Description of the Session 3: Noticing the features of explanations and unpacking student reasoning

In Session 3, participants will continue the work that they started in Sessions 1 and 2 on explaining and evaluating approaches to solving mathematics problems. First, they will solve the Three-Coin Problem and explain and evaluate the approaches they used. Then, they will focus specifically on noticing which features of their explanations—including the language, representations, and mathematical structure they used—were helpful for sharing their thinking with each other. Participants will then discuss the ways that the professional development content connects with the Common Core State Standards for Mathematics—particularly the standards for mathematical practice. Finally, participants will watch a video of a third-grade classroom and will use evidence from the video to consider students' approaches to reasoning and to identify ways teachers can nurture environments that support students' reasoning.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
Conversation about a CCA from the last session	5 minutes		• Participants will share the work they did with their students on the Pool Border Problem.
I. Preview	5 minutes	Part 1	• Participants will be oriented to the work of the session.
II. The Three-Coin Problem	30 minutes	Parts 2 & 3	• Participants will make use of mathematical practices including making sense of problems; constructing valid arguments; and looking for and making use of structure. • Participants will provide justifications using different approaches. • Participants will recognize and understand that there are multiple approaches to solving the Three-Coin Problem.
III. Features of explanations	10 minutes	Part 4	• Participants will begin to notice features of "good" explanations. • Participants will begin to notice how language, representations, and mathematical structure are used when making mathematical arguments.
IV. Connecting with the Common Core State Standards for Mathematics	5 minutes	Part 5	• Participants will understand the context of the Common Core State Standards. • Participants will begin to become familiar with the standards for mathematical practice.
V. Analyzing and supporting students' reasoning	25 minutes	Part 6	• Participants will use video evidence to notice how students reason about a problem. • Participants will use video evidence to notice and discuss how students support and explain their approaches using words, drawings and/or tools. • Participants will describe what teachers can do to establish/maintain environments that nurture mathematical practice and collective work on mathematics.
VI. Wrap up	10 minutes	Part 7	• Participants will understand ways of connecting the session content to their classroom.

Classroom Connection Activities

Required	Optional
Type of task: Video workshop preparation Description: Those sharing their practice with a video workshop group should select a short segment of video to share and bring examples of student work and their reaction to focus questions and background prompts. Everyone (regardless of whether or not they are sharing their video this week) should submit their records of practice and write-up to the facilitator. Type of task: Reading and extension of in-class work Description: Review the handout of key points about the Common Core State Standards mathematical practices and look at the information shared on the CCSS-M website (http://www.corestandards.org/the-standards/mathematics/introduction/standards-for-mathematical-practice/). Identify the practices that you see in the video segment that you selected from your teaching of the Pool Border Problem and that you saw in work in this session on the Three-Coin Problem.	Type of task: Using mathematical practices Description: Solve the 8s Problem and connect work on that problem with the Three-Coin Problem. Type of task: Mathematics Reading Description: The Three-Coin Problem Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.

Preparing for the session

☐ Make copies as needed:

- *Resources:* Handout: The Three-Coin Problem (Part 2); Handout: The mathematical practices (Part 5); Handout: 21¢ Problem (Part 6); Transcript: Classroom video – 21¢ Problem (Part 6)
- *Supplements:* Math notes: Analysis of the Three-Coin Problem (Part 3)

☐ Customize and make copies of the Classroom Connection Activities

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
    a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
    b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
    c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

*Scope of the module (focal content of this session in **bold**)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **making and justifying/refuting conjectures and generalizations** • **recognizing and using multiple approaches to solve mathematics problems** • understanding features of a "good" mathematical explanation and producing "good" explanations • identifying foundations of mathematical reasoning • **using and knowing the mathematical practices identified in the CCSS**	• **monitoring students' mathematical reasoning** • noticing collective elements of mathematical reasoning	• supporting students' engagement in mathematical practices by teaching them explicitly • supporting students in explaining their mathematical reasoning • **establishing and maintaining an environment that emphasizes reasoning** • adapting tasks to nurture mathematical reasoning	• using norms that support engagement in video workshop • understanding the video workshop process • learning to analyze teaching and learning in the context of video workshop

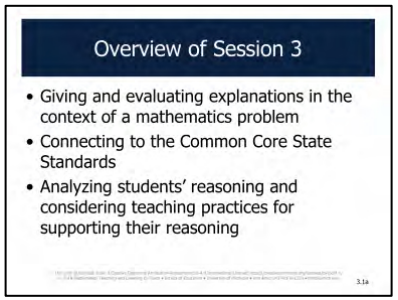
Conversation about a Classroom Connection Activity from last session (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will share the work they did with their students on the Pool Border Problem	1. Participants share their work from the Pool Border Problem. 2. Participants share about the logistics of video recording.	

Detailed description of activity	Comments & other resources
1. Ask a few participants to share their work on the Pool Border Problem with their students. • What version of the problem did you use? • What kinds of strategies, solutions, and questions did the students come up with? 2. Ask participants about the logistics of video recording. Explain that, though video recording can be challenging (e.g., helping kids understand why the class is being video recorded, setting up the camera, and saving and sharing the video), it is worth the effort because it will allow participants to study the mathematical practices in action in their own classrooms. You might ask questions such as: • How did video recording the teaching and learning work out? • Where did you position your camera? • What were you able see in your video? • How well were you able to hear what was being said?	*Use this discussion as an opportunity to reinforce several ideas:* • *the point of using the problems with students is to provide opportunities to learn about the mathematical practices and to have a common context for talking with colleagues about support students' learning* • *it is possible to use this problem with students meaningfully in 15 minutes of class time (while the problem could certainly be the basis for longer engagement)* • *video recording the teaching and learning provide really helpful records of practice that can support learning.* *Don't spend too much time of this time having multiple people talk about the teaching of the problem because that will be the focus of the video workshop for next session.*

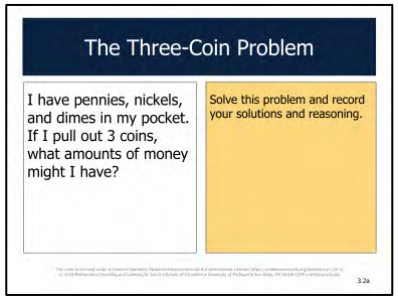
Part 1: Preview (~5 minutes)

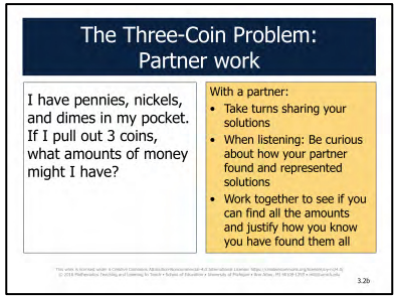
Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the session.	1. Introduce the session.	• Video A (01:31): Session overview

Detailed description of activity	Comments & other resources
1. Introduce the session: This session builds on the work started in Session 2. In this session, participants work on three core elements of module content: • Mathematics: Giving and evaluating explanations in the context of a mathematics problem • Teaching practice: Connecting to the Common Core State Standards, focusing specifically on the standards for mathematical practice • Student thinking: Analyzing students' reasoning in the context of a classroom video and considering teaching practices for supporting their reasoning  Have participants watch the *video* in which Dr. Ball frames the work of the session.	

Part 2: Exploring the Three-Coin Problem (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will make use of mathematical practices including making sense of problems; constructing valid arguments; and looking for and making use of structure. • Participants will provide justifications using different approaches. • Participants will recognize and understand that there are multiple approaches to solving the Three-Coin Problem.	1. Introduce Part 2 including watching Video A and launching the problem. 2. Work independently on the problem. 3. Watch Video C and the work with a partner on the problem.	• Video A (00:50): Launching the problem • Video B (01:51): Testing an initial solution • Video C (00:57): Initiating partner work on the problem • Handout: The Three-Coin Problem

Detailed description of activity	Comments & other resources
1. Introduce Part 2: This part launches work on a mathematics problem that will be used throughout the session. Watch *Video A* in which Dr. Ball provides rationale for working on a mathematics problem and frames how mathematics problems will be used in the module. Introduce the Three-Coin Problem and explain how work will proceed (individual, partner, then whole-group discussion). Distribute the *handout* with the problem.  *I have pennies, nickels, and dimes in my pocket. If I pull out 3 coins, what amounts of money might I have?* Watch *Video B* in which a teacher in the professional development series offers a solution and other teachers comment on why it is or is not a solution to the problem. Alternatively, invite one participant to share **one amount** that is solution and ask other participants to try to explain why it is or is not a solution.	*Even though this is a problem that could be done with students, the focus in this activity is on our own work on mathematics and our own thinking about the problem.*
2. Allow about 3 minutes for participants to work individually on the Three-Coin Problem.	

Detailed description of activity	Comments & other resources
3. Watch *Video C* in which Dr. Ball launches partner work on the mathematics problem. Then, have participants work with a partner. Partners should take turns sharing their solution. Encourage them to be curious about how their partner found and represented solutions. They should work together to see if they can find all the amounts and to justify how they know that they found them all. 	*Consider making a comment about "being neutral" when listening or "being curious" about what the other person thinks or found.* *Note the different approaches that participants are using to justify their solution.*

Part 3: Discussing Three-Coin Problem explanations (~15 minutes)

Goals

- Participants will make use of mathematical practices including making sense of problems; constructing valid arguments; and looking for and making use of structure.
- Participants will provide justifications using different approaches.
- Participants will recognize and understand that there are multiple approaches to solving the Three-Coin Problem.

Instructional sequence

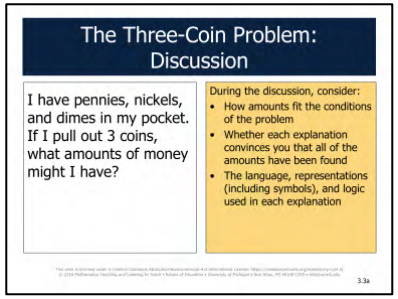
1. Introduce Part 3 by showing Video A.
2. Share solutions to the problem.
3. Watch a selection of Videos B-D and discuss.

Resources

- Video A (01:14): Initiating whole group discussion
- Video B (01:43): Approach 1 – Using an organized list
- Video C (01:10): Approach 2 – Considering whether the order of coins matters
- Video D (01:10): Approach 3 – Making an analog to types of triangles

Supplements

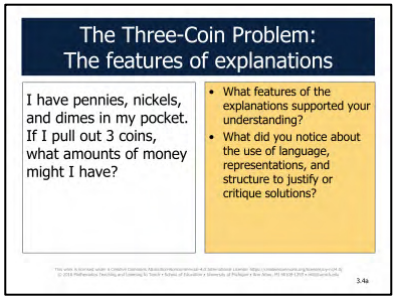
- Math notes: Analysis of the Three-Coin Problem

Detailed description of activity	Comments & other resources
1. Introduce Part 3: This part launches whole group discussion of the Three-Coin Problem. Watch *Video A* in which Dr. Ball frames the whole group discussion. 	
2. Invite several participants to share their methods and solutions to the problem. Have participants **justify how they know that they found <u>all</u> of the amounts**. Encourage both partners to come up and explain their methods and solutions. Elicit several different methods. During the discussion, consider: • how amounts fit the conditions of the problem • whether each explanation convinces you that all of the amounts have been found • the language, representations (including symbols), and logic used in each explanation.	*If the conversation goes off topic, direct the participants back to the focus questions listed.*

Detailed description of activity	Comments & other resources
3. Have participants watch a selection of *Videos B-D* in which teachers in the professional development course share their methods and solutions to the problem. Be sure to watch *Video D,* as it will be referenced in a video in the next part of the session. As participants watch each clip, have them consider the questions described above. • *Video B*: Approach 1 – Using an organized list • *Video C:* Approach 2 – Considering whether the order of the coins matters • *Video D:* Approach 3 – Making an analog to types of triangles Consider distributing the *Math notes* document so that participants can read more about the task.	*The explanation shared in Video D will be referenced in a video in the next part of the session. This video represents an unusual method for solving the problem.* *Video B* *A teacher uses an organized list to find the solution to the problem. First, he finds all combinations containing pennies (first 3 pennies, then 2 pennies, and then 1 penny). Then, he finds all combinations that contain nickels and dimes but no pennies. Finally, he records the combination that only contains nickels and the combination that only contains dimes.* *Video C* *A teacher explains that, because there are three different coins (pennies, nickels, and dimes) that could be pulled for each of the three coin pulls, she multiplied 3 x 3 and found that there were 27 different ways to pull 3 coins. After she shares her solution, the group discusses why there are fewer than 27 possible amounts of money even though there are 27 different ways to arrange the coins.* *Video D* *A teacher uses his knowledge of classification of triangles by sides (isosceles, scalene, and equilateral) to create an organizational structure for listing the possible combinations of coins.* *As an extension of this discussion, consider asking participants how this problem is like the 2, 4, and 7 problem and how it is different. The purpose of this discussion would be to highlight the difference between permutations problems (like the 2, 4, and 7 Problem, where the order of digits matters) and combinations problems (like the Three-Coin Problem, where the order of the digits doesn't matter).*

Part 4: Noticing the features of explanations (~10 minutes)

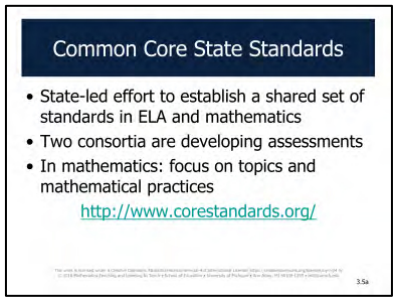
Goals	***Instructional sequence***	***Resources***
• Participants will begin to notice features of "good" explanations. • Participants will begin to notice how language, representations, and mathematical structure are used when making mathematical arguments.	1. Introduce Part 4 and watch Video A. 2. Discuss focus questions with a partner and then in whole group. 3. Watch and discuss Video B.	• Video A (00:38): Noticing the features of explanations • Video B (01:21): Teacher insight – Orienting explanations to listeners

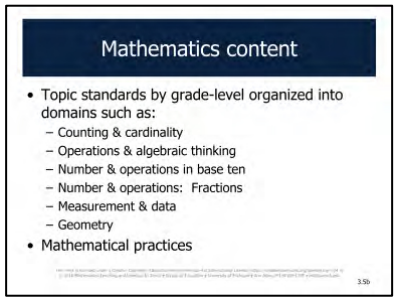
Detailed description of activity	Comments & other resources
1. Introduce Part 4: This part focuses on features of explanations and the use of representations in the work on the Three-Coin Problem. Watch *Video A* in which Dr. Ball introduces the focus of the part and asks participants to consider: • What features of the explanations supported your understanding? • What did you notice about the use of language, representations, and structure to justify or critique solutions? 	
2. Have participants discuss the two focus questions with a partner. Then, elicit responses in whole group.	*In Session 5, four features of "good" mathematical explanations will be introduced.* • *Has a clear purpose* • *Has a logical structure* • *Uses representations and language clearly and carefully* • *Focuses on meaning and is oriented to the listener(s)* *Do not explicitly name these features in this session.*
3. Watch *Video B* in which a teacher raises the notion that a good explanation is oriented to the listener. Invite participants to comment on the idea.	*When teaching new ideas to a particular group of students, it is helpful to have a sense of the related ideas that the group is already familiar with. When teachers attempt to make explanations more meaningful to their students by relating new ideas to familiar ones, they are "orienting their explanations to the listener".*

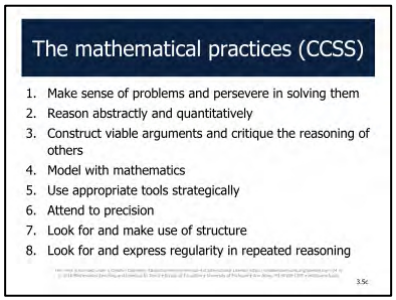
Part 5: Connecting with the Common Core State Standards (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand the context of the Common Core State Standards. • Participants will begin to become familiar with the standards for mathematical practice.	1. Introduce Part 5 and watch the video.	• Video (04:18): Connecting to the Common Core State Standards

Detailed description of activity	Comments & other resources
1. Introduce Part 5: The mathematical practices articulated in the Common Core State Standards are at the heart of the problem-solving work that is the focus of in this module. Before playing the video, share that Dr. Ball will be providing a recap of the context for the Common Core State Standards and then make an interesting point that the mathematical practices are essential **content**, just like grade-level standards for topics in domains like number and operations. After this introduction, play *Video A.*	*Giving and critiquing explanations and the use of mathematical language, representations, and structure are all examples of mathematical practices that came up in this session and that likely came up in previous sessions as well.* *If participants are quite familiar with the standards for mathematical practice, it is likely that the video will still be of use because of the points made about how the mathematics content is comprised of **both** topics and practice. Practices **are content.*** *If participants teach in settings where the Common Core State Standards have not been adopted, emphasize that the mathematical practices reflect general aspects of reasoning and problem solving that are important for students to engage in regardless of the specific set of standards their school is using.* *The slide labeled "mathematical practices" should be understood as providing space to elaborate the practices on the previous slide, not as separating practices from the content.*







After the video, encourage the participants to talk to a colleague about the idea that the mathematics practices are content.

Part 6: Considering students' approaches to reasoning (~25 minutes)

Goals

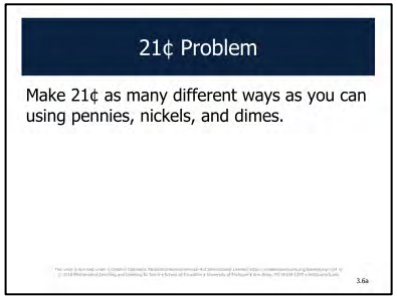
- Participants will use video evidence to notice how students reason about a problem.
- Participants will use video evidence to notice and discuss how students support and explain their approaches using words, drawings and/or tools.
- Participants will describe what teachers can do to establish/maintain environments that nurture mathematical practice and collective work on mathematics.

Instructional sequence

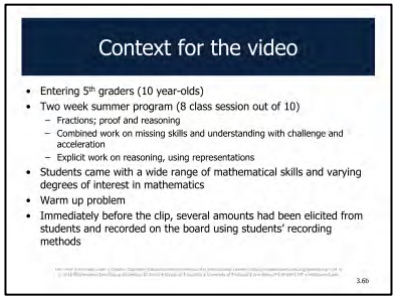
1. Introduce Part 5 and watch Video A.
2. Watch Videos B-C.
3. Discuss focus questions.
4. Watch a selection of Videos D-G.

Resources

- Video A (00:36): Introducing the 21¢ Problem
- Video B (01:04): Setting the context for the video
- Video C (06:13): Classroom video – 21¢ Problem
- Video D (01:02): Teacher insight – Representing information in the problem
- Video E (01:01): Teacher insight – Distinguishing between methods and solutions
- Video F (01:24): Teacher insight – Appreciating multiple methods
- Video G (01:33): Teacher insight – Connecting to literacy
- Handout: 21¢ Problem

Detailed description of activity	Comments & other resources
1. Introduce Part 6: This part focuses on students' reasoning and what a teacher can do to establish and maintain an environment that nurtures students' reasoning. Watch *Video A* in which Dr. Ball introduces the 21¢ Problem.  Have participants briefly work on the problem and anticipate how students might reason about the problem.	*You may want to signal that this problem has an interesting relationship to the problem participants just worked on. They will be exploring that relationship and the work that teachers do to create connected problems in the next session.* *The purpose here is to briefly consider the mathematics problem and the ways in which students might reason about the problem.*

Detailed description of activity	Comments & other resources
2. Watch *Video B*, which provides context for the classroom video and introduces the focus questions. 1) How are students reasoning about the problem? 2) How are students supporting/explaining their approaches using words, drawings, or tools? 3) What is the teacher doing to establish and maintain an environment that nurtures reasoning practices and collective work on mathematics? What else could the teacher be doing? Distribute the *transcript* and view the slide with the focus questions. Then, watch the classroom *video* (Video C) with these focus questions in mind.	*Continue to establish norms for studying mathematics teaching together in the session (records of practice; close attention to talk, student thinking, teacher's moves and comments; detail and evidence; learning to see and hear practices of teaching).* *This is not a standard video workshop, but it does model the video workshop norms.* *For example, Video B includes a model of how to set up a video. When participants share video in video workshops, they should also share the grade level, the task, lesson goals and the goal that is being worked on in the segment, and what has happened immediately before the segment.*
3. Invite participants to share their responses to the focus questions.	***How are students reasoning about the problem?*** • *Jael tries different coin combinations, recording symbols for coins and adding their amounts until she reaches 21 cents. "Three nickels equals 15 cents, so I just added 5 pennies to make 20 cents and then I added one more penny to make 21." (lines 47-49)* • *Most students discuss their methods for finding individual solutions—they don't discuss if/how they organized their solutions to make sure they weren't repeating any.* • *At the end of the clip (lines 140-141), Karina says that she numbered her solutions to keep track of how many she had.* ***How are students supporting/explaining their approaches using words, drawings, or tools?*** • *Some use symbols (e.g., different shapes, representations of coins, coin amounts) to record different combinations that were solutions to the problem. (lines 52-54, 75).* • *After recording the symbols for a particular solution, students count up the coin amounts to show they equal 21¢ (lines 47-49).* • *Some use a table to keep track of the number of pennies, nickels, and dimes in each solution.*

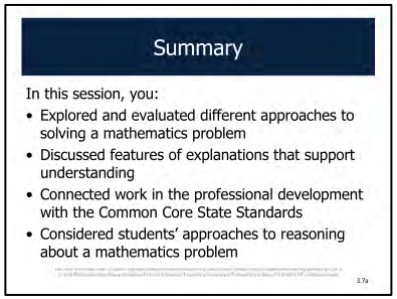


Detailed description of activity	Comments & other resources
	What is the teacher doing to establish and maintain an environment that nurtures student reasoning practices? • *The teacher records students' methods on the board or asks them to record (lines 21-23, 43, 75-76, 94-95, 119-120).* • *After Jael presents her method, the teacher asks another student to describe how to use Jael's method to record a different solution (lines 34-35).* • *When students present a solution, the teacher asks them to show how it fits the conditions of the problem: "And how can you explain that that's 21 cents?" (lines 45-46).* • *The teacher focuses on discussing different <u>methods</u> students used (rather than just focusing on different solutions): "I think we have at least one more method that people used in their notebooks that is different from anything on the* board…. [summarizes methods already shared.] Who has a different method of recording?" (lines 50-61) • She implicitly acknowledges the need for a structure to organize solutions: "Give us a solution that's different…It's getting harder to tell since we're all using different methods; it's harder to find one that's different." (lines 82-85) • She encourages Shawn to point out a mistake she (the teacher) made in recording a solution (lines 136-139).
4. Show a selection of *Videos D-G* to spur discussion about what a teacher can do to establish and maintain an environment that nurtures and supports students' reasoning. • Video D: Teacher insight – Representing information in the problem • Video E: Teacher insight – Distinguishing between methods and solutions • Video F: Teacher insight – Appreciating multiple methods • Video G: Teacher insight – Connecting to literacy After showing each video, ask participants to take up the points that are shared by asking for restatements, examples from their own teaching that connect to the point being raised, or new questions that the videos raise.	*The work to establish environments that nurture the mathematical practices is something that will be a theme across the module sessions (e.g. in the classroom videos) and also something that can be tracked on as participants work on CCAs. You will return to this point, and also to the 21¢ Problem and Three-Coin Problem, in the next session.* *Video D: Teacher insight – Representing information in the problem* *A teacher explains how she came to understand why it might be beneficial for some students to use abstract symbols (e.g., shapes) to represent information in a problem (e.g., coin values)* *Video E: Teacher insight – Distinguishing between methods and solutions* *A teacher notes that it is important to help students learn distinguish between "methods" for solving problems and "solutions" to problems, and it is also important to provide students with opportunities to share both "methods" and "solutions" when discussing mathematics problems. Doing so enables all students to have opportunities to contribute to the discussion—even if they do not have a complete solution to the problem.*

Detailed description of activity	Comments & other resources
	Video F: Teacher insight – Appreciating multiple methods *A teacher reflects that it is important for students to understand that there are multiple methods to solve problems so that they will feel comfortable figuring out a method that works for them. Dr. Ball also points out that it is important for students to learn that there are multiple methods for solving problems because this is part of the content of mathematics.* *Video G: Teacher insight – Connecting to literacy* *A teacher notices a connection between methods for solving mathematics problems and reading strategies. She notes that teachers can help support students in finding and using methods for solving math problems in the same way that they support students in using specific strategies in reading.*

Part 7: Wrap up (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom.	1. Summarize the work of the session. 2. Explain and distribute the Classroom Connection Activities.	

Detailed description of activity	Comments & other resources
1. Summarize the session by emphasizing that participants: • Worked on a mathematics problem, the Three-Coin Problem, with a focus on different approaches to solving the problem and explaining the solution • Discussed features of explanations that support understanding • Considered the Common Core State Standards for Mathematical Practice • Analyzed a classroom video with a focus on noticing students' reasoning and the work that a teacher does to establish and maintain an environment that supports and nurtures student reasoning practices 	
2. Distribute the *handout* you customized with selected Classroom Connection Activities and accompanying documents described below. Required: • Make sure that participants know who is sharing in the video workshop groups. Also be clear that those not sharing in the video workshop groups at the next session should still submit their materials. Provide/confirm directions about how your group will be sharing examples from teaching with colleagues. • Review the handout of key points about the Common Core State Standards Mathematical Practices and look at the information shared on the CCSS-M website (http://www.corestandards.org/the-standards/mathematics/introduction/standards-for-mathematical-practice/). Identify the practices that you see in the video segment that you selected from your teaching of the Pool Border Problem and in the work in this session on the Three-Coin Problem. Optional: • Write the solution for the following problem: Use as many 8's and plus signs as you want to write a statement that equals 1000. *(You may not use -, x, or ÷).* How many different* statements can you write that use only 8's and plus signs to equal 1000? **Exchanging the places of the terms does not count as "different" for this problem.* How is the problem similar to, and different from, the Three-Coin Problem (the problem you worked on at	**** If you will need teachers to bring laptops with them to the next session in order to show their videos in the video workshop groups, ask or remind them to do this during this session.* *Much of the work in this module is connected with the mathematical practice standards found in the Common Core State Standards for Mathematics. For this reason, it may be useful for participants to look at some of the resources provided on the CCSS-M website. You may also want to include specific local and state web resources on the Common Core that your participants should have access to.*

the beginning of the session)? • Read the Three-Coin Problem Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.	

List of Common Core State Standards Mathematical Practices

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision.
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.