Description of the Session 5: Producing "good" mathematical explanations

Session 5 extends the work on the module on mathematical explanations. First, participants will explore a new mathematics problem, the Sum of Consecutive Odd Numbers Problem, with a focus on producing mathematical conjectures and then justifying or refuting those conjectures. This work will also include consideration of the explanations of others with a focus on critiquing the justifications provided for the conjectures. Then, participants will consider a set of features of "good" explanations and discuss ways in which teachers can teach mathematical practices, like explanation, explicitly to children. The session will conclude with an examination of problems from curriculum materials that provide opportunities for students to work on producing "good" explanations and engaging in reasoning and mathematical practices more generally.

Activities and goals of the session

Activities*	Times	Corresponding parts of the session	Goals
I. Preview	5 minutes	Part 1	• Participants will be oriented to the work of the session.
II. Sum of Consecutive Odd Numbers Problem	50 minutes	Parts 2 & 3	• Participants will make and justify a conjecture using different approaches. • Participants will identify the mathematical practices involved in working on a particular mathematics problem. • Participants will practice talking about the mathematical practices. • Participants will evaluate and question justifications shared by colleagues.
III. Explanations	20 minutes	Part 4	• Participants will consider features of "good" explanations. • Participants will consider the rationale for and moves that support making the CCSS mathematical practices explicit to students.
IV. Classroom Connection Activity	10 minutes	Part 5	• Participants will connect the work on reasoning to their own curriculum and classroom teaching by identifying tasks with rich potential for mathematical reasoning.
V. Wrap up	5 minutes	Part 6	• Participants will understand ways of connecting the session content to their classroom.

*A conversation about a CCA from the last session is integrated into the session.

Classroom Connection Activities

Required	Optional
Type of task: Video workshop preparation Description: Rewatch the segments of video previously selected from teaching the Three-Coin Problem and identify one 3-5 minute video clip that would be interesting to discuss in a video workshop focused on students' mathematical reasoning and justification. Type of task: Practice and extension of in-class work on mathematical practices Description: Exploration of the sums of three consecutive natural numbers a) Explore the sums of three consecutive natural numbers. Make a conjecture about these sums. Why is your conjecture true? b) Evaluate a given conjecture about the sums of consecutive natural numbers.	Type of task: Mathematics Reading Description: The Sum of Consecutive Odd Numbers Problem Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.

Preparing for the session

☐ Make copies as needed:

- *Resources:* Handout: Sum of Consecutive Odd Numbers Problem (Part 2); Handout: The mathematical practices (Part 2); Handout: Approach 1 – Using a T-chart to find a pattern (Video B) (Part 3); Handout: Approach 2 – Representing the solution with nested squares (Video C) (Part 3); Handout: Approach 3 – Representing the solution geometrically (Video D) (Part 3); Handout: Features of a "good" mathematical explanation (Part 4); Handout: Being explicit about mathematical practices (Part 4)
- *Supplements:* Math notes: Sum of Consecutive Odd Numbers Problem (Part 3)

☐ Customize and make copies of the Classroom Connection Activities

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

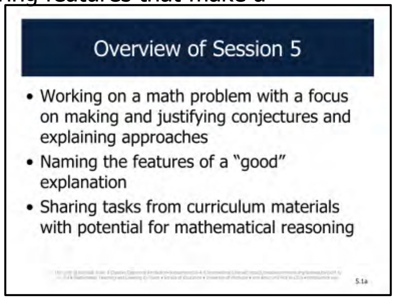
1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

*Scope of the module (focal content of this session in **bold**)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **making and justifying/refuting conjectures and generalizations** • **recognizing and using multiple approaches to solve mathematics problems** • **understanding features of a "good" mathematical explanation and producing "good" explanations** • identifying foundations of mathematical reasoning • **using and knowing the mathematical practices identified in the CCSS**	• monitoring students' mathematical reasoning • noticing collective elements of mathematical reasoning	• **supporting students' engagement in mathematical practices by teaching them explicitly** • **supporting students in explaining their mathematical reasoning** • establishing and maintaining an environment that emphasizes reasoning • **adapting tasks to nurture mathematical reasoning**	• using norms that support engagement in video workshop • understanding the video workshop process • learning to analyze teaching and learning in the context of video workshop

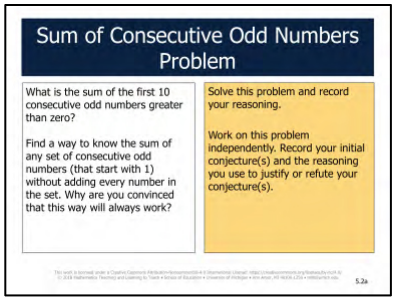
Part 1: Preview (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the session.	1. Introduce the session and watch the introductory video.	• Video A (01:18): Session overview

Detailed description of activity	Comments & other resources
1. Introduce the session: This session returns to work on the nature of mathematical explanations launched earlier in the module with a focus on explicitly considering features that make a mathematical explanation "good." Specifically, participants will engage in the following work: • Mathematics: Producing mathematical conjectures and justifications for the Sum of Consecutive Odd Numbers Problem and considering the critiquing the conjectures and justifications of others • Mathematics and teaching practices: Considering a set of features of "good" explanations • Teaching practice: Considering problems from curriculum materials that provide opportunities for students to work on producing "good" explanations  Have participants watch the *video* in which Dr. Ball frames the work of the session, including why work on explaining and discussing the features of explanations is useful.	*Discussion of a CCA activity will happen in Part 5 of the session.*

Part 2: Exploring the Sum of Consecutive Odd Numbers Problem (~25 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will make and justify a conjecture using different approaches. • Participants will identify the mathematical practices involved in working on a particular mathematics problem. • Participants will practice talking about the mathematical practices.	1. Introduce Part 2 by viewing Video A; then have participants work individually on the problem. 2. Watch Video B in which Dr. Ball sets up the partner work, and then have participants work with a partner.	• Video A (00:44): Launching the problem • Video B (00:56): Initiating partner work on the problem • Handout: Sum of Consecutive Odd Numbers Problem • Handout: The mathematical practices

Detailed description of activity	Comments & other resources
1. Introduce Part 2: This part launches work on a mathematics problem, the Sum of Consecutive Odd Numbers Problem.  Distribute the *Handout: Sum of Consecutive Odd Numbers Problem*. Watch *Video A* in which Dr. Ball launches work on the Sum of Consecutive Odd Numbers Problem by introducing a focus on mathematical explanations. Allow about 10 minutes for participants to work individually on the problem. Encourage participants to generate a conjecture about a way to find the sum of any set of consecutive odd numbers (that start with 1) and then to begin to try to justify or refute the conjecture.	*In its module "Using definitions in learning and teaching mathematics" (2009)*[1]*, the mod4 materials development project provides a useful way of thinking about conjectures,* *A "conjecture" refers to a claim for which one has fairly convincing reasons to believe its truth but does not yet have an entirely compelling argument or mathematical proof. In order to keep such a proposition in play, it is important to have a concise way to refer to it that acknowledges its state of uncertainty. There is no other word in mathematics that serves this important function.* *In learning any subject, it is important to understand where its knowledge comes from, why things are accepted or considered to be "true," and with what authority. Each field (science, history, literary analysis, etc.) has its own methods for this. In mathematics, that method is called proof. Making sense of mathematics depends on knowing something about its method for showing why the things we learn are true. Investigating the conjecture that participants generate in this part will afford the opportunity to engage in proving, the fundamental mathematical process of generating and verifying new knowledge.* *Emphasize that finding the sum of the first 10 consecutive odd numbers starting with 1 is not sufficient. It may be helpful to have a participant share the sum of three consecutive numbers to establish shared understanding of the problem.* *Participants may have questions about the conditions of the problem. For instance, participants may wonder whether the phrase "starts with one" refers to the number 1 or to any number with 1 in the largest place value. Clarify that the problem is asking for sets of numbers that start with the number 1.*

[1] Used with permission from the mod4 materials development project: http://mod4.soe.umich.edu. Adapted from the *Using definitions in teaching and learning* strand, Fall 2009.

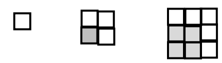
<table>
<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
<tr><td></td><td>

It is essential for participants to have had time to work on the problem before they begin work with a partner.

Participants might make calculation errors that prevent them from seeing patterns that would help them solve the problem. You may want to provide calculators to minimize these errors.

If you notice that few or none of the participants are thinking geometrically about the problem, you may want to introduce that possibility (e.g., sums of the sequences as shapes).

1, 1, 3 1, 3, 5



Totals: 1 4 9

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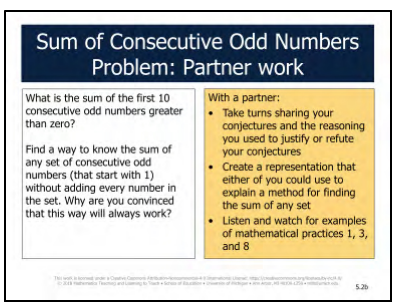
2. Watch *Video B* in which Dr. Ball launches partner work on the problem.

   Have participants work with a partner. Partners should:

   - Take turns sharing their conjectures and the reasoning used to justify or refute them
   - Create a representation that either partner could use to explain a method for finding the sum of any set
   - Listen and watch for examples of mathematical practices 1, 3, and 8

   Before participants start working in pairs, remind them about the mathematical practices introduced during the last session.



   Distribute the *Handout: The mathematical practices.*

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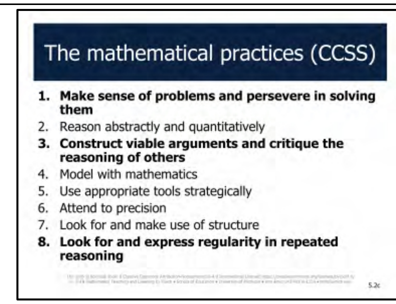
In the supplements section in Part 3, there is a document called Math notes: Sum of Consecutive Odd Numbers Problem *that describes a range of approaches that can be taken to solve this problem.*



Examples of mathematical practices that might be noticed include:

Mathematical Practice #1: Make sense of problems and persevere in solving them.

- *Participants might notice that their partners tried multiple different approaches in order to make progress in their work on the problem. They might also see from their partners' work that their partners have understood the conditions to the problem.*

Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

- *Participants might notice that their partners are using tables or pictures to support the arguments they are making.*

Mathematical Practice #8: Look for and express regularity in repeated reasoning

- *Participants might see repetition in early drafts of their own approaches and realize that such repetition could be expressed more efficiently*

The resources section for this part contains a slide with all of the mathematical practices and the focal mathematical practices for this session are bolded.

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Part 3: Discussing the Sum of Consecutive Odd Numbers Problem (~25 minutes)

Goals

- Participants will make and justify a conjecture using different approaches.
- Participants will identify the mathematical practices involved in working on a particular mathematics problem.
- Participants will practice talking about the mathematical practices.
- Participants will evaluate and question justifications shared by colleagues.

Instructional sequence

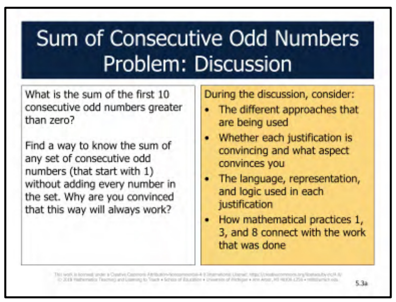
1. Introduce Part 3 by showing Video A.
2. Have participants share their explanations.
3. Watch and discuss Videos B-E as time and interest permit.
4. Consider how the explanations would function with different audiences.

Resources

- Video A (01:01): Initiating whole group discussion
- Video B (01:55): Approach 1 – Using a T-chart to find a pattern
- Video C (00:51): Approach 2 – Representing the solution with nested squares
- Video D (00:31): Approach 3 – Representing the solution geometrically
- Video E (01:14): Teacher insight – Connecting geometric and algebraic approaches
- Handout: Approach 1 – Using a T-chart to find a pattern (Video B)
- Handout: Approach 2 – Representing the solution with nested squares (Video C)
- Handout: Approach 3 – Representing the solution geometrically (Video D)

Supplements

- Math notes: Sum of Consecutive Odd Numbers Problem

Detailed description of activity	Comments & other resources
1. Introduce Part 3: This part continues work on the Sum of Consecutive Odd Numbers Problem by having participants share their conjectures and approaches in whole group. When sharing, participants should provide justifications for their conjectures. Watch *Video A* in which Dr. Ball launches the whole group discussion by providing a frame for listening to the explanations shared by others.  Participants should consider: • the different approaches that are being used • whether each justification is convincing and what aspect convinces you	*The explanations in this part should be justifications for conjectures about a way to know the sum of any set of consecutive odd numbers (that start with 1). One crucial part of the justification is showing that something is true for ALL cases. Often participants will present "justifications" that only show the conjecture is true for particular cases.* *A justification of a conjecture is a "proof" of the conjecture. In its module "Using definitions in learning and teaching mathematics" (2009)*[2]*, the mod4 materials development project provides a useful way of thinking about proofs,* *"Proof" is an important but difficult notion in mathematics. It is difficult in this context because teachers will have had experiences doing proof that may not align with the work here. Much of their work with proofs was likely in high school geometry, doing two-column proofs that they have to memorize and reproduce; that is not the object here. Proof in this context is less formal. But it must be mathematically sound, as well as understandable and convincing to the participants in this group (i.e., the present community of peers). Also, it must be grounded only in knowledge that is shared and available to everyone in the group. The strength of the proof depends on (i) the soundness of its logic; (ii) its use only of shared prior knowledge; and (iii) its accessibility and persuasiveness to one's peers.*

[2] Used with permission from the mod4 materials development project: http://mod4.soe.umich.edu. Adapted from the *Using definitions in teaching and learning* strand, Fall 2009.

<table>
<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
<tr><td>the language, representations, and logic used in each justification
how mathematical practices 1, 3, and 8 connect with the work that was done</td><td>Mathematical Practice #1: Make sense of problems and persevere in solving them.
Mathematical Practice #3: Construct viable arguments and critique the reasoning of others
Mathematical Practice #8: Look for and express regularity in repeated reasoning</td></tr>
<tr><td>2. Invite several participants to share their conjectures and justifications for them with the whole group. Use the following format for sharing:
Invite a participant to share a conjecture and a justification
Invite the rest of the group to comment on:
whether or not the justification is convincing and if so, what aspect is convincing
the language, representations, and logic used in the justification
Have the participant sharing explain how their work connected with mathematical practices 1, 3, or 8
Invite the rest of the group to comment on how they see mathematical practices 1, 3, and 8 connecting with the work that was done
Repeat for several justifications that represent diverse approaches.</td><td>Encourage participants to share their thinking about both parts of the task—finding the sum of the first ten consecutive odd numbers as well as finding the sum of any set of consecutive odd numbers beginning with 1.

Participants may use the following strategies:
Pairing numbers (e.g., 1 with 19, 2 with 17, and so forth) when summing and multiplying the number of pairs by the sum of the first and last number
Noticing that each sum is the square of the number of numbers
Taking the number of odd numbers in the set and dividing it by 2; then multiplying times the sum of the first and last number in the set
Note: If time is tight in this part, have 2-3 participants share their approaches before beginning to consider the conjectures and approaches shared by the teachers in the professional development. At least half of the time in this part should be devoted to participants' conjectures and justifications and discussion around them.

As participants share, listen for the features of "good" explanations. Also, consider recording the mathematical practices that are raised. If you do not have a good opportunity to acknowledge features of good explanations or participants' use of mathematical practices in this part, you can do so in the next part.</td></tr>
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3. As time and interest permit, watch *Videos B-F* in which teachers in the professional development share their approaches. After viewing each video, ask participants to consider the video-specific focus questions posed in the right column. Consider selecting videos that represent approaches that had not already been shared. For each of these videos, there is an accompanying handout in the resources section containing the participant's written work. • Video B (01:55): Approach 1 – Using a T-chart to find a pattern • Video C (00:51): Approach 2 – Representing the solution with nested squares • Video D (00:31): Approach 3 – Representing the solution geometrically • Video E (01:14): Teacher insight – Connecting geometric and algebraic approaches	*There will not be time in the session to share all of these videos. If it appears that participants are having trouble understanding an approach used by a participant, you may want to use a video that illustrates a similar approach, as there may be turns of phrase or uses of representations that could help the ideas move forward. If participants seem to have a good grasp of the approaches that have been shared, then show a video that features a different approach. When picking video, focus on depth rather than breadth. Select a small subset of videos and ask participants to consider the focus questions posed below.* *Video B: Approach 1 – Using a T-chart to find a pattern* *In this video, the teacher's conjecture is that the number of consecutive odd numbers in the set squared equals the sum of the numbers. Participants may note that the justification is not convincing because the teacher in the video gave examples that showed that her approach worked in particular cases, but she did not explain why the approach ALWAYS works. Consider asking your participants if they can justify or refute the teacher's conjecture using her method.* *Video C: Approach 2 – Representing the solution with nested squares* *In this video, the teacher's conjecture is a geometric approach that illustrates each consecutive number and shows that the sum of consecutive odd numbers equals the square of the number of consecutive odd numbers. Consider asking your participants to explain where the odd numbers are located in the figure and how they can use the representation to explain that the sum of consecutive of odd numbers is the square of the number of consecutive odd numbers.* *Video D: Approach 3 – Representing the solution geometrically* *In this video, the teacher represents the solution geometrically. Ask participants to consider how this representation and explanation is similar to and different from the explanation shared in Video C.* *Video E: Teacher insight – Connecting geometric and algebraic approaches* *In this video, a participant comments on the importance of connecting algebra and geometry. Ask participants to comment on what connections between algebraic and geometric approaches they noticed in the work today.*

4. To conclude the work on justifications in this part of the session, introduce the notion that, when producing an explanation, it can be useful to consider whether the explanation would be convincing to various kinds of audiences, such as: • A friend • A skeptic • A colleague • A group of students who have many shared understandings • A group of students who do not have many shared understandings Ask participants to consider how the justifications shared might hold up with these different audiences and to think about which features of "good" explanations would be important when trying to convince these different audiences.	*A justification is a particular type of explanation. In this part, we focus on the broader meaning of explanation.* *The notion of considering audiences for explanations is drawn from the work of Mason, Burton, & Stacey (1982)*[3] *and Barrett (personal communication, 2013).* *Participants might notice that "shorthand" can often be used in explanations given to friends, colleagues, and students who have many shared understandings. When giving an explanation to a skeptic or a group of students who do not have many shared understandings, it is important to be more thorough (e.g., defining terms clearly, demonstrating how each statement in the explanation flows logically from the previous statement, etc.).*

[3] Mason, J., L. Burton & K. Stacey. (1982) *Thinking mathematically.* Addison-Wesley Publishers Ltd. See also: Stylianides, A. (2007). Proof and Proving in School Mathematics. *Journal for Research in Mathematics Education, 38*(3), 289-321.

Part 4: Naming the features of a "good" explanation (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will consider features of a "good" explanation. • Participants will consider the rationale for and moves that support making the CCSS mathematical practices explicit to students.	1. Introduce Part 4 and watch Video A in which features of "good" explanations are introduced. 2. Watch Video B and invite participants to comment on ways that teachers can make facets of mathematical practices explicit for students.	• Video A (01:57): Features of a "good" mathematical explanation • Video B (02:40): Teaching students to explain • Handout: Features of a "good" mathematical explanation • Handout: Being explicit about mathematical practices

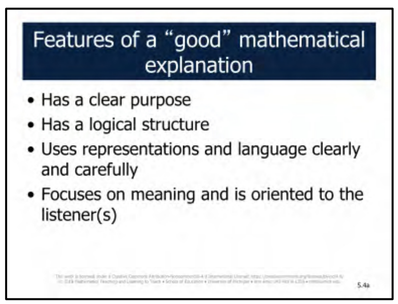
Detailed description of activity

1. Introduce Part 4: This part focuses on naming and describing the features of "good" explanations. There is also an opportunity to consider why it is crucial to teach students to explain.

   Distribute the *Handout: Features of a "good" mathematical explanation.* Watch *Video A* in which Dr. Ball introduces four features of "good" explanations.

   - Has a clear purpose
   - Has a logical structure
   - Uses representation and language clearly and carefully
   - Focuses on meaning and is oriented to the listener(s)

   Note several instances in which you saw participants attending to these features in their explanations in the previous part and invite participants to share a few additional instances.



Comments & other resources

This use of work on the Sum of Consecutive Odd Numbers Problem to further investigate mathematics and teaching practices is different from the discussions following work on previous mathematics problems in the module. In the past, the discussions were fairly open-ended. In this case, there is an explicit focus on a mathematical goal (examining features of "good" explanations) and a teaching practice goal (considering moves that would support students in providing good explanations).

A justification is a particular type of explanation. In this part, we focus on the broader meaning of explanation. It is important to emphasize that explanations are more than simply describing the steps taken to solve a problem—they explain ***why*** *a particular method works.*

One way that an explanation can be oriented to the listener is by being clear about assumptions, definitions, previously established knowledge and procedures, and the logic that links these ideas. This is an explicit part of Mathematical Practice #3: Construct viable arguments and critique the reasoning of others.

Participants might note that the list of features of a "good" explanation does not explicitly reference "mathematical validity". This is intentional. While the integrity of the mathematics is important, the integrity of the act of explaining is also important, and explanations can be "good" (e.g., clear, targeted toward the audience) even when they include flaws in the mathematics. The ultimate goal is to have an elegant explanation that is also mathematically valid, but it is important for teachers to

Detailed description of activity	Comments & other resources
	recognize students' ability to give good mathematical explanations of their thinking even when the mathematics included in the explanations contains flaws. This is also important to keep in mind when working with participants on explanations in the module. *There are several reasons for being explicit about the features of explanations in this professional development series, including:* • *providing a set of ideas that will support teachers in listening to and guiding students' explanations* • *supporting teachers in planning explanations they will provide* • *developing professional language that supports dialog with colleagues*

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<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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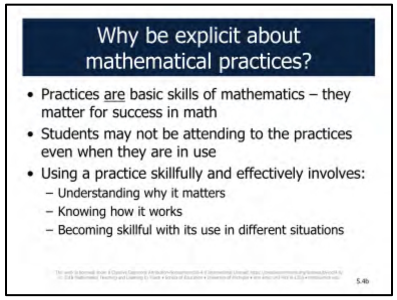
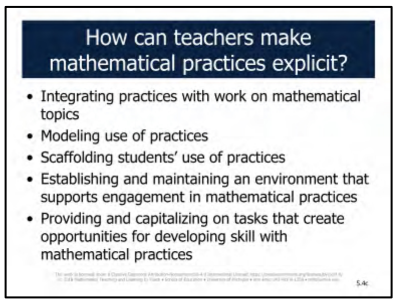
2. The Common Core State Standards put an increased emphasis on teaching children to explain and these features also apply to the explanations produced by children. Watch *Video B* in which Dr. Ball presents a rationale for teaching students to explain, including:



- Practices <u>are</u> basic skills of mathematics
- Students may not be attending to the practices even when they are in use
- Using a practice skillfully and effectively requires understanding why it matters; knowing how it works; and becoming skilled with its use in different situations.

Dr. Ball then provides examples of how to make mathematical practices such as explaining explicit, including:



- Integrating practices with work on mathematics topics
- Modeling use of practices
- Scaffolding students' use of practices
- Establishing and maintaining an environment that supports engagement in mathematical practices
- Providing and capitalizing on tasks that create opportunities for developing skill with mathematical practices

Invite participants to comment on the rationale for teaching students to explain and then see if they have additional ideas about how teachers can make mathematical practices explicit for students.

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Begin to consider the questions of why it is important to be explicit about mathematical practices in teaching in order to make a link to the work that the teachers tried to do in connecting the practices to their work in this session.

Video B is dense. Consider making the slides full screen and let the video play in the background. Alternatively, participants can use the Handout: *Being explicit about the mathematical practices to jot notes as they watch the video.*

You may want to provide participants with the Handout: Being explicit about mathematical practices*, which lists the points Dr. Ball makes in Video B.*

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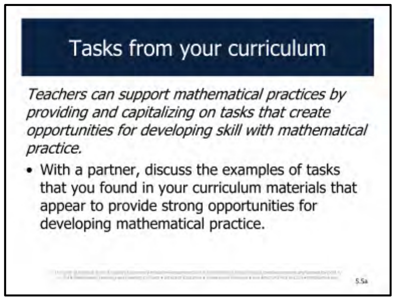
Part 5: Sharing reasoning tasks from curriculum materials (~10 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will connect the work on reasoning to their own curriculum and classroom teaching by identifying tasks with rich potential for mathematical reasoning.	1. Introduce Part 5 and watch the video. 2. Have participants share and discuss the examples of tasks that they found in their curriculum materials.	• Video A (00:42): Sharing examples from curriculum materials

Detailed description of activity	Comments & other resources
1. Introduce Part 5: In the prior parts of the session, participants considered features that make a mathematical explanation "good." This part focuses how teachers can support mathematical practices by providing and capitalizing on tasks that create opportunities for developing skill with mathematical practice. Specifically, problems from curriculum materials that provide opportunities for students to work on producing "good" explanations and working on mathematical reasoning more generally. Watch *Video A* in which Dr. Ball introduces the activity.  	*Tasks that provide opportunities for reasoning and engaging in mathematical practices have features such as:* • *Multiple entry points into the problem exist* • *The problem is open ended* • *Connections are required* • *No particular solution approach is suggested* • *Generalization and extended exploration are necessary* • *Multiple steps are involved in solving the problem* *(drawing on: Smith, 2013; Stein & Smith, 1998; Hsu, Kysh, & Resek, 2007)*[4] *Such tasks can be found in many curriculum materials and/or tasks in existing curriculum materials can be tweaked to support opportunities for reasoning and engaging in mathematical practices.* *Caution: Participants should not leave this module thinking that they need to invent their own curriculum in order to engage children in reasoning and mathematical practices. In this activity, focus on helping participants notice ways they can take advantage of tasks that already exist in their curriculum materials in order to create/enhance opportunities for work on reasoning.*

[4] Smith, M. E. (2013). *Tasks, Tools, and Talk: A Framework for Enacting the CCSS Mathematical Practices.* Teachers Development Group Leadership Seminar.
Stein, M. K. & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3,* 268-275.
Hsu, E., Kysh, J., & Resek, D. (2007). Differentiated instruction through rich problems. *New England Mathematics Journal. 39*, 6-13.

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<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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2. Have participants work with a partner to share the examples of tasks that they found in their curriculum materials that they thought were likely to provide strong opportunities for developing skill with mathematical practice.

As participants share, they should explain WHY they believe that each task has rich potential for mathematical reasoning.
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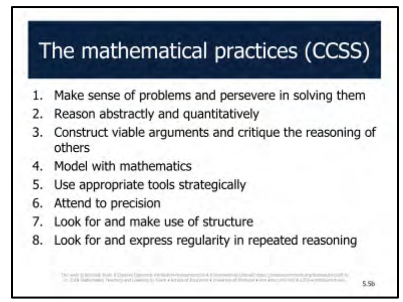
Consider having participants work in grade level groups for this activity.

The point here is to help participants connect the work on reasoning to their own curriculum and classroom teaching.

Consider noting that for a video workshop later in the module, each participant will identify a mathematics task in his or her curriculum materials that is likely to provide opportunities for developing skill with mathematical practice and use this task with his or her students. Records from this work will be discussed in Session 10 as a part of the last video workshop.

A slide displaying the mathematical practices is included as a resource. You may want to display this slide or a poster of the mathematical practices for reference during this activity.



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Part 6: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom.	1. Summarize the work of the session. 2. Explain and distribute the Classroom Connection Activities.	

<table>
<tr><th>Detailed description of activity</th><th>Comments & other resources</th></tr>
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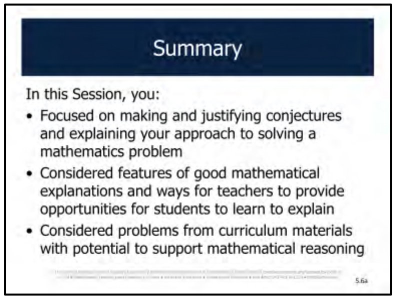
1. Summarize the session by emphasizing that participants:
    - Worked on a mathematics problem with a focus on making and justifying a conjecture and explaining their approach
    - Considered features of "good" explanations and ways for teachers to make mathematical practices explicit to students
    - Considered problems from curriculum materials with the potential to support mathematical reasoning



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2. Distribute the handout you customized with selected Classroom Connection Activities and accompanying documents described below.
    - Rewatch the segments of video previously selected from teaching the Three-Coin Problem and identify one 3-5 minute video clip that would be interesting to discuss in a video workshop focused on students' mathematical reasoning and justification. Respond to the set of context and focus questions.
    - Extend work on mathematical reasoning by exploring the sums of three consecutive natural numbers.
        a) Explore the sums of three consecutive natural numbers. Make a conjecture about these sums. Why is your conjecture true?
        b) Evaluate a given conjecture about the sums of consecutive natural numbers.

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Emphasize that the mathematical problems participants work in the professional development sessions and in their CCAs are helpful for developing their own reasoning skills. One of the goals of the module is to support participants' growth in mathematical knowledge.

Participants may wonder if it is necessary for everyone in the professional development to write up their analysis of their work with a scaled version of the Three-Coin Problem. All participants should produce a write up because:

- *The write up helps the facilitator know how the work with the module content is going for ALL participants*
- *Doing so is an important part of engaging in this professional development community – all participants are engaging in the work.*

If a participant is not satisfied with the quality of the video of the scaled version of the Three-Coin Problem, suggest one of the following:

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Optional: • Read the Sum of Consecutive Odd Numbers Problem Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.	• *watching the video together (outside of the session) to think together about the teaching and learning shown and the quality of the audio/visuals in the video to see if there is a clip that will work for Session 6.* • *using a scaled version of the that problem as another opportunity support student reasoning and justification (and to capture that work on video).*

List of Common Core State Standards Mathematical Practices

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision.
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.