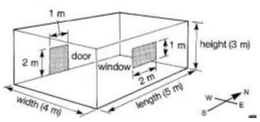
Session 9: Volume learning trajectory – Instructional activities



Overview of Session 9

- Your volume assessments and anecdotal notes
  - Learning from Practice Protocol
- The third part of learning trajectories—Instructional Activities
  - Examples along the learning trajectory
  - Activities from your curriculum
- Classroom Connection Activity

Test ourselves!

Three videos of the same student
at pre-K, kindergarten, grade 1

(Pre?) Volume Quantity Recognizer

Volume Quantifier

Initial Composite 3-D Structurer

Learning from practice protocol

Why we are working on the note taking and using the Learning from Practice Protocol...

- It is important to your teaching
- Teachers rarely get a chance to get better at it

Discussing student performance

Use the Learning from Practice Protocol

- Getting started — norm setting
- Using notes to share about student performance
- Discussion
- Focus on teaching practice by reflecting on
  - students
  - teaching
  - note taking

Using notes to describe student performance

- Sharing the tasks
- Discussing and recording small group responses to the focus questions
  - What was consistent or not with the learning trajectory?
  - How did or could the trajectory provide a framework for understanding students' responses and strategies?
  - How could the trajectory help plan "next steps" (formative assessment)?
  - How could the use of notes be improved to support descriptions of the students' measurement?

Learning from practice protocol – Debriefing

Debrief in whole group:

- Insights gained into the learning trajectory for volume
- The process of talking with colleagues using notes to support the discussion
- Ideas for enhancing the taking and use of notes
- Ways to enhance the protocol for next time

Learning trajectories approach

- Goal
- Developmental Progression
- Instruction

Common Core State Standards

- Suggests the idea of volume in Grade 2 (building, drawing, analyzing 3-D figures)
- Grade 5 begins with counting the number of cubes and quickly moves to calculating volume using linear dimensions.

Analyzing instructional activities

- Often teachers need to create their own activities or modify tasks.
- This time, instead of showing a video, we will show a picture or text and describe an activity.
- After each example, discuss:
  - What level is this developing?
  - If appropriate, how would you adapt and implement in your own classroom?

What level is this activity developing?

How would you adapt and implement the activity in your own classroom?

Volume Quantity Recognizer (VQR)

Filling Scheme

Identifies capacity as attribute
I can pour lots of sand into this can.

General language without comparison
This jar is big.

Packing Scheme

Identifies volume as attribute
This box doesn't hold many toys.

General language without comparison
This box is big.

Building Scheme

Builds with blocks: *I made a big house.*

May not recreate a given shape in size or dimensions

Counts on only one face of cube building

Comparing

May compare containers, but makes no reference to dimensions:
This glass is big. This one is tiny.

Eventually may compare 1 dimension:
This one's bigger because it's taller.

Comparing volumes

Children compare how much sand or water about eight containers will hold.

The teacher asks children to show which holds more and how they know.

Eventually, the teacher asks which holds the most.

What level is this activity developing?

How would you adapt and implement the activity in your own classroom?

Volume Filler (VF)

Filling Scheme

Fills container & counts, but may not recognize need for equal size units

Smaller container, fewer scoops - no quantification

Attends to space *filled*, not capacity

Packing Scheme

Fills box with cubes, but leaves gaps. Sometimes only one layer

Eventually fills, but doesn't quantify or use equal-size units

May not recognize "half full"

Building Scheme

May recreate, attending to 1-2 dimensions, but not pattern / plan

Counts multiple faces of cube building without pattern

Comparing Scheme

Compares by aligning 1-2 dimensions
This one holds more, it's longer and wider.

Compares counts, but without accurate recognition of unit size or number
This is big; that is small. Two scoops for this one; one scoop for that one.

Using cubes to fill a small box

Students use cubes to fill boxes constructed so a small number of cubes fit well.

They eventually predict how many cubes they will need, fill the box, and count to check.

What level is this activity developing?

How would you adapt and implement the activity in your own classroom?

Volume Quantifier (VQ)

Filling Scheme

Estimates number of scoops, but may not explicitly maintain unit size

Partitions space (capacity); can recognize "half full"

Packing Scheme

Limited spatial structuring: *counts single units*

Does not recognize need for equal-size units

Recognizes "half full," but may not visualize or calculate total

Building Scheme

Partial understanding of cubes as filling space: *Initially may double-count cubes at corners and ignore internal cubes*

Piaget's "coordination" (integration) of dimensions.

Comparing Scheme

Compares, recognizes 3 dimensions

Directly compares capacity

Attempts to compare count of cubes

Predict and test

The teacher provides three half-gallon containers (labeled "A," "B," and "C" in three different colors, cut to hold two, four, and eight cups), a one-cup measuring cup, and water or sand. Students find the one that holds only four cups.

They predict and test how many half-cups would fill the container.

What level is this activity developing?

How would you adapt and implement the activity in your own classroom?

Volume Unit Relater and Repeater (VURR)

Filling Scheme

Accurately counts number of scoops

Relates size and number of units explicitly

Packing Scheme

Accurate packing and counting

Relates size and number of units

Able to iterate a unit throughout volume; although may ignore internal cubes

Building Scheme

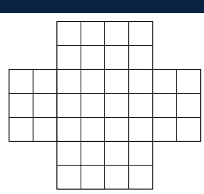
Begins to understand cubes as filling space

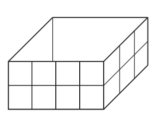
Counts cubes, not faces

Comparing

Begins to relate number and size of units to volume

Rectangular prism "nets"





Predict how many cubes will be needed to fill the box, then count and check.

Students first get a net, or pattern (on the left), and then get a picture.

What level is this activity developing?

How would you adapt and implement the activity in your own classroom?

Initial Composite 3-D Structurer (VICS)

Filling Scheme

Relates number of cubes to cubic units as measured by capacity

Sand filled to the 10 in graduated cylinder would fill a box that holds 10, inch cubes

Packing Scheme

"Sees" rows and columns (but not layers)

Fills/iterates unit to fill space (including internal)

Partitions space; uses units or subunits; visualizes remaining rows or columns

Building Scheme

Understands cubes as filling a space, moves to more sophisticated strategies and additive reasoning

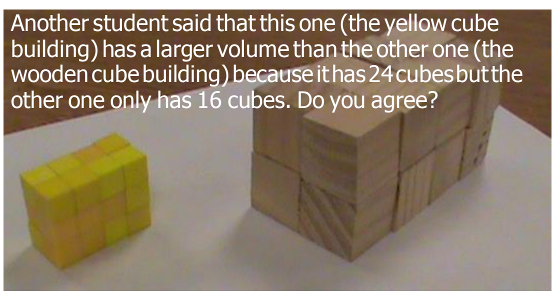
Counts number of cubes in one row/column of 3-D structure, skip counts to get total

Comparing Scheme

Explicitly relates number and size of units to volume

Recognizes that buildings of different shapes but made from same number of cubes could be packed into the same size box

Reflections on a Relating Units task



Predict

- How would a Volume Quantifier respond to this question?
- How would a Volume Unit Relater and Repeater respond to this question?

How would you respond?

A student thinks that the shape made of the smaller blocks has a greater volume.

Response option 1: Provide a new context

A student thinks that the shape made of the smaller blocks has a greater volume.

- Option #1 - provide a new context: "Imagine that these two things are made out of ice and we let them melt. Each would make a puddle on the table. Which would make a bigger puddle?"

Response option 2: Shift to a related length units question

A student thinks that the shape made of the smaller blocks has a greater volume.

- Option #1 - provide a new context: "Imagine that these two things are made out of ice and we let them melt. Each would make a puddle on the table. Which would make a bigger puddle?"
- Option #2 – shift to length units: What if I told you I measured the length of something and it was 8 inches long and this thing over here was 2 feet long. Which one is longer?

Our results (Grade 3)

- 6 out of 7 students initially agreed with the original statement
- However, they correctly answered the ice melting question or comparison by talking about the amount of space taken up

The mathematics of the Relating Units task

- Units: Centimeter and inch cubes
- Task representation: 3D

Connections to CCSSM

Grade 4

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Know relative sizes of measurement units within one system of units.

Grade 5

- Recognize volume as an attribute of 3D space.

Calculating volume using linear dimensions

- Grade 5 quickly moves to calculating volume using linear dimensions

  V = l x w x h

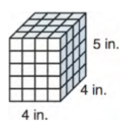
  Or V = Bh

- Which activities represent a better learning trajectory?

Improving a typical Grade 5 lesson

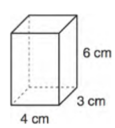
Activity 1: Fold and tape boxes then fill with cubes

- Have children fill the box one layer at a time
- Ask children how many cubes are in a layer
- Ask children how many layers are in a box



Activity 2: Building by layers

- Have students build a rectangular prism out of cubes (start with one layer, such as 3 x 4 x 1)
- Ask children to add 2nd and 3rd layers, asking the volume after each addition.
- Follow-up Tasks…



How could we improve these further?

Volume curriculum activity

In grade-level small groups, share the curriculum activities you brought in

- Which level(s) do they teach?
- Are they appropriate—based on your assessments?
- How might you improve them?
- What is the activity doing (or not) to establish and maintain an environment that nurtures math learning and practices and collective work on mathematics
- What should the teacher be doing?

Discussion

- How can we modify routine volume measurement tasks?
- Based on your experience, what have you seen that is similar to or different from our results in volume measurement activities?
- How do you think you would modify volume activities in your school?
- What challenges have you experienced in your teaching volume measurement?

Summary

In this session you:

- Engaged in a workshop
  - Connecting students' performance on volume measurement tasks with a learning trajectory
  - Considering ways to enhance the use of anecdotal notes
- Analyzed instructional activities in terms of the learning trajectory for volume measurement

Session 9 CCA

- Make a short video of a measurement instructional activity (or some portion of it)
- Could be something from your curriculum (even the activity you brought in and modified) or something completely different
- Try using the anecdotal notes form that is relevant to the content of the task (length, area, or volume)

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