Session 1: Studying mathematics teaching with a focus on reasoning and explanation

Description of the session

In this session, participants will be introduced to the goals of the module, which include (a) learning mathematics; (b) developing skill with high-leverage teaching practices; (c) developing skill in anticipating, interpreting, managing, and using students' ideas and ways of thinking; and (d) developing ways of analyzing and learning from practice. Participants are oriented to records of practice (such as video) as a means to study mathematics teaching. Participants watch a video in which sixth-grade students are discussing a three-digit permutation, and then analyze this video using three different lenses: mathematics, student thinking, and teaching practice. Participants then begin to consider a mathematics problem to launch work on engaging in reasoning and establishing the foundations for work on reasoning, the problem will be continued in Session 2. The session closes with an overview of the Classroom Connection Activities that will be completed prior to the next session.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
I. Module preview	25 minutes	Part 1	• Participants will be oriented to the work of the module and develop the sense that this module will be useful in helping to improve their mathematics teaching, knowledge of mathematics, understanding of student thinking, and ability to learn from teaching. • Participants will be oriented to the work of the session.
II. Studying mathematics teaching	25 minutes	Part 2	• Participants will be able observe mathematics teaching attending to mathematics, student thinking, and teaching practices (the core elements of module content). • Participants will develop norms for discussing records of mathematics teaching (a video).
III. The Pool Border Problem	30 minutes	Parts 3 and 4	• Participants will be able to make and justify conjectures. • Participants will begin to recognize and understand that that there are multiple approaches to solving the Pool Border Problem and there are several equivalent equations/expressions to determine the number of tiles in any border. • Participants will begin to develop an understanding of the role that shared knowledge plays for a community in their work on reasoning.
IV. Wrap up	10 minutes	Part 5	• Participants will understand the function of the Classroom Connection Activities and will understand the assignment.

Classroom Connection Activities

Required	Optional
Type of task: Practice and extension of work Description: Represent an approach for solving the Pool Border Problem. Write an explanation of how that problem would work with any size pool. Type of task: Reading Description: Ferrini-Mundy et al. (1997) on the use of the Pool Border Problem at different grade levels. (*Note: This reading is not available in the module.)* Type of task: Practice and extension of in-class work Description: Consider how the reasoning involved in the Pool Border Problem is connected with a problem you have used with your students. Describe the connection between the problem you have used and the Pool Border Problem.	Type of task: Using mathematical practices Description: Solve a book arranging problem and connect work on that problem with the problem used at the beginning of Session 1 (creating three digit numbers from 2, 4, and 7).

Preparing for the session

☐ Make copies as needed:

- *Resources:* Transcript: Classroom video – Permutations of 2, 4, and 7 (Part 2); Handout: The Pool Border Problem (Part 3)
- *Supplements:* Handout: Pool Border poster (Parts 3 & 4)

☐ Customize and make copies of the Classroom Connection Activities

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.

b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.

c) Carefully using mathematical language.

5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Scope of the module *(focal content of this session in* ***bold****)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **making and justifying/refuting conjectures and generalizations** • **recognizing and using multiple approaches to solve mathematics problems** • understanding features of a "good" mathematical explanation and producing "good" explanations • **identifying foundations of mathematical reasoning** • using and knowing the mathematical practices identified in the CCSS	• **monitoring students' mathematical reasoning** • noticing collective elements of mathematical reasoning	• supporting students' engagement in mathematical practices by teaching them explicitly • supporting students in explaining their mathematical reasoning • **establishing and maintaining an environment that emphasizes reasoning** • adapting tasks to nurture mathematical reasoning	• using norms that support engagement in video workshop • understanding the video workshop process • learning to analyze teaching and learning in the context of video workshop

Part 1: Module preview (~25 minutes)

Goals

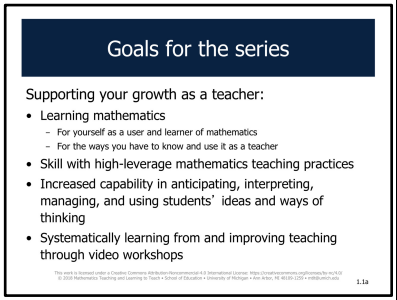
- Participants will be oriented to the work of the module and develop the sense that this module will be useful in helping to improve their mathematics teaching, knowledge of mathematics, understanding of student thinking, and ability to learn from teaching.
- Participants will be oriented to the work of the session.

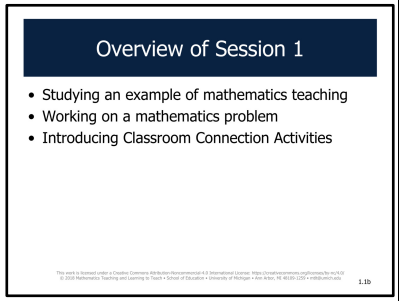
Instructional sequence

1. Have participants introduce themselves.
2. Introduce the module by having participants watch Video A.
3. Introduce the session by showing Video B.

Resources

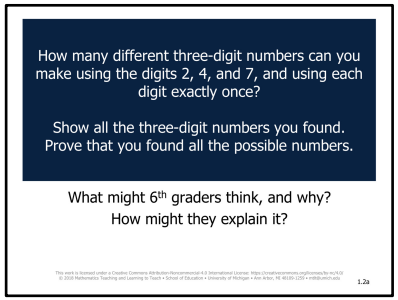
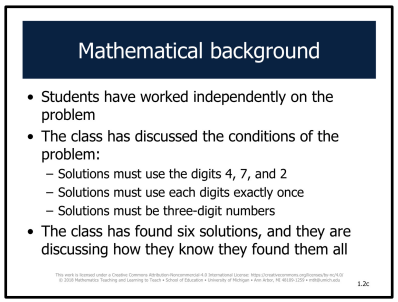
- Video A (05:47): Module overview
- Video B (01:17): Session overview

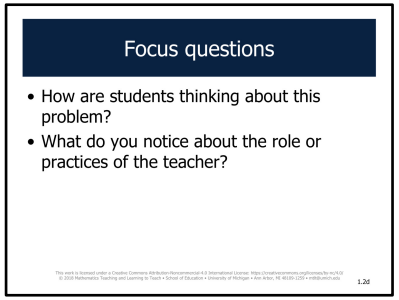
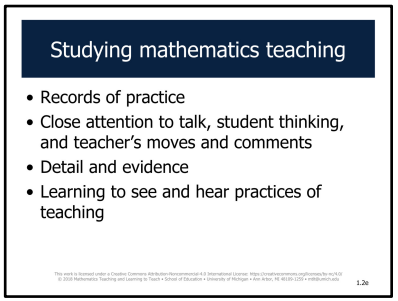
Detailed description of activity	Comments & other resources
1. Have participants introduce themselves.	*This could be done in many ways. One relatively quick way, that would provide useful information for you, would be to ask participants to say their names, grade levels, years of experience teaching elementary grades, and the topic that they have recently been working on with students in math.*
2. Introduce the module by having participants watch *Video A* in which Dr. Deborah Loewenberg Ball, a professor at the University of Michigan, provides an overview of the module. Each Dev-TE@M module focuses on four core elements of the work of elementary teaching: • Mathematics geared to the demands of teaching • Student thinking about mathematics • High-leverage mathematics teaching practices • Approaches for systematically learning from and improving teaching Work on these elements is integrated across the ten sessions of a module, providing opportunities to practice, build on, and extend ideas over time. In addition, simultaneously working on the four core elements is important because the work of elementary mathematics teaching requires integrated attention to these elements in practice. The content of the module is applicable across grade levels and strands of mathematics. The module includes examples from elementary classrooms and professional development sessions. Many of the videos used in this module are taken from a professional development workshop for teachers led by Dr.	 *To play video in the main viewer, simply move your cursor over the video image to reveal the video player controls – including the "play" button, volume control, full-screen/normal view toggle, and progress bar for viewing and controlling where, in the video, you are located. The slides are synchronized with the video and will change at the appropriate time.* *To display the video or slides in full-screen view, move your cursor over the image and select the "full-screen" icon in the lower right corner:* *When in full-screen, you can toggle between displaying the video or slides (if present) by clicking on the image to reveal the video or slides as desired. Clicking on the image allows for display options such as single screen, and picture-in-picture. Clicking on the image in any of these viewing modes will switch the position of the video and slides.*

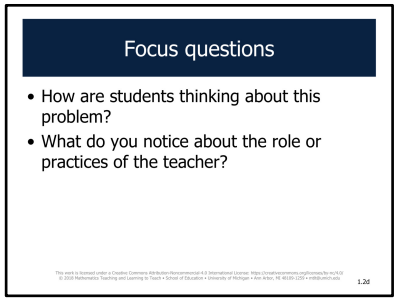
Detailed description of activity	Comments & other resources
Ball. The teachers in the professional development workshop focused on the same content and many of the same activities as are used in this module. These videos are important resources that will support the work in the module. Video clips from the professional development session are often used to frame and summarize activities in the module. These clips are also used to provide opportunities to listen and respond to ideas raised by the teachers in the clips, as well as to analyze and discuss many of the issues with which they grappled during the course. The module includes Classroom Connection Activities because learning about teaching is an extended process that needs to be connected with and supported by learning in and from one's own teaching. These activities provide significant opportunities to learn. Routine tasks in the activities encourage participants to use their teaching as a context for learning, connect professional development content with common classroom resources, support feedback on learning and teaching, as well as extend thinking about mathematics and teaching that arose in the session.	*To exit full-screen view, press the "esc" key on your computer keyboard or click on the normal view icon in the video player controls:* *Participants may have questions about the use of video from their own classrooms in the module.*
3. Introduce the session by having participants watch *Video B.* Session 1 begins work on studying mathematics teaching and launches work on a mathematics problem that is the basis for the first two sessions. 	

Part 2: Analyzing a video from a sixth grade-classroom (~25 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be able observe mathematics teaching attending to mathematics, student thinking, and teaching practices (the core elements of module content). • Participants will develop norms for discussing records of mathematics teaching (a video).	1. Introduce Part 2 and have participants anticipate solutions and explanations of sixth-grade students. 2. Watch Video A to provide context for the classroom video. 3. Watch the classroom video (Video B). 4. Discuss the video using the focus questions.	• Video A (02:26): Setting the context for the video • Video B (04:45): Classroom video – Permutations of 2, 4, and 7 • Transcript: Classroom video – Permutations of 2, 4, and 7

Detailed description of activity	Comments & other resources
1. Introduce Part 2: This part uses video from a sixth-grade mathematics class to begin to explore core elements of elementary teaching: mathematics, student thinking, and teaching practice. Have participants solve the problem themselves and anticipate how sixth graders might think about the problem: *How many different three-digit numbers can you make using the digits 2, 4, and 7, and using each digit exactly once? Show all the three-digit numbers you found. Prove that you found all the possible numbers.* 	*Anticipating student thinking is a routine activity in the module. Participants are likely to know many ways in which students may think about math problems, and may also hold strong opinions about whether or not problems are appropriate and the extent to which students are likely to succeed when presented with particular problems. You may need to persist or redirect participants to move past initial judgments and to engage in predicting what students might do with problems that were actually used with students.*
2. Show *Video A* to provide context on the classroom in the video and context around the mathematics that the class has been working on. 	

Detailed description of activity	Comments & other resources
The focus questions are also introduced as a part of the video: • What are students thinking about this problem? • What do you notice about the role or practices of the teacher? 	
3. Establish norms for discussions of video records of practice by showing and discussing *Slide: Studying mathematics teaching.* Remind participants that the classroom video is an example of teaching, not an instance of exemplary teaching. Participants should treat the video as a concrete instances of teaching that can be seen, heard, and used to support description and analysis. The video provides an opportunity to focus on "teach**ing**" so try not to get involved with statements that focus on points about the "teach**er.**"  Distribute *transcripts* and have participants watch *Video B: Classroom video – Permutations of 2, 4, and 7*. Tell participants to think about the focus questions as they watch the video and suggest using the transcript to keep track of things that stand out to them in the video.	*Watching a video of teaching in a professional development context may be a new experience for some participants. Some participants may think that the teaching in the videos is intended as an examples of "good teaching" to be emulated. Try to make clear that the video is not intended to model "good teaching," rather the video is being used to study the practice of teaching mathematics.* *You may need to help participants navigate between noting ideas in the transcript and attending to the video. The video includes more information than the transcript, so it is important to watch the action. However, making some notations in the transcript will help participants in the next part when they are asked to refer to specific examples in the video to support their ideas.* *This video, like others in the module, is text tracked. To see the text tracking, move your cursor off of the video so that the tool bar disappears.*

Detailed description of activity	Comments & other resources
4. With these points in mind, have participants discuss the focus questions in either small or whole group.  	*Possible observations:* *Student thinking about the problem* • *Kevin is considering different possible positions for the digits and how each digit must be in each possible position. His explanation is not a complete explanation and may not fully reflect his understanding.* • *Shandel is noticing mathematical structure ("it's like a third-grade line")* *Role or practices of the teacher* • *Asks questions to clarify explanations shared by students (e.g., asking Kevin how he got 6 solutions when he shared reasoning for 3 solutions)* • *Asking students to explain a solution shared by a classmate* • *Identifies a connection between the problem and a third-grade line, initially shared by a student (Shandel), and asks students to explain why lining up is like the numbers* • *The teacher never records on the board during the clip; recording is done by students* *Participants may make general comments about the teaching and learning in the video that are focused on the causes/reasons for particular moves. Encourage participants to respond to the focus questions by listening for and, if needed, asking about specific facets of the video related to mathematics, teaching, and student thinking. Support norms of attending to detail and supporting claims with evidence from the video/transcript.* *Participants may also be curious about what happened before and after the action in this particular clip. Support them in focusing on the teaching captured in the clip itself.*

Part 3: Exploring the Pool Border Problem (~15 minutes)

Goals

- Participants will be able to make and justify conjectures
- Participants will generate and/or interpret multiple approaches to solving the problem *(several expressions/equations, series of progressively larger problems, etc.)*

Instructional sequence

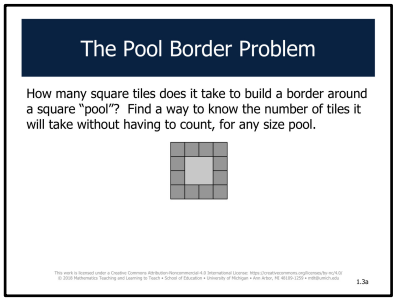
1. Introduce Part 3 and watch Video A.
2. Watch Video B and have participants work independently on the problem.

Resources

- Video A (00:54): Rationale for working on mathematics problems
- Video B (01:56): Launching the problem
- Handout: The Pool Border Problem

Supplements

- Handout: Pool Border poster

Detailed description of activity	Comments & other resources
1. Introduce Part 3: This part launches work on the mathematics element of the module. Have participants watch *Video A* in which Dr. Ball provides a rationale for working on mathematics problems.	
2. Have participants watch *Video B* in which Dr. Ball launches work on the Pool Border Problem.  Check to see if participants understand the problem by seeing if they know, in the example on the slide, what would be referred to as the "border", "square tile", "square pool", and well as the nature of the solution that is sought. Have participants work independently on the problem for approximately 10 minutes.	*You may want to organize this section of work in the following way:* *1) Show the slide with the problem full screen and clarify the meaning of the problem with participants.* *2) Show* Video B. *3) Ask if there are any questions.* *4) Have participants work alone on the problem for 7-10 minutes.* *Don't spend time in the introduction on different ways of describing how to get the total number of tiles. Just make clear that the point of the problem is to be able to describe how to get the total number of tiles in the border for any size pool. Make it clear that the pool must be a square.* *Let participants come up with their own methods of recording. You may want to provide graph paper for their use. Do not provide manipulatives to participants at this point. As participants are working, you may want to encourage participants to try drawing a diagram of a pool and its border if participants appear to be stuck.* *If you have access to a document camera, encourage participants to record their work using pens or markers so that they can later project their work for the group.*

Detailed description of activity	Comments & other resources
	Encourage participants to work on their own. They might want to work with colleagues immediately. Reassure them that they will get to work on the problem with colleagues later in the session. Make the point that participants' small group and whole group discussions of the problem will be enhanced if they first have the opportunity to work independently on the problem. *Participants may describe the size of the pool in ways that are different from those in the video. In the video, the pool is described in terms of its area (e.g., "the pool is size 4"), but it could be described in terms of its side length (e.g., "the pool has a side length of 2.")* *When working on this problem, some participants may confound area and perimeter. Clarify the meaning of these terms as needed.*

Part 4: Considering others' explanations for the Pool Border Problem (~15 minutes)

Goals

- Participants will be able to make and justify conjectures.
- Participants will begin to recognize and understand that that there are multiple approaches to solving the Pool Border Problem and there are several equivalent equations/expressions to determine the number of tiles in any border.
- Participants will begin to develop an understanding of the role that shared knowledge plays for a community in their work on reasoning.

Instructional sequence

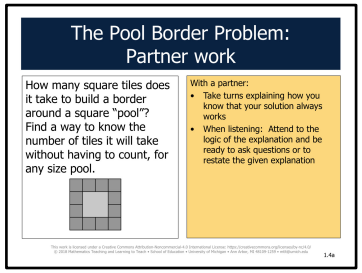
1. Introduce Part 4 and watch the video.
2. Have participants work with a partner on the problem.
3. Share 1-2 approaches in whole group.

Resources

- Video A (00:55): Initiating partner work on the problem

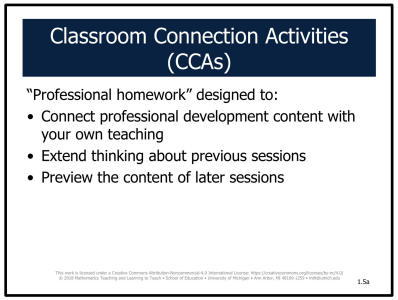
Supplements

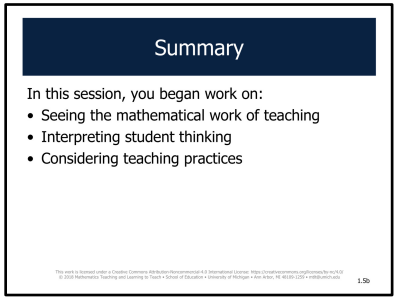
- Handout: Pool Border poster

Detailed description of activity	Comments & other resources
1. Introduce Part 4: In this part, continue work on the Pool Border Problem by conferring with a partner. Have participants watch the *video* in which Dr. Ball launches partner work on the Pool Border Problem. 	*Consider ordering this section of work in the following way:* *1) Show the slide full screen.* *2) Show the video.* *3) Return to the slide while partners work.*
2. Have participants work with a partner on the problem. • Take turns explaining how you know that your solution always works • When listening: Attend to the logic of the explanation and be ready to ask questions or to restate the given explanation.	*Emphasize that this is a time for participants to do mathematics together. They should not focus at this point on thinking about how students would solve the problem. Instead, they should focus on understanding how their partner approached the problem and should ask clarification questions as needed.*
3. Invite 1-2 participants to share their methods and solution to the problem. During the discussion, consider: • Whether each approach will allow you to know the number of tiles for any size pool • How solutions connect back onto the problem (is the explanation complete?) • How solutions/approaches map onto each other *(if there is time to share two approaches)* • The language, representations, and logic used in each explanation After 1-2 participants have shared, summarize what has already been discussed and indicate to participants that they will continue their discussion of the Pool Border Problem in the next session.	*Participants can share work using a document camera, or they can recreate their work on a whiteboard or on a piece of poster paper.* *Depending on how the work proceeds, you might choose to ask someone to present who has tried some different approaches, but who has <u>not</u> yet solved the problem.* *The goal is <u>not</u> to get all approaches out on the table; rather, the goal is to share 1-2 approaches. Participants will write up an explanation of their method as part of their Classroom Connection Activities assignment and will discuss the problem further in the next session.*

Part 5: Wrap up (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand the function of the Classroom Connection Activities and will understand the assignment.	1. Watch Video A. 2. Distribute the Classroom Connection Activities. 3. Summarize the work of the session.	• Video (01:54): Classroom Connection Activities

Detailed description of activity	Comments & other resources
1. Introduce Classroom Connection Activities. These activities are professional homework designed to: • Connect the professional development content with classroom teaching • Supporting feedback on teaching • Extend thinking about the content of previous sessions • Preview the content of later sessions Watch the *video* in which Dr. Ball describes the use of Classroom Connection Activities in the module. 	*Because learning about teaching is an extended process that can be supported by learning in and from ones own teaching, each session includes a "Classroom Connection Activities." These activities provide significant opportunities to learn. Routine tasks in the activities encourage participants to use their teaching as a context for learning, connect professional development content with common classroom resources, support feedback on learning and teaching, as well as extend thinking about mathematics and teaching that arose in the session.*
2. Distribute the handout you customized with selected Classroom Connection Activities and accompanying documents described below. <u>Required</u>: • Represent an approach for solving the Pool Border Problem. Write an explanation of how that approach would work with any size pool. • Read the article "Experiences with Patterning" by Ferrini-Mundy et al. (1997) from *Teaching Children Mathematics* to begin thinking about how the Pool Border Problem could be used with students at different grade levels. • Consider how the reasoning involved in the Pool Border Problem is connected with a problem you have used with your students. Write a brief explanation of the connection you see between the problem you have chosen and the Pool Border Problem. <u>Optional</u>: • Solve a book arranging problem and connect work on that problem with the problem used at the beginning of Session 1 (creating three-digit numbers from 2, 4, and 7).	*To instill the idea that the CCAs will be an important part of participating in the module, make sure that participants know that there will be time at the beginning of the next session to talk about one or more of the CCA responses. For instance, participants should be ready to share their unpacking of a colleague's approach suggested by the first task or to share the problem suggested in the third task.* *It may be helpful to ask participants to upload their responses or materials related to the Classroom Connection Activities so that you review what participants have been thinking and trying prior to the next sessions. In addition, uploading responses would allow participants to easily share their ideas with each other.*

Detailed description of activity	Comments & other resources
3. Summarize the work of the session for participants. Note that ideas and examples from the Classroom Connection Activities will be taken up during the next session and that work on the Pool Border Problem will also be continued. 	

List of Common Core State Standards Mathematical Practices

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision.
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.