Overview of Session 8

- Exploring geometry statements and developing clear justifications
- Responding to students' explanations

8.1a

Geometry Statement: Squares & Rectangles

Decide whether the following statement is **true** or **false** and develop a clear justification or refutation for the statement.

All squares are rectangles, but this does not mean that all rectangles are squares.

8.2a

Geometry Statements

Consider the following statements:

- All polygons with four straight connected sides are quadrilaterals.
- Any parallelogram with at least one right angle is a rectangle.
- If the statement "A square is a parallelogram" is true, then which of the following are true as well?
  - A parallelogram is a square. (converse)
  - If a shape is not a square, it is not a parallelogram. (inverse)
  - If a shape is not a parallelogram, it is not a square. (contrapositive)

Decide whether the statements shown on the left are **true** or **false** and develop a clear justification or refutation for each statement. Use the glossary to support writing justifications/refutations.

8.2b

Geometry Statements: Partner work

Consider the following statements:

- All polygons with four straight connected sides are quadrilaterals.
- Any parallelogram with at least one right angle is a rectangle.
- If the statement "A square is a parallelogram" is true, then which of the following are true as well?
  - A parallelogram is a square. (converse)
  - If a shape is not a square, it is not a parallelogram. (inverse)
  - If a shape is not a parallelogram, it is not a square. (contrapositive)

With a partner:

- Take turns sharing your justifications/refutations
- Attend to the features of a "good" explanation:
  - Has a clear purpose
  - Has a logical structure
  - Uses representations and language clearly and carefully (including the selection of examples and definitions)
  - Focuses on meaning and is oriented to the listener(s)
- Work together to see if you can develop a complete justification or refutation for each statement

8.2c

Features of a "good" mathematical explanation

- Has a clear purpose
- Has a logical structure
- Uses representations and language clearly and carefully
- Focuses on meaning and is oriented to the listener(s)

8.2d

Geometry statement: Quadrilaterals

All polygons with four straight connected sides are quadrilaterals.

Attend to the features of a "good" explanation:

- Has a clear purpose
- Has a logical structure
- Uses representations and language clearly and carefully (including the selection of useful examples and definitions)
- Focuses on meaning and is oriented to the listener(s)

8.3a

Approach 1 – Finding a counterexample

All polygons with four straight connected sides are quadrilaterals.

Definition used by teachers:

- **Polygon:** A closed figure consisting of line segments (sides) connected endpoint to endpoint. The sides of a polygon may not cross.

8.3b

Approach 2 – A pentagon as a possible counterexample

All polygons with four straight connected sides are quadrilaterals.

Definitions used by teachers:

- **Polygon:** A closed figure consisting of line segments (sides) connected endpoint to endpoint. The sides of a polygon may not cross.
- **Parallelogram:** A 4-sided polygon whose opposite sides are parallel. The opposite sides of a parallelogram are also the same length. And the opposite angles in a parallelogram have the same measure.

8.3c

Geometry Statement: Parallelograms & Rectangles

Decide whether the following statement is **true** or **false** and develop a clear justification or refutation for your conclusion.

Any parallelogram with at least one right angle is a rectangle.

8.3d

Geometry Statement: Parallelograms & Squares

Decide whether the following statement is **true** or **false** and develop a clear justification or refutation for your conclusion.

If the statement "A square is a parallelogram" is true, then which of the following are true as well?

1) A parallelogram is a square.
2) If a shape is not a square, it is not a parallelogram.
3) If a shape is not a parallelogram, it is not a square.

8.3e

Analyzing student explanations

Consider each explanation with particular attention to:

- Does the explanation have a logical structure?
- Does the explanation use representations and language clearly and carefully?
- Is the explanation focused on meaning and oriented to the listener(s)? What background knowledge is assumed?

8.4a

Responding to student explanations

Identify important/useful teaching moves to ask the student or the class.

- How do these questions and teaching moves connect with what is noticed in the student explanation?
- What do these questions and teaching moves accomplish mathematically?
  - for the student who gave the explanation?
  - for the other students in the class?

8.5a

Summary

In this session, you:

- Justified and refuted conjectures using different approaches, and considered features of "good explanations" in the context of geometry
- Used analyses of students' explanations to design teaching moves, including moves that make mathematical practices explicit to the class

8.6a