## *Description of the Session 4: Scaling mathematics problems and engaging in a video workshop*

Session 4 extends the work from Session 3 that focused on: giving and evaluating explanations for the Three-Coin Problem, noticing features of good explanations, noticing students' reasoning, and identifying what a teacher can do to establish an environment that supports students' reasoning practices. Participants will begin this session by considering similarities and differences between the Three-Coin Problem and the 21¢ Problem, which will help them think about the features of problems that make them mathematically similar. Then, they will have opportunities to practice adapting mathematics problems in ways that maintain a focus on mathematical reasoning. At the end of the session, participants will engage in a video workshop—the first workshop in which participants share and analyze videos from their own classrooms.

## *Activities and goals of the session*

| Activities* | Times | Corresponding parts of the session | Goals |
|---|---|---|---|
| I. Preview | 5 minutes | Part 1 | • Participants will be oriented to the work of the session. |
| II. Scaling mathematics problems | 25 minutes | Part 2 | • Participants will see the similarities and differences between two reasoning problems.<br>• Participants will consider how a particular problem might be scaled yet maintain a focus on mathematical reasoning.<br>• Participants will leave with a version of the Three-Coin Problem that can be used with their students. |
| III. A video workshop | 55 minutes | Parts 3 & 4 | • Participants will use video evidence to notice and discuss how students support and explain their approaches using words, drawings and/or tools.<br>• Participants will continue to notice what a teacher can do to establish and maintain an environment that nurtures mathematical practice.<br>Participants will engage in an early experience with a video workshop.<br>• Participants will consider aspects of the context that are important to know when viewing an instance of teaching.<br>• Participants will consider how focus questions can support discussion that is focused on specific aspects of teaching.<br>• Participants will reflect on their reasons for selecting specific instances in the video to share. |
| IV. Wrap up | 5 minutes | Part 5 | • Participants will understand ways of connecting the session content to their classroom. |

*A conversation about a CCA from the last session is integrated into the session.

## *Classroom Connection Activities*

| Required |
|---|
| Type of task: Extension of in-class work<br>Description: In local curriculum materials, locate problems that are likely to provide strong opportunities for mathematical reasoning. Bring copies of a few of these problems to the next session and be prepared to discuss the potential of the tasks for mathematical reasoning.<br><br>Type of task: Reading<br>Description: Linsenmeier & Sherin (2009) to support thinking about the selection of video clips for video workshops.<br><br>Type of task: Video workshop preparation<br>Description: Teach a scaled version of the Three-Coin Problem developed during the last session. Video record the entire activity and collect student work samples. Pick one or two 3-5 minute video segments that are focused on student reasoning using the ideas in the article by Linsenmeier & Sherin. |

## *Preparing for the session*

☐ Make copies as needed:

- *Resources:* Handout: Comparing two problems (Part 2); Handout: Exploring the scaling of problems (Part 2); Handout: Video workshop agenda (Part 3)

☐ Customize and make copies of the Classroom Connection Activities

☐ Test technical setups: Internet connection, speakers, projector

☐ Set up spaces for the video workshops

## *Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)*

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.

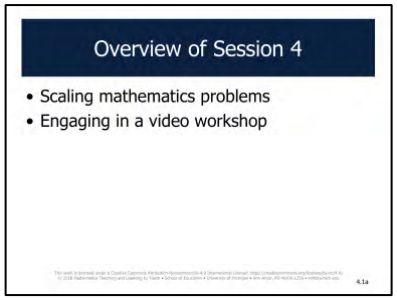
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

***Scope of the module*** *(focal content of this session in* ***bold****)*

| Mathematics | Student thinking | Teaching practice | Learning from practice |
|---|---|---|---|
| • making and justifying/refuting conjectures and generalizations<br>• recognizing and using multiple approaches to solve mathematics problems<br>• understanding features of a "good" mathematical explanation and producing "good" explanations<br>• identifying foundations of mathematical reasoning<br>• using and knowing the mathematical practices identified in the CCSS | • **monitoring students' mathematical reasoning**<br>• noticing collective elements of mathematical reasoning | • supporting students' engagement in mathematical practices by teaching them explicitly<br>• supporting students in explaining their mathematical reasoning<br>• **establishing and maintaining an environment that emphasizes reasoning**<br>• **adapting tasks to nurture mathematical reasoning** | • **using norms that support engagement in video workshop**<br>• **understanding the video workshop process**<br>• **learning to analyze teaching and learning in the context of video workshop** |

## *Part 1: Preview (~5 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will be oriented to the work of the session. | 1. Introduce the session and watch video. | • Video (02:04): Session overview |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce the session: The previous session focused on giving and evaluating explanations, noticing features of good explanations, and seeing explaining as a core component of expectations for student learning. Teaching practices that support students' engagement in reasoning and working to understand students' reasoning were also discussed.<br><br>  <br><br><br>Session 4 builds on the work started in Session 3 with the Three-Coin Problem and the 21¢ Problem as well as classroom teaching focused on using different versions of the Pool Border Problem. This session focuses on:<br><br>• Mathematics and teaching practices: Scaling mathematics problems<br>• Learning from practice: Engaging in a video workshop<br><br>Have participants watch the *video* in which Dr. Ball frames the work of the session. | *An opportunity to take up work on a Classroom Connection Activity is built into the session through participants' engagement in a video workshop. While only some participants will be able to share video from their classroom in the session, the conversations that happen in video workshop will benefit from the work that all participants did as a part of their Classroom Connection Activities.*<br><br>*Because this is the first session in which participants share records from their own classrooms during video workshop, consider framing the video workshop in ways that will generate buy in and interest among your participants. When talking about the video workshop, make it clear that video workshop is also an opportunity to learn more about the mathematical practices, student thinking, and ideas for enhancing practice. It could also be helpful to comment about what a special opportunity this is to really be able to see each other's students learn and to see each other teach. Participants also have the chance to talk about teaching in a PD context in ways that are closely tied to the contexts in which they teach. Emphasizing these benefits of video workshop is important for helping participants see the value of video workshops and buy in to the process.* |

## *Part 2: Scaling mathematics problems (~25 minutes)*

***Goals***

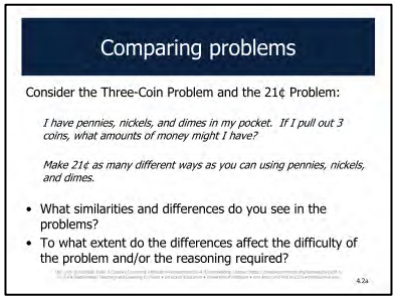
- Participants will see the similarities and differences between two reasoning problems.
- Participants will consider how a particular problem might be scaled yet maintain a focus on mathematical reasoning.
- Participants will leave with a version of the Three-Coin Problem that can be used with their students.

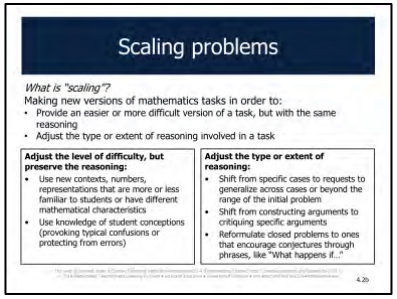
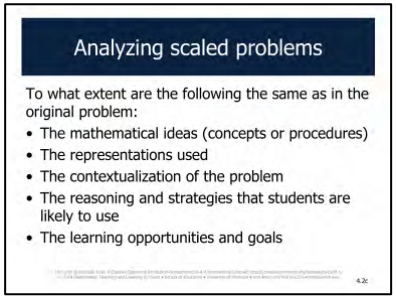
***Instructional sequence***

1. Introduce Part 2, including analyzing the Three-Coin Problem and the 21¢ Problem.
2. Watch the video in which the idea of scaling mathematics problems is introduced.
3. Consider scaled versions of the Three-Coin Problem.

***Resources***

- Video A (03:42): Introducing scaling tasks
- Handout: Comparing two problems
- Handout: Exploring the scaling of problems

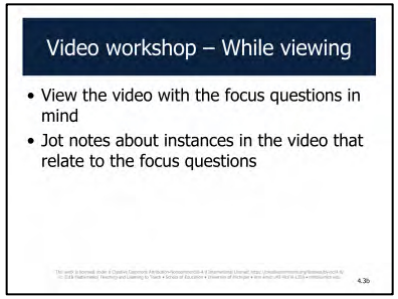
| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 2: This part launches work on scaling mathematics problems.<br><br>Distribute the Handout: *Comparing two problems* and ask participants to consider the Three-Coin Problem and the 21¢ Problem.<br><br>• What similarities and differences do you see in the problems?<br>• Which of these similarities and differences focus on or affect the reasoning required by the problems?<br><br>Allow 5 minutes for participants to discuss with a partner.<br><br>Discuss in whole group for 5 minutes.<br><br>  <br> | *This task is designed to connect work that participants have already done in the module with the work they will do in this part on scaling mathematics problems. An unpacking of the term "scaling" will be provided when the video is shown later in the part.*<br><br>*Try to establish the following in the whole group discussion:*<br><br>*Similarities:*<br><br>• *Both problems are about combinations*<br>• *Both involve some addition work*<br>• *Both entail proving **all** of the solutions have been found*<br><br>*Differences:*<br><br>• *The Three-Coin Problem requires picking 3 coins while the number of coins is not fixed in the 21¢ Problem*<br>• *The goal of the Three-Coin Problem is to find all possible amounts while the goal of the 21¢ Problem is to make a given amount.*<br><br>*Participants may notice other similarities and differences. For instance, they may notice that both problems involve money. The point here is to move past surface level connections, such as "involving money."* |

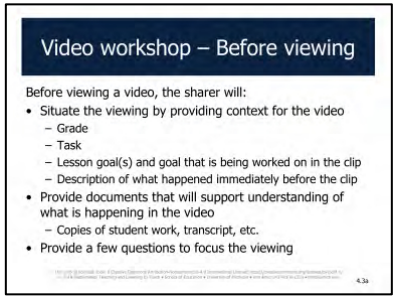
| Detailed description of activity | Comments & other resources |
|---|---|
| 2. Watch the *video* in which Dr. Ball introduces the idea of scaling mathematics problems and describes different ways in which problems can be scaled.<br>Scaling involves creating mathematically similar problems that may vary in difficulty, framing, and context. Teachers may call this kind of work by another name, such as modifying or adapting. In this module, the term scaling is used purposefully to emphasize the work to preserve particular mathematical connections between the original and new versions of the problem.<br><br>  <br><br><br>Scaling is necessary for a number of reasons, including:<br>• supporting students who encounter initial difficulties, need more practice, or would benefit from an extension<br>• focusing more precisely on a particular mathematical issue.<br><br>Scaling well is crucial because it provides access for all students to the same mathematics even when engaging with different forms of a problem. There are ways in which teachers can scale problems and there are many factors that teachers consider when scaling, but primary emphasis must be given to the mathematics that can be learned. | |
| 3. Distribute *Handout: Exploring the scaling of problems* and view the "Analyzing scaled problems" slide.<br><br>  <br><br><br>Have participants work in grade-level similar groups to consider scaled versions of the Three-Coin Problem or versions that are at about the same level as the Three-Coin Problem. As they look at the problems, participants should consider and discuss the extent to which the following features are the same as the original problem:<br><br>• The mathematical ideas (concepts or procedures<br>• The representations used<br>• The contextualization of the problem<br>• The reasoning and strategies that students are likely to use<br>• The learning opportunities and goals<br><br>Participants should either select one of the problems or modify it to use with their students before the next professional development session. | *The goal is for participants to leave the session with a problem to use with their students. This problem should be different from the Three-Coin Problem.*<br><br>*Participants will continue to work on scaling later in the module (see Session 9).* |

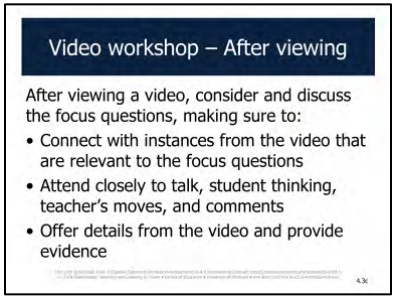
## *Part 3: Engaging in Video Workshop #1 (~40 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will use video evidence to notice and discuss how students support and explain their approaches using words, drawings and/or tools.<br>• Participants will continue to notice what a teacher can do to establish and maintain an environment that nurtures mathematical practice.<br>• Participants will engage in an early experience with a video workshop. | 1. Introduce Part 3 by showing the video.<br>2. Engage in the video workshop in small groups. | • Video (01:15): Launching Video Workshop #1<br>• Handout: Video workshop agenda |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 3: This part launches the first full video workshop in the module. Watch the *video* in which Dr. Ball launches the video workshop work in this session. This video includes review of the parts of the video workshop and an introduction to the focus questions for the video workshop. | *It is important to continue to reinforce the idea that the purpose of the video workshop is to look at <u>teaching</u> rather than the teacher. This is particularly important in this video workshop experience as discussion norms are being established in the small groups. A focus on teaching rather than the teacher will help participants be more comfortable sharing excerpts of their teaching with the small group.*<br><br>*It is also important to encourage participants to focus on the evidence of student thinking they see in the videos. Learning about student thinking is an important goal of video workshop, and focusing on student thinking in the workshops may also help teachers feel more comfortable sharing their classroom videos.* |



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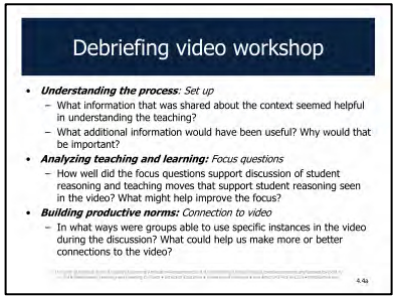
|---|---|
| 2. Distribute copies of the *Handout: Video workshop agenda*.<br><br>Have participants get into their video workshop groups. Depending on what was said during Session 3, you may need to tell participants with whom they are working. Suggest spaces for each of the groups to work and provide pointers about using the technology.<br><br>During the video workshop, participants should do the following:<br><br>1) Set context for video (5 minutes)<br><br>The sharer will:<br><br>• Situate the viewing by providing context for the video:<br>  ○ Grade<br>  ○ Task<br>  ○ Lesson goal(s) and goal that is being worked on in the segment<br>  ○ Description of what happened immediately before the segment<br>• Provide documents that will support the understanding of what is happening in the video (e.g., copies of student work, transcript, etc.)<br><br>2) View video with the focus questions in mind (5 minutes)<br><br>Focus questions:<br><br>• How are students supporting/explaining their approaches using words, drawings, and/or tools?<br>• What teaching moves are used to establish and maintain an environment that nurtures student reasoning practices?<br><br>3) Discuss the focus questions (20 minutes)<br><br>In the discussion, make sure to:<br><br>• Connect with instances from the video that are relevant to the focus questions<br>• Attend closely to talk, student thinking, teachers' moves and comments<br>• Offer details from the video and provide evidence<br><br>4) Determine who in the group will share video the next time. | *As the video workshops begin, circulate to all of the groups to make sure that each group has gotten started successfully. Then, pick one or two groups to focus in on during the video workshop. During the next video workshop, you can focus in on different groups.* |

## *Part 4: Debriefing video workshop (~15 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will consider aspects of the context that are important to know when viewing an instance of teaching.<br>• Participants will consider how focus questions can support discussion that is focused on specific aspects of teaching.<br>• Participants will reflect on their reasons for selecting specific instances in the video to share. | 1. Introduce Part 4 and have participants discuss the focus questions.<br>2. Watch and discuss Videos A-C as time and interest permit. | • Video A (00:59): Teacher insight – Seeing alternative teaching approaches<br>• Video B (00:48): Teacher insight – Noticing sequences of questions<br>• Video C (01:39): Teacher insight – Using video to support conversation about teaching |

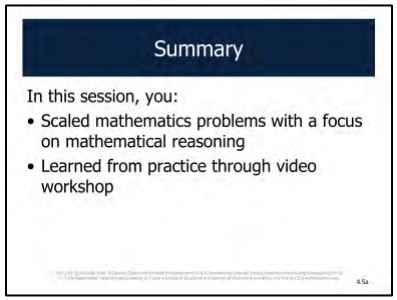
| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Introduce Part 4: This part focuses on debriefing the experience in the video workshops. Ask participants to consider the following three focus questions:<br>• ***Understanding the process****: Set up*<br>o What information that was shared about the context seemed helpful in understanding the teaching?<br>o What additional information would have been useful? Why would that be important?<br>• ***Analyzing teaching and learning:*** *Focus questions*<br>o How well did the focus questions support discussion of student reasoning and teaching moves that support student reasoning seen in the video? What might help improve the focus?<br>• ***Building productive norms:*** *Connection to video*<br>o In what ways were groups able to use specific instances in the video during discussion? What could help us make more or better connections to the video?<br><br>Elicit discussion of the focus questions in whole group. | *Make clear that the debriefing is being done to support learning about the process of video workshop. Knowing and being able to use the process is an important outcome of the professional development series. It will support learning over time in ways that typical professional development work is not able to do. By clarifying and deepening our knowledge of this process, participants will have a more "usable" understanding.*<br><br>*Before debriefing the video workshop process with the whole group, you may want to give participants an opportunity to write a brief reflection on how the video workshop was helpful to them in thinking about practice, and specifically the student reasoning.* |



| | |
|---|---|
| 2. As time and interest permit, show *Videos A-C* in which teachers in the professional development series reflect on their learning in the first video workshop experience.<br>• Video A: Teacher insight – Seeing alternative teaching approaches<br>• Video B: Teacher insight – Noticing sequences of questions<br>• Video C: Teacher insight – Using video to support conversation about teaching | *While the videos do not precisely map onto the focus questions, they do surface other kinds of insights that could be helpful as teachers pick new videos or think about rationale for why video workshops are useful.*<br><br>*It is crucial to build the "good vibes" about video workshop here, as the previous section probably surfaced a mixture of successes and struggles. Participants will be cycling through the process several more times in coming sessions, so emphasizing the value of video workshop in this session is important for helping participants buy in to the process.*<br><br>*Video A: Teacher insight – Seeing alternative teaching approaches:*<br>*A teacher notes that the use of video allows attention to the wording of questions and tone of voice used. These are aspects of teaching that cannot easily be noticed or discussed without video or direct observation. Video allows teachers to identify fine-grained aspects of others' practices that might be useful to adopt.*<br><br>*Video B: Teacher insight – Noticing sequences of questions.*<br>*A teacher reflects on how the video workshop enabled her to notice and consider a particularly effective pairing of questions that her colleague used to engage other students with what one student shared in his class.*<br><br>*Video C: Teacher insight – Using video to support conversation about teaching*<br>*Teachers discuss that video (and engagement in the video workshop) is supporting conversation about teaching in their school. It is rare that teachers get to see teaching happening in another classroom, and video provides this opportunity.* |

## *Part 5: Wrap up (~5 minutes)*

| ***Goals*** | ***Instructional sequence*** | ***Resources*** |
|---|---|---|
| • Participants will understand ways of connecting the session content to their classroom. | 1. Summarize the work of the session.<br>2. Explain and distribute the Classroom Connection Activities. | |

| Detailed description of activity | Comments & other resources |
|---|---|
| 1. Summarize the session by emphasizing that participants:<br>• Scaled mathematics problems with a focus on mathematical reasoning<br>• Learned from practice thought a video workshop, with a focus on noticing students' reasoning and teaching moves that establish and maintain an environment that nurtures mathematical reasoning<br><br>  | |
| 2. Distribute the *handout* you customized with selected Classroom Connection Activities and accompanying documents described below. Several of these will produce ideas or products that are necessary for subsequent sessions.<br><br><u>Required:</u><br>• Look in your curriculum for problems that are likely to provide strong opportunities for mathematical reasoning. Bring copies of a few of these problems with you to the next session and be prepared to talk with others about why you believe these tasks have rich potential for mathematical reasoning.<br>• Read the short *Teaching Children Mathematics* article "What makes a video clip interesting?" by Linsenmeier & Sherin (2009) to support your thinking about the selection of video clips for video workshops.<br>• To begin preparation for the video workshop in Session 6, spend 15-20 minutes in your classroom working on the scaled version of the Three-Coin Problem you developed during our last session. Video record the entire activity and collect student work samples (approximately 6 samples that represent a range in student reasoning). Watch the video and use the ideas from the Linsenmeier & Sherin article to identify one or two short segments in the video that show students engaged in reasoning and justification that would be interesting to share with colleagues. | *Remind the participants about the method for submitting classroom records of practice that will eventually be used for sharing examples from teaching with colleagues.*<br><br>*Depending on how much time you have scheduled between sessions, you could choose to assign the teaching of the Three-Coin Problem in Session 5. A potential drawback of waiting until that time is that the Three-Coin Problem, which participants initially discussed in Session 3, will not be as fresh in participants' minds by that time.* |

**List of Common Core State Standards Mathematical Practices**

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision.
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.