Description of the Session 8: Developing justifications and responding to students' explanations

Session 8 extends work in the module on making and justifying conjectures by considering this type of reasoning within a different mathematics strand. At the beginning of the session, participants share work from one of the Classroom Connection Activities for the last session in which they considered the types of explanations that they and their students might give for a topic/concept/idea in their current mathematics unit. Then, participants will consider True/False Geometry Statements with a focus on justifying and refuting conjectures in geometry using different approaches. In this work, they will consider features of "good" explanations in the context of a different mathematics topic. After working on their own explanations for these statements, participants will analyze students' explanations to the same statements and will design teaching moves they could use to follow up on these explanations, including moves that make mathematical practices explicit to the class.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
Conversation about a CCA from the last session	5 minutes		• Participants will share their planning for a key mathematical concept, process, or idea that will need to be explained in the unit that they are currently teaching.
I. Preview	5 minutes	Part 1	• Participants will be oriented to the work of the session.
II. Justifying True and False Geometry Statements	40 minutes	Parts 2 & 3	• Participants will justify or refute a conjecture using different approaches. • Participants will develop clear and convincing justifications or refutations for each statement. • Participants will consider features of "good" explanations in the context of geometry. • Participants will connect the work done on the statements with relevant CCSS mathematical practices.
III. Responding to students' explanations	35 minutes	Parts 4 & 5	• Participants will notice mathematically and pedagogically important features of a student's explanation. • Participants will use analysis of student explanations as the basis for developing teaching moves with the potential to enhance student explanations. • Participants will respond to student explanations in ways that move the learning of the class forward.
IV. Wrap up	5 minutes	Part 6	• Participants will understand ways of connecting the session content to their classroom.

Classroom Connection Activities

Required	Optional
Type of task: Video workshop preparation Description: Select a problem from your curriculum that you feel can be used to provide students with a strong opportunity to engage in mathematical reasoning. Recall from our work in previous sessions and CCAs that there are many problems in your curriculum that can be used for this purpose. We are attaching a resource, "Approaches to Modifying Tasks" that you can use to support your selection and revision of a task that could be used to provide opportunities for you students to reason and engage in mathematical practices. Provide rationale for the problem you select. We will discuss your selection at our next session.	Type of task: Professional reading Description: Choose one of the following articles that provides an example of how student reasoning can be encouraged in different topic areas in mathematics: Lehrer and Curtis's (2000) article pertaining to geometry, Nitaback and Lehrer's (1996) article pertaining to measurement, the Mathematical Sciences Education Board (1993) article pertaining to data, and Carpenter and Levi's (2000) article pertaining to algebra. Make connections between the article and our work to support students' reasoning. Type of task: Mathematics Reading Description: The Justifying True/False Statements Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.

Preparing for the session

☐ Make copies as needed:

- *Resources:* Handout: Justifying True/False Statements (Part 2); Handout: Glossary of relevant definitions (Part 2) Transcript: Approach 1 – Finding a counterexample (Part 3); Handout: Scaffolding explanations (Part 4)
- *Supplements:* Math notes: Justifying True/False Statements (Part 3)

☐ Customize and make copies of the Classroom Connection Activities

☐ Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others' ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people's thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
   a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others' ideas with interest and respect.
   b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
   c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

Scope of the module *(focal content of this session in **bold**)*

Mathematics	Student thinking	Teaching practice	Learning from practice
• **making and justifying/refuting conjectures and generalizations** • recognizing and using multiple approaches to solve mathematics problems • **understanding features of a "good" mathematical explanation and producing "good" explanations** • identifying foundations of mathematical reasoning • **using and knowing the mathematical practices identified in the CCSS**	• **monitoring students' mathematical reasoning** • noticing collective elements of mathematical reasoning	• **supporting students' engagement in mathematical practices by teaching them explicitly** • supporting students in explaining their mathematical reasoning • establishing and maintaining an environment that emphasizes reasoning • adapting tasks to nurture mathematical reasoning	• using norms that support engagement in video workshop • understanding the video workshop process • learning to analyze teaching and learning in the context of video workshop

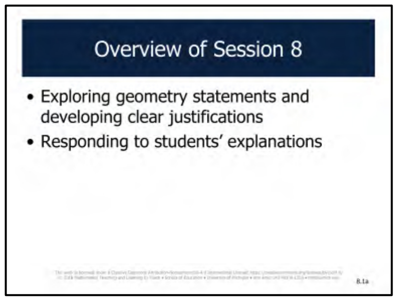
Conversation about a Classroom Connection Activity from last session (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will share their planning for a key mathematical concept, process, or idea that will need to be explained in the unit that they are currently teaching.	1. With a partner, participants share their work on planning for an explanation in the unit they are currently teaching.	

Detailed description of activity	Comments & other resources
1. Have participants share their work on one of the Classroom Connection Activities with a partner. Participants should share a key mathematical concept, process, or idea that will need to be explained in the unit that they are currently working on. In the CCA, participants identified a key concept/process/idea, anticipated features of the explanations that students might provide, and considered an explanation that they might provide. Specifically: a) Student explanation: How do students typically explain this mathematical concept/process/idea? What points do students typically include in their explanations? What is typically missing from student explanations and/or what is hard for students to describe or represent? What is challenging about supporting students in explaining? b) Teacher explanation: When you explain this concept/process/idea, what are the key components you make sure to include? What representations do you use (and why)? What terms do you try to use (or make sure to avoid)? What examples do you use (and why)? Bring this activity to a close by asking participants what struck them when listening to their colleagues share what they would explain or what their students were explaining.	*It is very likely that participants will find many things striking about the conversations they have with colleagues, such as:* • *Connections to content they are teaching* • *How the content they teach either sets the stage for what their colleagues will be working on OR how the content they teach is grounded in what their colleagues do* • *Similarities or differences in the way that they typically explain particular concepts* • *New appreciation for the work their colleagues do* • *A new insight into the curriculum*

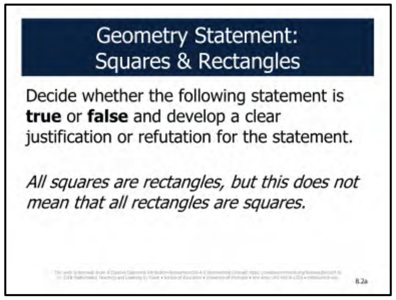
Part 1: Preview (~10 minutes)

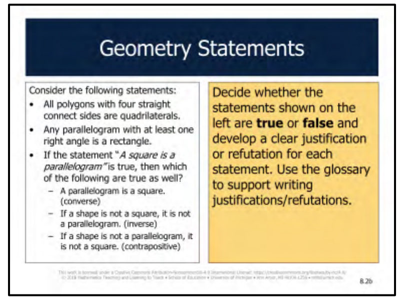
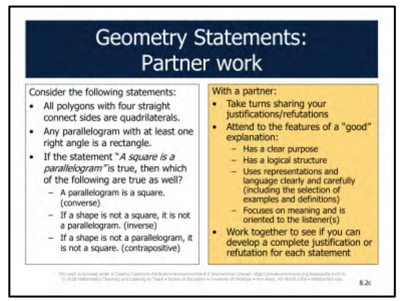
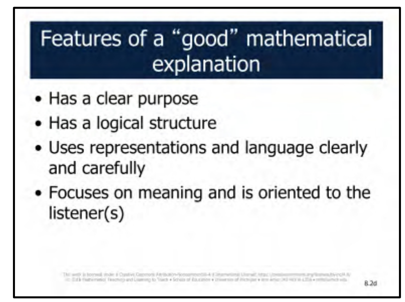
Goals	***Instructional sequence***	***Resources***
• Participants will be oriented to the work of the session.	1. Participants work with a partner to share their work on one of the Classroom Connection Activities. 2. Introduce the session and watch the introductory video.	• Video A (01:02): Session overview

Detailed description of activity	Comments & other resources
1. Introduce the session: This session extends work on making and justifying conjectures by considering this type of reasoning within a different mathematics strand. Geometry will provide a new context for appraising students' mathematical explanations and identifying teaching moves in response to students' explanations. Specifically, participants will engage in the following work:  • Mathematics & teaching practice: Justifying and refuting conjectures about geometry using different approaches and considering features of "good" explanations in the context of a different mathematics strand • Analyzing students' explanations and using what is learned to design teaching moves and consider how to make mathematical practices explicit to the class Have participants watch the *video* in which Dr. Ball frames the work of the session and describes why work on explaining and discussing the features of "good" explanations is useful.	

Part 2: Exploring True/False Geometry Statements (~20 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will justify or refute a conjecture using different approaches. • Participants will consider features of "good" explanations in the context of geometry.	1. Introduce Part 2 by viewing Video A, have participants consider one True/False Geometry Statement and then view Video B. 2. Work independently to consider the set of True/False Geometry Statements. 3. Work with a partner on the True/False Geometry Statements.	• Video A (01:34): Launching the problem • Video B (00:45): An example justification • Video C (01:10): Initiating partner work on the problem • Handout: Justifying True/False Statements • Handout: Glossary of relevant definitions

Detailed description of activity	Comments & other resources
1. Introduce Part 2: This part launches work on a mathematics problem set – considering True/False Geometry Statements. This problem set is different from previous mathematics problems both in terms of the topic and the type of task. This foray into another mathematics strand is designed to support consideration of how reasoning permeates mathematics. It will also illustrate in new ways the important of precision with mathematical language. Watch *Video A* in which Dr. Ball launches work on the True/False Geometry Statements problem set. Have participants work alone or with a partner to consider the following True/False Geometry Statement: Decide whether the following statement is **true** or **false** and develop a clear justification or refutation for the statement. *All squares are rectangles, but this does not mean that all rectangles are squares.*  After participants have had several minutes to consider the Geometry Statement, watch *Video B* in which a teacher from the professional development series provides a justification. Invite participants to comment on the justification and/or to present an alternative way to justify the Geometry Statement.	*This activity illustrates how a classroom environment that encourages reasoning can elevate the level of mathematical engagement, even with tasks that on the surface seem narrow, or even "closed."* *This statement is an example of an idea that is challenging to teach and also illustrates a very common kind of reasoning (thinking about converses) that is challenging for students.* *The Geometry Statements that participants will work on in this part are examples of ways to work on the following mathematics:* *a) identifying hidden/assumed quantifiers when justifying/refuting mathematical statements* *b) using a definition to justify/refute a statement* *c) using logic rules to evaluate statements*

Detailed description of activity	Comments & other resources
2. Distribute the *Handout: Justifying True/False Statements* and the *Handout: Glossary of relevant definitions.* Show the *Slide: Geometry Statements*. Explain that participants will work individually to consider the series of True/False Geometry Statements. Participants should decide if each statement is true or false and develop a clear justification or refutation for the statement. Participants will first work individually, then they will work with a partner, and finally they will share their justifications/refutations for the statements in whole group. Have participants work individually for 5 minutes. 	*As in previous activities, it is important to encourage participants to grapple with the problems independently so that there is more to take up in small group work and more need for participants to explain.* *The glossary provided in the module is the glossary that the teachers in the video used as they were working on these problems. It was drawn from the teachers' home district. You may choose to provide a different glossary that is drawn from your participants' curriculum materials.*
3. Watch *Video C* in which Dr. Ball launches partner work on the problem. Then, have participants work with a partner. Participants should take turns sharing their justifications/refutations. When listening, participants should listen for the features of a "good" explanation (has a clear purpose; has a logical structure; uses representations and language clearly and carefully, including selection of useful examples and definitions; focuses on meaning and is oriented to the listener(s)). After sharing, participants should work together to develop a complete justification or refutation for each statement. 	*Video C introduces the idea that quantifiers like* any, all, sometimes, *and* never *are key in mathematics. There is a need to pay attention to the language that the problem is framed with.* *A slide that lists the "Features of a 'good' explanation" is included as a supplement.* 

Part 3: Developing clear and convincing justifications and refutations (~20 minutes)

Goals

- Participants will develop clear and convincing justifications or refutations for each statement.
- Participants will consider features of "good explanations" in the context of geometry.
- Participants will connect the work done on the statements with relevant CCSS mathematical practices.

Instructional sequence

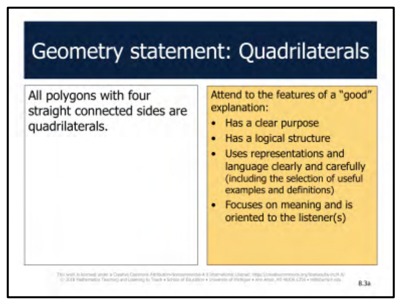
1. Introduce Part 3 by showing Video A.
2. Invite a pair of participants to share their justifications/refutations.
3. Watch and discuss Slide: Approach 1, Video B, and Video C.
4. *Optional:* Discuss additional statements

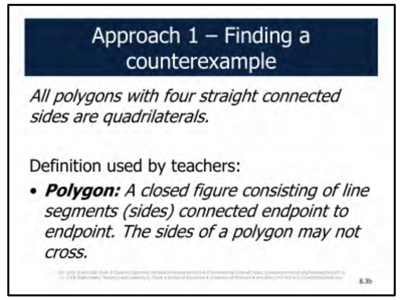
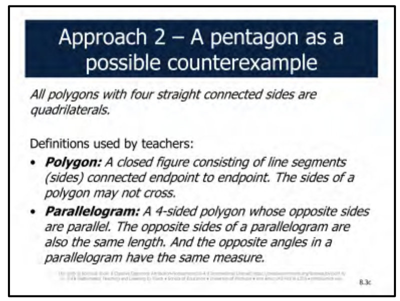
Resources

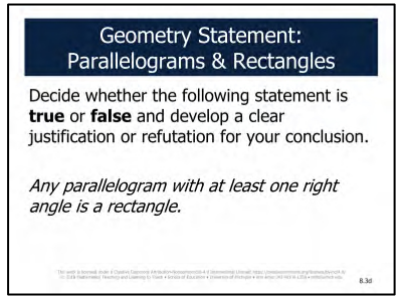
- Video A (01:05): Initiating whole group discussion
- Video B (03:03): Approach 2 – A pentagon as a possible counterexample
- Video C (00:49): Teacher insight – The importance of precision with language
- Transcript: Approach 1 – Finding a counter-example

Supplements

- Math notes: Justifying True/False Statements

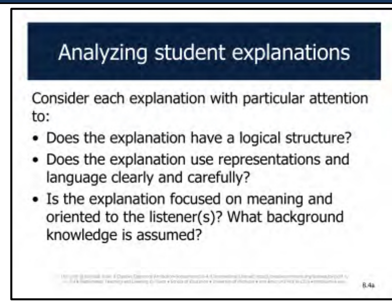
Detailed description of activity	Comments & other resources
1. Introduce Part 3: This part continues work to provide justifications or refutations for the True/False Geometry Statements with a focus on the following statement: *All polygons with four straight connected sides are quadrilaterals.*  Watch *Video A* in which Dr. Ball launches the whole group discussion work by asking participants to consider the features of "good" explanations as others share their solutions. • Has a clear purpose • Has a logical structure • Uses representations and language clearly and carefully (including the selection of useful examples and definitions) • Focuses on meaning and is oriented to the listener(s)	*Engagement in this activity will highlight the role of examples in reasoning, justification, and proving.*
2. Invite one participant (or pair of participants) to share a justification or refutation for the True/False Geometry Statement and invite other participants to comment on the justification/refutation using the frame of features of "good" explanations.	*Emphasize that the point of trying to notice the features of a "good" explanation is not to judge the person providing the explanation, but rather to enhance skill in recognizing these features and also to try more explicitly to use them as goals to guide the production of explanations.*

Detailed description of activity	Comments & other resources
3. Have participants consider a series of justifications/refutations offered by participants in the professional development series.  Distribute copies of *Transcript: Disproving a Geometry Statement* and have participants read the transcript. In this transcript, a participant in the professional development series explains how she tried to refute the statement by finding a counter-example. Show Slide: Approach 1 – Finding a counterexample (see the bottom portion of the viewer, where the videos are located), which is an image of the work referenced in the transcript. A slide with the Geometry Statement and a definition that was used by teachers in the video will also appear. Ask participants to comment on what they noticed related to features of "good" explanations. Be sure to note to participants that, though statements cannot be proven through the use of confirming examples (even if many examples are provided), a single counterexample is sufficient for refuting a (false) statement.  Then show *Video B* in which Dr. Ball offers a possible counter-example. Ask participants to comment on what they noticed with regard to features of "good" explanations and any additional insights that they have about the statement. Close by watching *Video C* in which a teacher in the professional development series raises questions about precision with mathematical language. Invite participants to share new insights that emerge after viewing the video.	*This is a different type of activity. The use of the transcript without an accompanying video helps participants to really focus in on the words and representations in an explanation.* *Video B: In this video, the teachers discuss whether a pentagon could be considered a "polygon with four straight connected sides", and they discuss whether the meaning of the original Geometry Statement would change if they added the word "exactly" to it. Participants might notice that there are multiple interpretations of the original statement, and that it is not clear which interpretation is correct.* *Video C: A teacher points out that, in school mathematics, definitions are not always precise, and quantifiers like "exactly" are often assumed. Participants might think of examples of definitions they use with their students that may be ambiguous in some way.*

Detailed description of activity	Comments & other resources
4. If there is time, continue work on justification by considering one of the other True/False Geometry Statements: *Any parallelogram with at least one right angle is a rectangle.* *If the statement, "A square is a parallelogram" is true, then which of the following are true as well?* *1) A parallelogram is not a square.* *2) If a shape is not a square, it is not a parallelogram.* *3) If a shape is not a parallelogram, it is not a square.*  Invite participants to share justifications or refutations of the Geometry Statements. As participants share their justifications or refutations, have all participants consider the features of "good" explanations and ask them to comment on each justification or refutation after it is shared. If there is time, ask participants to provide examples of the demands of this problem for precise use of mathematical language and/or other CCSS mathematical practices.	*If you have time to discuss a second Geometry Statement, select the one that you believe will be most useful in pushing your participants' work on making justifications/refutations forward. You might make this decision based on what you heard when participants were working with their partner on the statements or on participants' responses in the previous part.* *Slides with each statement are available as "Supplements."* *Participants may mention CCSS mathematical practices such as:* *1. Make sense of problems and persevere in solving them.* *2. Reason abstractly and quantitatively.* *3. Construct viable arguments and critique the reasoning of others.* *6. Attend to precision.*

Part 4: Analyzing student explanations (~20 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will notice mathematically and pedagogically important features of a student's explanation.	1. Introduce Part 4 by viewing Video A, have participants analyze students' explanations individually and then with a partner. 2. Participants share their analysis of the mathematics of the student explanations. 3. Watch and discussion Videos B-C as time and interest permit.	• Video A (02:16): Launching the analysis of student explanations • Video B (01:17): Teacher insight – Varying the standard for precision based on grade level • Video C (03:15): Different aspects of precision • Handout: Scaffolding explanations

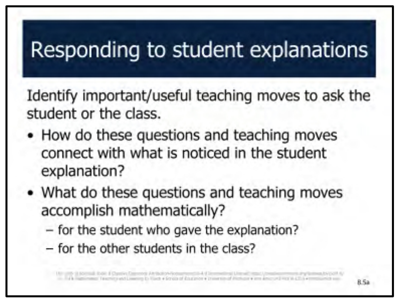
Detailed description of activity	Comments & other resources
1. Introduce Part 4: This part launches a focus on analyzing students' explanations. This provides practice in the work that teachers do everyday to respond to student reasoning in ways that help individuals and the class focus on key mathematics and make progress in light of instructional goals. Skill in noticing the aspects of a student's explanation can support the generation of follow up questions that surface important mathematical ideas and decisions about how to engage the rest of the class with what has been shared. Watch *Video A* in which Dr. Ball introduces the focus of this part.  Distribute the *Handout: Scaffolding Explanations*. This handout contains the three sample student explanations. Have participants work individually for five minutes to consider each explanation through the lenses of the features of "good" explanations. In particular, they should consider: • Does the explanation have a logical structure? • Does the explanation use representations and language clearly and carefully? • Is the explanation focused on meaning and oriented to the listener(s)? What background knowledge is assumed? Then have participants work with a partner and continue their work on analyzing the explanation.	*As in past sessions, you may need to reaffirm that there are many uses for the kind of knowledge that is being developed through this activity. Skill in perceiving these aspects of an explanation is useful when listening to what students say, in thinking about what follow up questions to use, in determining how to engage the rest of the class with that has been shared.*

Detailed description of activity	Comments & other resources
2. Ask a pair to present their analysis of the mathematics of one of student explanations. Invite a second pair with a different analysis of the explanation to share their analysis. Ask others to comment and extend. If there is time and it seems useful, examine the mathematics of a second student explanation.	*For the first explanation, participants might notice that the logic of the explanation seems pretty clear, but that the language and representations used in the explanation are not very precise.* *For the second explanation, participants might notice that the student uses precise language to explain his justification, but the logic seems somewhat incomplete because he does not provide a specific example of a polygon with four straight connected sides that is not a quadrilateral.*

3. As time and interest permit, watch and discuss *Videos B-C* in which teachers in the professional development series raise two important questions about precision.
• Video B: Teacher insight – Varying the standard for precision based on grade level.
  ○ Framing question: Are standards for precision grade-level specific?
• Video C: Teacher insight – Different aspects of precision
  ○ Framing question: Is attending to precision just about vocabulary?
 	
After viewing each video, ask participants whether they have additional thoughts about the framing question for each video.
	In Videos B and C, teachers in the professional development discuss the student's justification of the statement: "Any parallelogram with at least one right angle is a rectangle."
	
	In Video B, teachers notice that the student's drawing of the rectangle is not very precise. A teacher explains that, depending on the age of the students, precision with drawing rectangles may be more or less important for creating a "good" explanation. For example, for young students who are still learning what a rectangle is, it might be very important to make the drawing of the rectangle precise.
	
	In Video C, a teacher argues that, though the language in the student's explanation is not precise, the logic/structure of the explanation does seem "precise" in a sense. Other teachers discuss the idea that "attending to precision" includes not just using precise vocabulary, but also being precise with representations. They argue that attending to precision can make an argument more convincing.

Part 5: Responding to students' explanations (~15 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will use analysis of student explanations as the basis for developing teaching moves with the potential to enhance student explanations. • Participants will respond to student explanations in ways that move the learning of the class forward.	1. Introduce Part 5 by showing Video A and then have participants work with a partner to identify questions/teaching moves. 2. Invite a pair of participants to share the questions/moves that they identified. 3. Watch and discuss Videos B – E as time and interest permit.	• Video A (01:14): Launching work on responding to student explanations • Video B (03:29): Working collectively to revise an explanation • Video C (01:01): Helping students at different grade levels understand the features of a "good" explanation

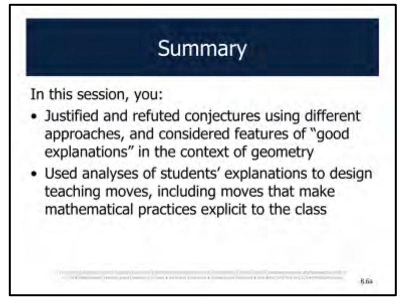
Detailed description of activity	Comments & other resources
1. Introduce Part 5: Analysis of explanations shared by students must be coupled with skill in formulating teaching responses that enhance opportunities to engage in reasoning and enhance the learning of mathematics. This part of the session fosters mathematics teaching that addresses the learning needs of individuals while advancing the learning of the class. Watch *Video A* in which Dr. Ball launches the work on responding to student explanations.  Have participants select one of the explanations and work with a partner to identify important/useful questions and teaching moves to ask the student or the class. When identifying questions/moves, participants should consider the following two questions: • How do these questions and teaching moves connect with what is noticed in the student explanation? • What do these questions and teaching moves accomplish mathematically? ○ for the student who gave the explanation? ○ for the other students in the class?	*If participants are focused on one aspect of the student's explanation (e.g., the language used), encourage them to think about another aspect of the explanation (e.g., the logical structure).* *For the first student explanation, participants might suggest that the teacher follow up with clarifying questions about the language in the explanation. For example, the teacher might ask, "What does it mean for lines to be "lined up" with other lines?"* *For the second student explanation, participants might ask the student who gave the explanation (or another student in the class) to give an example of a polygon with four straight connected sides that is not a quadrilateral.* *Participants may notice connections to practices students work on in other curriculum areas (e.g., revising writing in writer's workshop). Encourage these connections.*

Detailed description of activity	Comments & other resources
2. Invite one participant (or pair of participants) to share the teaching moves that they identified for one of the student explanations. Encourage the group to consider the following two questions as another participant (or pair of participants) shares a set of questions and teaching moves. • How do these questions and teaching moves connect with what is noticed in the student explanation? • What do these questions and teaching moves accomplish mathematically?	*Three variations on this sharing include:* • *Ask the participants to first share the teaching moves and to then talk about their connection with the analysis.* • *Invite one group could share their mathematical analysis and teaching moves and then open it up to those groups who had a similar analysis, but different moves OR similar moves that went along with a different mathematical analysis.* • *Make, or ask for, explicit connections between the student work and features of "good" explanations.*

3. As time and interest permit, watch and discuss *Videos B-C,* in which teachers in the professional development series discuss teaching moves related to two questions.
• Video B: Working collectively to revise an explanation
  ○ Framing question: How might collectively revising a student's explanation advance the learning of the student who gave the explanation as well as the learning of the rest of the class?
• Video C: Helping students at different grade levels understand the features of a "good" explanation
  ○ Framing question: What are the advantages of building a list of features of a "good" explanation with students?
After viewing each video, ask participants whether they have additional thoughts about teaching moves to respond to each question.	

Part 6: Wrap up (~5 minutes)

Goals	***Instructional sequence***	***Resources***
• Participants will understand ways of connecting the session content to their classroom.	1. Summarize the work of the session. 2. Explain and distribute the Classroom Connection Activities.	

Detailed description of activity	Comments & other resources
1. Summarize the session by emphasizing that participants: • Justified and refuted conjectures using different approaches and considered features of "good" explanations in the context of geometry • Used analyses of students' explanations to design teaching moves, including moves that make mathematical practices explicit to the class. 	
2. Distribute the *handout* you customized with selected Classroom Connection Activities and accompanying documents described below. Several of these will produce ideas or products that are necessary for subsequent sessions. <u>Required:</u> • Select a problem from your curriculum that you feel can be used to provide students with a strong opportunity to engage in mathematical reasoning. Recall from our work in previous sessions and CCAs that there are many problems in your curriculum that can be used for this purpose. We are attaching a resource, "Approaches to Modifying Tasks" that you can use to support your selection and revision of a task that could be used to provide opportunities for you students to reason and engage in mathematical practices. Provide rationale for the problem you select. We will discuss your selection at our next session. <u>Optional:</u> • Choose one of the following articles that provides an example of how student reasoning can be encouraged in different topic areas in mathematics: Lehrer and Curtis's (2000) article pertaining to geometry, Nitaback and Lehrer's (1996) article pertaining to measurement, the Mathematical Sciences Education Board (1993) article pertaining to data, and Carpenter and Levi's (2000) article pertaining to algebra. Make connections between the article and our work to support students' reasoning. • Read the Justifying True/False Statements Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.	

List of Common Core State Standards Mathematical Practices

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision.
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.