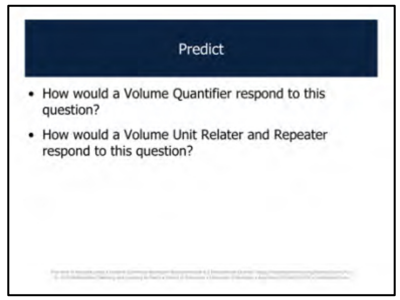
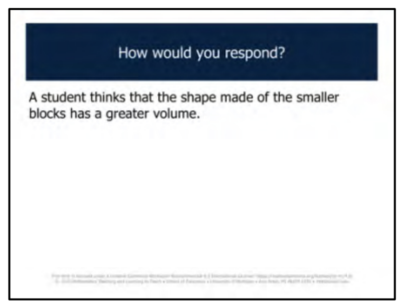
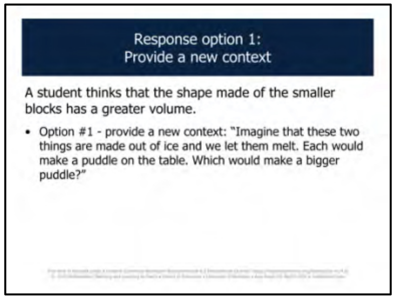
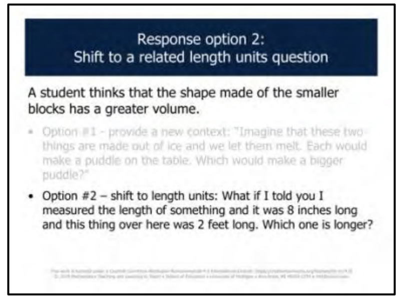
Detailed description of activity		Comments & other resources
2. Have participants watch Video C: Rectangular prism "nets." In this activity, students predict how many cubes would be needed to fill the box. Then, they count and check. When solving this problem, students first get a net, or pattern (on the left-hand side of the slide), and then they get a picture.		*In Video C, Dr. Clements explains that the origin of this usage of the term "net" is unknown.*
Give participants time to discuss • What level is this developing? • How would you adapt or implement this activity in your own classroom?		
After participants have had time to discuss, watch Video D, in which Dr. Sarama shares that this activity targets the Initial Composite 3-D Structurer level.		*By providing both a net and a picture of the box with the squares shown, students are encouraged to begin thinking about rows/columns of cubes. Children at the Volume Unit Relater and Repeater level may still visualize individual units and count one at a time, so teacher should encourage discussion focusing on how many cubes are in one row/column, then how many rows/columns are needed to fill the container. Further, teachers can encourage thinking at higher levels by encouraging discussion around how many cubes are in one layer and how many layers are needed to fill, relating these discussions to the dimensions displayed on the images.*

Detailed description of activity	Comments & other resources
3. Have participants watch Video E: Units relating task. In this task, students are shown two rectangular prisms—one with a volume of 24 small (yellow) cubes and the other with a volume of 16 large (wooden) cubes (pictured in Slide: Reflections on a Relating Units task). They are asked: "Another student said that this one (the yellow cube building) has a larger volume than the other one (the wooden cube building) because it has 24 cubes but the other one only has 16 cubes. Do you agree?" Ask participants to predict • How would a Volume Quantifier respond to this question? • How would a Volume Unit Relater and Repeater respond to this question?   After participants have had time to discuss, watch Video F: Commentary on "units relating task." In this video, Dr. Clements and Dr. Sarama discuss how students at different levels might answer this question. They also suggest a couple of ways teachers could respond to students who initially answer incorrectly in ways that might help advance their thinking, including  • Providing a new context in which to think about the problem (e.g., asking students to imagine that the blocks are made of ice and asking which one would make a bigger puddle when melted) • Offering an analogy to length (e.g., "What if I told you I measured the length of something and it was 8 inches long and this thing over here was 2 feet long? Which one is longer?")	*In Video E, teachers predict that Volume Quantifiers would agree that the prism with 24 cubes was larger because it had more cubes. They predict that students at the Volume Unit Relater and Repeater level would disagree because they would recognize that the size of the units are different.*

Detailed description of activity	Comments & other resources

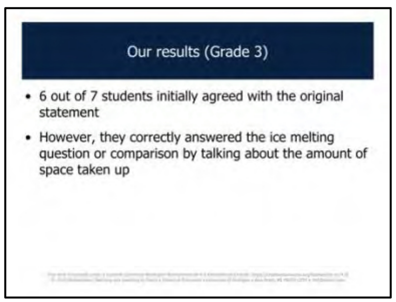


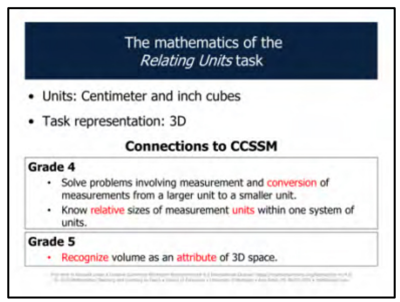


Dr. Clements then explains that, when they did this task with third graders, 6 out of 7 students originally agreed with the incorrect statement, but all changed their minds after follow-up questions using the ice melting analogy or about how much space each cube takes up.

At the end of the video, Dr. Clements explains that this task fits in with Common Core State Standards in Grades 4 and Grades 5. Dr. Sarama also points out how asking follow-up questions about students' work on tasks can be a powerful way to push their thinking forward.

After watching this video, provide participants with an opportunity to respond and/or to ask questions.





This problem aligns with the Grade 4 CCSS. In the CCSSM for Grade 4, students are expected to solve problems (in a single system of measurement – length, area, or volume) that involve the conversions of measurement from a larger unit to a smaller unit (e.g., a larger cube to a smaller cube). They are also asked to know the relative sizes of measurement units within one system of units.

In Grade 5, students are expected to recognize volume as an attribute of 3D space. Providing a context of ice cubes that melt supports thinking about volume in terms of 3-D space rather than just volume in terms of a number of 3-D units.

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<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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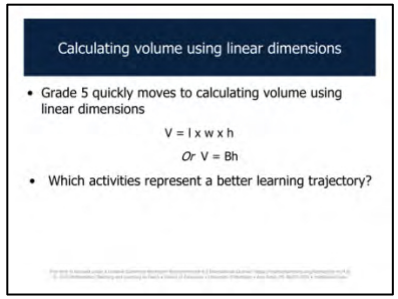
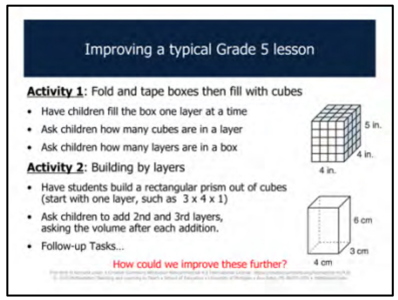
4. Watch Video G: Calculating volume using linear dimensions, where Dr. Clements explains that students in Grade 5 are expected to calculate volume of cubes by using the formulas V = l x w x h or V = Bh. He then introduces some modifications of this type of task that may better support students' learning. (These are shown on Slide: Improving a typical grade 5 lesson.)





Activity 1: Fold and tape boxes then fill with cubes

- Have students fill the box one layer at a time
- Ask children how many cubes are in a layer
- Ask children how many layers are in a box

Activity 2: Building by layers

- Have students build a rectangular prism out of cubes (start with one layer, such as 3 x 4 x 1)
- Ask children to add 2nd and 3rd layers, asking the volume after each addition
- Follow-up tasks….

Have participants discuss these tasks and how they might be improved further.

Conclude this part by watching and having participants respond to Video H: Commentary on "calculating volume".

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Encourage participants to think about the developmental progression of student thinking: children first identify volume as an attribute, then start seeing individual cubes, then rows/columns, then layers. When presented with the volume formula, students often begin to focus only on numbers.

Especially for students who have not yet developed a solid understanding of volume, it is important to continually reinforce thinking about individual units, rows/columns, and layers—and how each of these relates to the dimensions.

In Video H, Dr. Clements summarizes a comment of a teacher who recognizes that her students need more opportunities to work with cubes; however, the only cubes in her classroom are base ten blocks, which seem too small to work with for these tasks. Dr. Clements agrees that size matters, and he points out the importance of thinking about the nature of the task.

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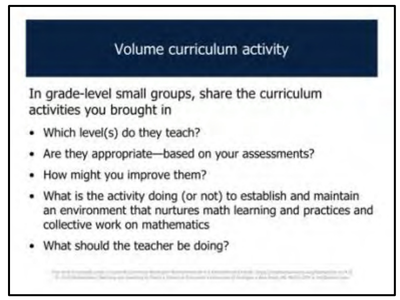
Part 5: Volume measurement in the school curriculum (10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will connect activities in their curriculum to the Learning Trajectory levels. • Participants will revise an activity to target the levels of the students in their classroom.	1. Introduce Part 5 by showing Video A; have participants discuss their curriculum activities in grade-level groups. 2. Share in whole group, watching Videos B and C as time and interest permit.	• Video A (01:23): Sharing curriculum activities • Video B (00:34): Example task 1: Constructing prisms • Video C (01:00): Example task 2: Liquid volume in containers • Handout: Content cube – Volume Learning Trajectory

Detailed description of activity	Comments & other resources
1. Introduce Part 5: Show Video A, which explains that participants will discuss activities in their curriculum and the ways they could improve them.  Have participants discuss the activities they brought in grade-level small groups, focusing on the following questions: • What Learning Trajectory level(s) do they teach? • Are they appropriate for your students—based on insights from your assessments? • How might you improve the activities? • What is the activity doing (or not) to establish and maintain an environment that nurtures learning, mathematical practices, and collective work on mathematics? What should the teacher be doing to help establish this environment? Give participants about five minutes to work in their grade-level groups.	*If running short on time, encourage participants to* • *Start with a group member who did not have an opportunity to share the last time the group met together* • *Choose a time keeper who can keep the discussion moving* • *Share in partners first and then come together as a larger group* *Participants can continue to use Handout: Content cube – Volume Learning Trajectory as a reference during this part.*

<table>
<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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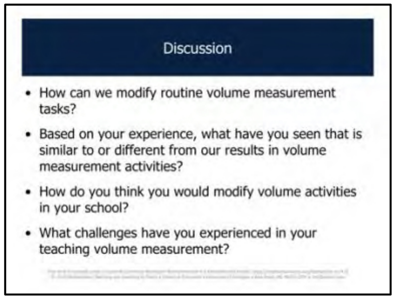
2. Invite participants to share what they discussed in their grade-level groups with the whole group.

   Focus the discussion on the following questions:

   - How can we modify routine volume measurement tasks?
     - asdf
   - Based on your experience, what have you seen that is similar to or different from our results in volume measurement activities?
   - How do you think you would modify volume activities in your school?
   - What challenges have you experienced in your teaching volume measurement?

   If it would be useful to support the discussion, show one or both of the following videos:

   - Video B: Example task 1: Constructing prisms
   - Video C: Example task 2: Liquid volume in containers

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Video B: Example task 1: Constructing prisms

In this video, a teacher says she would give students a certain number of cubes and then ask what kinds of prisms they could construct with them. Dr. Clements then suggests a natural follow-up question: "How do you know you have all of the combinations?"

Video C: Example task 2: Liquid volume in containers

In this video, a teacher describes an activity where she asked students to predict whether water in one container would fit into another container of a different shape. She also used water to show them how big a milliliter was in comparison to a liter.

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Part 6: Wrap up (5 minutes)

Goals

- Participants will recall the work they have done in the three sessions that have focused on the Learning Trajectory for volume measurement.
- Participants will understand the ways of connecting session content to their classroom

Instructional sequence

1. Summarize the work of the session.
2. Watch Video A and distribute the Classroom Connection Activities.

Resources

- Video A (01:29): Previewing the CCA and the next session
- Handout: Classroom Connection Activity 3 – Volume
- Handout: Anecdotal notes form – Volume Learning Trajectories
- Handout: Anecdotal notes form – Length Learning Trajectories
- Handout: Anecdotal notes form – Area Learning Trajectories

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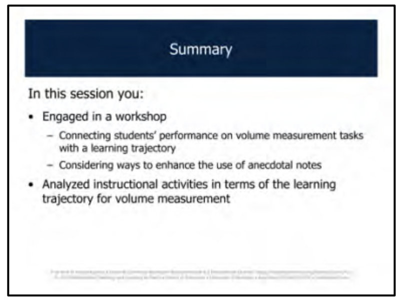
<tr><td>

1. Summarize the session by emphasizing that participants:

- Engaged in a workshop
  - Connecting students' performance on volume measurement tasks with a Learning Trajectory
  - Considering ways to enhance the use of anecdotal notes
- Analyzed instructional activities in terms of the Learning Trajectories for volume measurement



Note that participants have now completed sessions focused on the three components of the volume Learning Trajectory: the mathematics, the developmental progression of students' thinking, and instructional activities designed to advance students' thinking about volume measurement.

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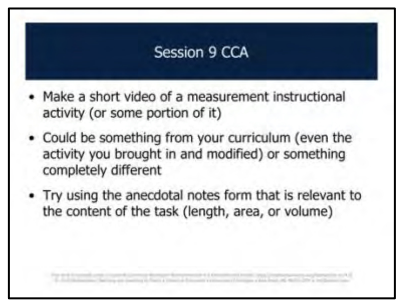
2. Watch Video A, which previews the next session and introduces the Classroom Connection Activities. In this video, Dr. Sarama explains that, in preparation for Session 10, participants should

- Make a short video of a measurement instructional activity (which could be from the curriculum or something completely different)
- Try using the anecdotal notes form that is relevant to the content of the task (length, area, or volume)



In Session 10, participants will share and discuss their videos. Then, they will spend time making mathematical connections between length, area, and volume measurement. To prepare for this session, participants should also reflect on the professional development series (using the prompts on the handout) and bring their ideas with them. Distribute the handout you customized with the Classroom Connection Activities.

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Along with the CCA for Session 10, it may be helpful to provide participants with copies of

- *Session 9 Handout: Anecdotal notes form – Volume Learning Trajectories*
- *Session 6 Handout: Anecdotal notes form – Area Learning Trajectories*
- *Session 3 Handout: Anecdotal notes form – Length Learning Trajectories*

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