

Description of the Session 9: Planning instruction to target reasoning and engagement in mathematical practices

Session 9 provides an opportunity to consolidate learning about reasoning, explanation, and mathematical practices through work with one last mathematics problem. The session also provides a chance to re-engage with the idea of “scaling” and modifying mathematical tasks. The session begins with work related to one of the Classroom Connection Activities from the last session, for which, participants will describe and discuss their selection of a task/problem that will be the basis for teaching considered in the video workshop in Session 10. Then, participants will consider a mathematical object, known as Pascal’s Triangle, and unpack some of the mathematics contained in mathematical object. Specifically, participants will notice patterns contained in the mathematical object, explain how the patterns are produced, and use the patterns to identify the next row of numbers in the triangle. Throughout, participants will make use of features of “good” explanations and mathematical practices. In the last part of the session, participants will scale and modify mathematics tasks that are commonly found in curriculum materials and simultaneously think about how to make mathematical practices more explicit within tasks.

Activities and goals of the session

Activities	Times	Corresponding parts of the session	Goals
Conversation about a CCA from the last session	5 minutes		<ul style="list-style-type: none"> Participants will share their thinking about the problems/tasks they have chosen as the basis for the teaching that will be considered in the last video workshop session.
I. Preview	5 minutes	Part 1	<ul style="list-style-type: none"> Participants will be oriented to the work of the session.
II. Pascal’s Triangle	35 minutes	Parts 2 & 3	<ul style="list-style-type: none"> Participants will make and justify a conjecture using different approaches. Participants will consider features of “good explanations” in a new context. Participants will connect the CCSS mathematical practices to the work the group has just completed. Participants will consider ways of using a problem such as Pascal’s Triangle with integrity at different grade levels to learn about reasoning.
III. Planning Instruction for Reasoning and Engagement in Mathematical Practices	40 minutes	Part 4	<ul style="list-style-type: none"> Participants will consider multiple approaches to infusing instruction with reasoning Participants will modify tasks in ways that enhance students’ opportunities to reason and engage in the CCSS mathematical practices. Participants will consider how modifications affect the mathematics of tasks. Participants will formulate ways of making mathematical practices explicit to students through modeling.
IV. Wrap up	5 minutes	Part 5	<ul style="list-style-type: none"> Participants will understand ways of connecting the session content to their classroom.

Classroom Connection Activities

Required	Optional
<p>Type of task: Video workshop preparation Description: Video record the teaching of the task that you selected for working on reasoning with students. Pick a 3-5 minute segment and reflect on it.</p> <p>Type of task: Video workshop reflection Description: You have engaged in multiple cycles of learning through engaging in video workshops. Look across the records and reflections you have generated and take stock of what you have learned about your own teaching and the process of video workshop.</p>	<p>Type of task: Mathematics Reading Description: The Analysis of Pascal’s Triangle Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices.</p>

Preparing for the session

- Make copies as needed:
 - *Resources:* Handout: Features of a “good” explanation (Part 2); Handout: Pascal’s Triangle Problem (Part 2); Handout: Pascal’s Triangle poster (Part 2); Handout: The mathematical practices (Part 3); Handout: Infusing reasoning (Part 4); Handout: Tasks from curriculum materials (Part 4); Handout: Approaches to modifying tasks (Part 4)
 - *Supplements:* Math notes: Analysis of Pascal’s Triangle
- Customize and make copies of the Classroom Connection Activities
- Test technical setups: Internet connection, speakers, projector

Developing a culture for professional work on mathematics teaching (ongoing work of the facilitator throughout the module)

1. Encourage participation: talking in whole-group discussions; rehearsing teaching practices; coming up to the board as appropriate.
2. Develop habits of speaking and listening: speaking so that others can hear; responding to others’ ideas, statements, questions, and teaching practices.
3. Develop norms for talking about teaching practice: close and detailed talk about the practice of teaching; supporting claims with specific examples and evidence; curiosity and interest in other people’s thinking; serious engagement with problems of mathematics learning and teaching.
4. Develop norms for mathematical work:
 - a) Reasoning: explaining in detail; probing reasons, ideas, and justifications; expectation that justification is part of the work; attending to others’ ideas with interest and respect.
 - b) Representing: building correspondences and making sense of representations, as well as the ways others construct and explain them.
 - c) Carefully using mathematical language.
5. Help participants make connections among module content and develop the sense that this module will be useful in helping them improve their mathematics teaching, their knowledge of mathematics, their understanding of student thinking, and their ability to learn from their own teaching.
6. Help participants understand connections between module content and the Common Core State Standards.

*Scope of the module (focal content of this session in **bold**)*

Mathematics	Student thinking	Teaching practice	Learning from practice
<ul style="list-style-type: none"> • making and justifying/refuting conjectures and generalizations • recognizing and using multiple approaches to solve mathematics problems • understanding features of a “good” mathematical explanation and producing “good” explanations • identifying foundations of mathematical reasoning • using and knowing the mathematical practices identified in the CCSS 	<ul style="list-style-type: none"> • monitoring students’ mathematical reasoning • noticing collective elements of mathematical reasoning 	<ul style="list-style-type: none"> • supporting students’ engagement in mathematical practices by teaching them explicitly • supporting students in explaining their mathematical reasoning • establishing and maintaining an environment that emphasizes reasoning • adapting tasks to nurture mathematical reasoning 	<ul style="list-style-type: none"> • using norms that support engagement in video workshop • understanding the video workshop process • learning to analyze teaching and learning in the context of video workshop

Conversation about a Classroom Connection Activity from last session (~5 minutes)

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> Participants will share their thinking about the problems/tasks they have chosen as the basis for the teaching that will be considered in the last video workshop session. 	<ol style="list-style-type: none"> With a partner, participants share the problems/tasks they have selected. 	

Detailed description of activity	Comments & other resources
<ol style="list-style-type: none"> With a partner, have participants share the tasks/problems (one per person) they have chosen as the basis for the teaching they will analyze in the last video workshop. Have participants discuss their reasons for selecting particular tasks, focusing on the potential of these tasks to support engagement in reasoning and mathematical practices, specifically: <ol style="list-style-type: none"> Working on reasoning collectively in a whole class setting Making mathematical practices explicit to students Modifying tasks to enhance the opportunities they provide for working on reasoning and mathematical practices <p>Bring this activity to a close by asking participants what struck them when thinking about the reasoning involved in the tasks that were chosen.</p> <p>Make clear to all participants that everyone will be sharing videos of their teaching in Session 10.</p> 	<p><i>It is possible that participants used ideas or methods described in the document "Approaches to modifying tasks" to adjust the tasks they are using with their students. Listen for the ways in which participants used these ideas in their task selection and how they are talking about them with colleagues.</i></p> <p><i>This information will support your facilitation of Part 4 of this session.</i></p> <p><i>Make sure that all participants have sound tasks for using as the basis of teaching that will be shared in the last video workshop.</i></p> <p><i>All participants will have the opportunity to share segments from their videos, which makes the video workshop activity in Session 10 a bit different from previous experiences. Session 10 was designed this way to provide a culmination of video workshop in the module.</i></p>

Part 1: Preview (~5 minutes)

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> Participants will be oriented to the work of the session. 	1. Introduce the session and watch the introductory video.	<ul style="list-style-type: none"> Video A (01:17): Session overview

Detailed description of activity	Comments & other resources
<p>1. Introduce the session: This session provides an opportunity to consolidate learning about reasoning, explanation, and mathematical practices through work with one last mathematics problem. The session also provides a chance to re-engage with the idea of “scaling” and modifying mathematical tasks. The session focuses on:</p> <ul style="list-style-type: none"> Unpacking some of the mathematics within a mathematical object known as Pascal’s Triangle Scaling and modifying mathematical tasks that are typically found in curriculum materials and simultaneously thinking about how to make mathematical practices more explicit within tasks 	

Overview of Session 9

- Recognizing the mathematical practices in action
- Scaling mathematics tasks and considering how to make mathematical practices more explicit with tasks

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Part 2: Exploring Pascal's Triangle (~25 minutes)

Goals

- Participants will make and justify a conjecture using different approaches.
- Participants will consider features of "good" explanations in a new context.

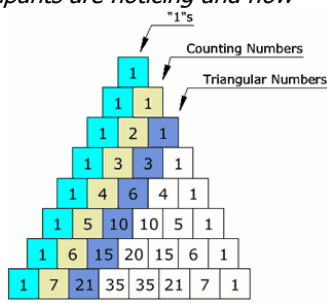
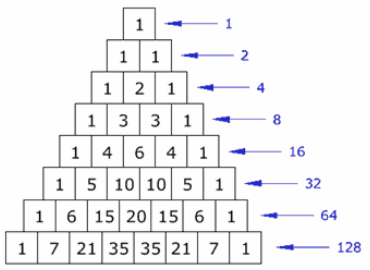
Instructional sequence

1. Introduce Part 2 by reading and showing the problem and clarifying questions.
2. Work independently to determine patterns and the "rules" those patterns follow.
3. Work in partners on explanation and noting the use of representations when explaining.
4. Discuss patterns in whole group.

Resources

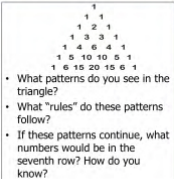
- Handout: Features of a "good" explanation
- Handout: Pascal's Triangle Problem
- Handout: Pascal's Triangle poster

Detailed description of activity	Comments & other resources																																																																									
<p>1. Introduce Part 2: This part launches work on a mathematics problem – Pascal's Triangle. This problem is different from previous problems in the module because the focus is on explaining how particular solutions, in this case patterns, are produced. This work is designed to support different methods of explaining and different uses of the representation to support explanations. Later it is used as the basis for summarizing some of the work on the CCSS mathematical practices.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p style="text-align: center; background-color: #1a3d4d; color: white; padding: 2px;">Pascal's Triangle</p> <table style="width: 100%; border-collapse: collapse;"> <tr><td style="text-align: center;">1</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">1</td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">2</td><td style="text-align: center;">1</td><td></td><td></td><td></td><td></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">3</td><td style="text-align: center;">3</td><td style="text-align: center;">1</td><td></td><td></td><td></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">4</td><td style="text-align: center;">6</td><td style="text-align: center;">4</td><td style="text-align: center;">1</td><td></td><td></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">5</td><td style="text-align: center;">10</td><td style="text-align: center;">10</td><td style="text-align: center;">5</td><td style="text-align: center;">1</td><td></td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">6</td><td style="text-align: center;">15</td><td style="text-align: center;">20</td><td style="text-align: center;">15</td><td style="text-align: center;">6</td><td style="text-align: center;">1</td></tr> </table> <div style="background-color: #fff9c4; padding: 5px; margin-top: 5px;"> <ul style="list-style-type: none"> What patterns do you see in the triangle? What "rules" do these patterns follow? If these patterns continue, what numbers would be in the seventh row? How do you know? </div> </div> <p>Distribute the <i>Handout: Pascal's Triangle</i>. Use the <i>Slide: Pascal's Triangle</i> to show the image of Pascal's Triangle and pose the three questions for participants to explore.</p> <ul style="list-style-type: none"> • What patterns do you see in the triangle? • What "rules" do these patterns follow? • If these patterns continue, what numbers would be in the seventh row? How do you know? <p>Clarify any terminology that may be confusing. If it seems generative, have a participant name one pattern that he or she sees, point to one or more places where the pattern occurs, and then explain "the rule" that the pattern follows.</p>	1							1	1						1	2	1					1	3	3	1				1	4	6	4	1			1	5	10	10	5	1		1	6	15	20	15	6	1	<p><i>The Pascal's Triangle Problem is different than many of the problems that participants have explored in the module that have focused on how many solutions there are or on finding all solutions.</i></p> <p><i>One of the first uses of Pascal's Triangle (in the 11th century) was in determining the coefficients of a binomial expansion. If the expression $(x + y)$ is raised to any power, its coefficients are the numbers in that row of the triangle.</i></p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr style="background-color: #d3d3d3;"> <th>Row</th> <th>Power</th> <th>Binomial Expansion</th> <th>Pascal's Triangle</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>$(x + y)^0$</td> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>$(x + y)^1$</td> <td>1x + 1y</td> <td>1, 1</td> </tr> <tr> <td>2</td> <td>$(x + y)^2$</td> <td>1x² + 2xy + 1y²</td> <td>1, 2, 1</td> </tr> <tr> <td>3</td> <td>$(x + y)^3$</td> <td>1x³ + 3x²y + 3xy² + 1y³</td> <td>1, 3, 3, 1</td> </tr> <tr> <td>4</td> <td>$(x + y)^4$</td> <td>1x⁴ + 4x³y + 6x²y² + 4xy³ + 1y⁴</td> <td>1, 4, 6, 4, 1</td> </tr> </tbody> </table>	Row	Power	Binomial Expansion	Pascal's Triangle	0	$(x + y)^0$	1	1	1	$(x + y)^1$	1x + 1y	1, 1	2	$(x + y)^2$	1x² + 2xy + 1y²	1, 2, 1	3	$(x + y)^3$	1x³ + 3x²y + 3xy² + 1y³	1, 3, 3, 1	4	$(x + y)^4$	1x⁴ + 4x³y + 6x²y² + 4xy³ + 1y⁴	1, 4, 6, 4, 1
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Detailed description of activity	Comments & other resources
<p>2. Have participants work individually for about five minutes.</p> <p>Participants may notice that:</p> <ul style="list-style-type: none"> every row has one more number each term is generated by adding the two above the sum of every row is a power of 2 (e.g. the sum of row zero is 2^0, the sum of row one is 2^1 (see figure in the right hand column) the table is symmetric each diagonal row follows a different rule going along one diagonal and adding the terms will result in a sum that is the next term heading in the opposite diagonal direction (which is sometimes called the "Hockey Stick Rule") the middle number is repeated every other row <p>The values for the seventh row: 1, 7, 21, 35, 35, 21, 7, 1.</p>	<p><i>Circulate to see the patterns that participants are noticing and how they are documenting those patterns.</i></p> <p><i>Ask participants to first show a pattern using numbers with two examples. Then, they should attempt to describe the pattern in words. This gives participants opportunities to try to represent an idea in different ways. By describing patterns in words, participants are working on providing explanations.</i></p>  <p><i>Encourage participants not to move on to the last question about the seventh row until they have grappled for with noticing and articulating patterns. Waiting will allow participants to test those patterns and ways of thinking about how they are generated to discover and (later) to justify why certain numbers must be in the next row.</i></p> <p><i>Emphasize the difference between understanding and being able to articulate an idea. Participants may be able to understand certain patterns without being able to articulate them.</i></p>  <p><i>There are multiple approaches to working on the Pascal's Triangle Problem. Highlight this idea with participants.</i></p>
<p>3. After participants have had sufficient time to notice patterns and think about the seventh row, have participants work with a partner to share what they have noticed, and more importantly to listen to what others found, noticing the qualities of the explanations given and the ways in which representations are used as a part of the explanations. Show <i>Slide: Pascal's Triangle - Partner work</i>. Specifically, participants should consider the following:</p> <ul style="list-style-type: none"> whether their explanations: 	<p>The <i>Handout: Features of a "good" explanation</i> could be distributed here as a resource.</p>

Detailed description of activity	Comments & other resources
<ul style="list-style-type: none"> ○ have a clear purpose ○ have a logical structure ○ use representations and language clearly and carefully ○ have a focus on meaning and an orientation to the listener(s) <ul style="list-style-type: none"> ● how they and their partners are using the Pascal's Triangle representations in their explanations. 	
<p>4. Use the question about which terms will be in the seventh row to orchestrate a whole group discussion. As participants share which numbers should appear, ask participants to justify those numbers using reasoning connected with multiple patterns they found in the table. Ask questions like:</p> <ul style="list-style-type: none"> ● How do you know that number should appear there? (What patterns helps you know that number should appear there?) ● Is there another pattern that would confirm that number should appear there? ● How would you describe the pattern? Does someone have a different way of describing that pattern? ● How did you determine that pattern? How is the pattern generated? What makes you confident about that pattern? (e.g. it happens in lots of places, you tried lots of other ways to generate that pattern) ● Is there a pattern that you feel less confident about? Why? <p>As participants explain the values, take opportunities to notice the logic, completeness, language/representation that are being used. Ask participants to comment on effective uses of representations to support explanations (gesturing, for providing examples, etc.).</p>	<p><i>Every number can be justified in many ways. For instance, when someone says that the fourth number should be 35, it could be confirmed by talking about the patterns: on the diagonal, the sum of the two numbers above it, that it must be the same as the fifth number to preserve symmetry, that when you use it with the other numbers it makes the right row sum, etc.</i></p> <p><i>You may consider using the Handout: Pascal's Triangle poster, a document camera, or other visual aids to support this discussion.</i></p>

Pascal's Triangle: Partner work



What patterns do you see in the triangle?

What "rules" do these patterns follow?

If these patterns continue, what numbers would be in the seventh row? How do you know?

As you discuss the questions, consider:

- Whether your explanations:
 - Have a clear purpose
 - Have a logical structure
 - Use representations and language clearly and carefully
 - Have a focus on meaning and an orientation to the listener(s)
- How you and your partner are using the Pascal's triangle representation in your explanations.

Discussion of solutions to the Pascal's Triangle Problem

To what extent does the explanation:

- Have a clear purpose
- Have a logical structure
- Use representations and language clearly and carefully
- Have a focus on meaning and an orientation to the listener(s)

Part 3: Identifying the mathematical practices in the work on Pascal's Triangle (~10 minutes)

Goals

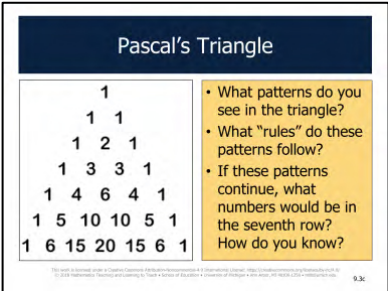
- Participants will connect the CCSS mathematical practices to the work the group has just completed.
- Participants will consider ways of using a problem such as Pascal's Triangle with integrity at different grade levels to learn about reasoning.

Instructional sequence

1. Discuss the mathematical practices relevant to the Pascal's Triangle Problem using the focus questions.
2. Invite participants to consider what might be required to adjust this problem for use in different contexts.

Resources

- Handout: The mathematical practices

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 3: This part extends the work on the Pascal's Triangle Problem by explicitly noting where different mathematical practices were illustrated as participants worked on the problem and shared their insights. Being able to notice and comment upon engagement in mathematical practices is an important skill in mathematics teaching. It supports assessing student learning of this set of educational outcomes and awareness about the practices that can be the basis for enhancing the learning of mathematics through the practices as well as learning about the practices themselves.</p> <p>Use the focus questions on the left side of the viewer to launch a whole group discussion about the ways in which the mathematical practices were used or illustrated in the work on the Pascal's Triangle Problem.</p> <ul style="list-style-type: none"> • Which of the mathematical practices are particularly relevant to work on this problem? • Which of the mathematical practices are less connected to work on this problem? <p>In addition to the questions on the slide, which frame the consideration in terms of the entire set of practices, also ask questions that:</p> <ul style="list-style-type: none"> • follow up on the mathematical practices that participants mention like, "what is an 	<p><i>It is likely that participants will be able to see mathematical practices #1, #3, #7, and #8 in use as they either worked on the problems themselves or as they share with and listened to colleagues. Practices related to using tools and modeling (MP #4 and #5) may be seen as less evident. Noticing this distinction is important because not all problems are places where all practices will get worked on.</i></p> <p><i>Overall, the point is to have participants talking about the meaning of the mathematical practices, how the practices support work on mathematics, and very practically on what it sounds like/looks like to engage in these practices.</i></p> <p><i>The Slide: Pascal's Triangle is included as a resource.</i></p> 

Detailed description of activity	Comments & other resources
<p>example of how you attended to that one?" or "how are others thinking about that mathematical practice in relation to this problem?"</p> <ul style="list-style-type: none"> • elicit discussion of mathematical practices that are not mentioned, like "In what ways did we attend to precision in our work on this problem?" or "What was challenging about 'making sense' when working on this problem?" 	
<p>2. If there is time, have participants consider how they might use a problem like this with integrity at different grade levels. This is a rich problem with many possibilities in terms of how it might be staged and what might be expected of students as they work on the problem.</p> <p>For instance, in the version of the problem participants worked on in this session, there were three questions that guided the initial work. Consider asking participants:</p> <ul style="list-style-type: none"> • What are the purposes of these questions? • How might those questions, or others, be posed in ways that engage students in the mathematical practices? <p>Another example of possibilities that would affect the work on this problem is what kind of recording sheet could be used. The recording sheet participants used had multiple iterations of the triangle. Consider asking participants:</p> <ul style="list-style-type: none"> • How would work have been different if the recording sheet had just one triangle? • How would work have been different if the recording sheet had a version of the triangle with "fill-in-the-blank" dashes to be used when figuring out the 7th row of numbers? 	<div data-bbox="774 485 1157 776" style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p style="text-align: center;">Using Pascal's Triangle</p> <p>How might you use this problem at your grade level?</p> <ul style="list-style-type: none"> • What recording sheet would you use? • What questions would you pose? • How could you make the mathematical practices explicit? </div> <p><i>Connect this with the work participants did as they thought about the tasks they used in their classrooms for the last video workshop.</i></p> <p><i>Providing blank spaces for the row(s) they need to figure out (this would reveal how many terms are there, but after they see that every row has one more number than having a template might help.</i></p> <p><i>There are many ways in which teachers can shape The Pascal's Triangle Problem so that it is appropriate for different ages while still mathematically robust. For example, features of the handouts, the questions posed, or the timing (e.g., doing the problem in one session or as a "problem of the week") all have an impact on what can be learned and done with reasoning and the mathematical practices in this problem. Participants can consider how they could design each of these features to use the problem with integrity at various grade levels.</i></p>

Part 4: Planning instruction for reasoning and engaging in mathematical practices (~40 minutes)

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> Participants will consider multiple approaches to infusing instruction with reasoning. Participants will modify tasks in ways that enhance students' opportunities to reason and engage in the CCSS mathematical practices. Participants will consider how modifications affect the mathematics of tasks. Participants will formulate ways of making mathematical practices explicit to students through modeling. 	<ol style="list-style-type: none"> Introduce Part 4 and launch the activity on modifying tasks. Have participants work on modifying the "Guess My Rule" task with a partner. Invite multiple groups to share their modifications of the task, and show and discuss Video A. Have participants work on modifying a second task with a partner, with a focus on making mathematical practices explicit. Watch and discuss Video B. 	<ul style="list-style-type: none"> Video A (01:03): Teacher insight – Making "Guess My Rule" easier and harder Video B (02:46): Modified tasks and working on mathematical practices Handout: Infusing reasoning Handout: Tasks from curriculum materials Handout: Approaches to modifying tasks

Detailed description of activity	Comments & other resources
<p>1. Introduce Part 4: This part continues work maintaining and extending opportunities for students to reason. Specifically, scaling mathematics tasks is reconsidered through various approaches that can have very practical and meaningful impact on preparing for instruction in ways that enhance students' opportunities to reason and engage in mathematical practices. In this part, tasks commonly found in curriculum materials are used to explore different modification strategies that can enhance opportunities to engage in mathematical reasoning.</p> <p>Remind participants that throughout the module, they have been engaged in working on three focal teaching practices (see <i>Slide: Focal teaching practices</i>).</p> <ul style="list-style-type: none"> Establishing an environment that emphasizes sense-making, justifying, and collective mathematical work Scaling and infusing tasks with reasoning opportunities Making mathematical practices explicit 	<p><i>Facilitators should use the tasks provided if this is the first time that they are using the Dev-TE@M materials.</i></p> <p><i>These tasks were selected because they are relatively common across curriculum materials and often are not used in ways that bring reasoning to the forefront.</i></p> <p><i>Participants looked at the approaches to modifying tasks when they selected the task for their final video workshop.</i></p> <p><i>It may help to quickly preview the tasks that are in the packet, noting that, while all have potential to be rich opportunities for working on reasoning and engaging in mathematical practices, they are often used in ways that do not realize their mathematical potential.</i></p>

Focal teaching practices

- Establishing an environment that emphasizes sense-making, justifying, and collective mathematical work
- Scaling tasks and infusing tasks with reasoning opportunities
- Making mathematical practices explicit

Infusing reasoning into Guess My Rule

- Modify the task in different ways
 - What, specifically, did you change about the task?
 - How do you think each modification enhances the potential to engage students in mathematical reasoning or practices?

Detailed description of activity	Comments & other resources
<p>This part provides additional opportunities to work on all of these practices, with particular attention on the second and third of these practices. The work of this part connects with earlier work on scaling in Session 4. The term “scale” means creating problems that stay the same mathematically but are different in other ways that meet a particular need. In this part, participants will be modifying tasks (rather than scaling tasks) and may be changing the mathematics in the problem in order to bring out the reasoning potential. Distribute the <i>Handout: Approaches to modifying tasks</i>. Review the different approaches to modifying problems (<i>Slide: Approaches to modifying problems</i>):</p> <ul style="list-style-type: none"> • Changing it to have more than one right answer • Reversing the task to work backwards • Making it into a multi-step problem • Making it into one where students have to find all solutions • Turning it into an analytic task (e.g., analysis of an alternative method, a general claim, evaluation of correctness or validity; if incorrect, error analysis) 	
<p>2. Distribute the <i>Handout: Tasks from curriculum materials</i>. Explain to participants that you want them to work with a partner to modify the “Guess My Rule” problem to provide additional opportunities to engage in mathematical practices. The “Guess My Rule” problem is one that is common to many mathematics curricula. Participants should work with a grade-level colleague, if possible, and modify the task in ways that allow it to be used at their grade level and provide opportunities to engage in mathematical practices.</p> <p>Participants might create several different modifications of the task. As participants work, they should consider the following questions:</p> <ul style="list-style-type: none"> • What, specifically, did you change about the task? • How do you think that each modification enhances the potential to engage students in mathematical reasoning or practices? <p>Have participants work for about 10 minutes.</p>	<p><i>While circulating among the small groups, you might ask participants: “Which mathematical practices would the modified version of the task provide opportunities to work on?”</i></p>
<p>3. Discuss modifications of the task for about 10 minutes. Invite several groups to share their modifications of the problem. Use this as an opportunity to help participants attend to what is being changed about the problem and how that impacts the opportunity to reason and engage in mathematical practices. As participants share, ask the following focus questions:</p> <ul style="list-style-type: none"> • What, specifically, did you change about the task? 	<p><i>Try to have participants working at different grade levels share their modifications of the task.</i></p> <p><i>Participants might have the following ideas about ways to modify the tasks from curriculum materials:</i></p> <ul style="list-style-type: none"> • <i>Guess My Rule – Ask students to create two rules and</i>

Detailed description of activity	Comments & other resources
<ul style="list-style-type: none"> How do you think that each modification enhances the potential to engage students in mathematical reasoning or practices? <p>If it seems useful, show <i>Video A</i> in which two teachers in the professional development series talk about how they would scale the <i>Guess My Rule</i> task to make it either easier or harder.</p>	<p><i>identify the shapes that fit both rules, one rule, or neither rule.</i></p> <ul style="list-style-type: none"> <i>Estimation Jar – After students estimate the number of cubes in a jar, fill a second jar to the same level with a smaller or larger object (e.g., paper clips or ping pong balls). Ask students estimate whether the second jar has more or fewer objects than the first jar and to explain the reason for their answers.</i> <i>Name-Collection Boxes – Constraints could be added to the problem (e.g., write different names for 8 that include division; write different names for 50 that include the number 5)</i> <i>Dividing Whole Numbers – Students could write story problems for the division exercises; Students could answer questions such as: "What happens to the quotient if you add a zero to the dividend?"</i> <p><i>Consider showing Video A if some of the potential ideas for scaling shared by the teachers have not emerged in your discussion. Focus participants on the question that is asked at the end of the video: "What practice would you be creating occasions to work on?"</i></p> <p><i>Many scholars have focused on the way in which modifications of tasks and problems influence the mathematics that is available to learn. For instance, Stein & Smith (1998)¹ reported on the ways in which modifications impact the cognitive demand of tasks. In this module, participants are encouraged to notice many ways in which modifications change the mathematics in tasks and problems, including the complexity of the mathematics and the opportunity for generalization and representation.</i></p>

¹ Stein, M. K. & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268-275.

Detailed description of activity	Comments & other resources
<p>4. Explain to participants that they will now be considering an added layer to the modification work – making mathematical practices explicit to students. Have participants select one of the tasks in the <i>Handout: Tasks from curriculum materials</i>. All of these tasks are commonly found in curriculum materials. Participants should work with a grade level colleagues to do the following:</p> <ul style="list-style-type: none"> • Modify the task in different ways: <ul style="list-style-type: none"> ○ What, specifically, did you change about the task? ○ How do you think that each modification enhances the potential to engage students in mathematical reasoning or practices? • Select one of the modified tasks to use as the context for making a mathematical practice explicit <ul style="list-style-type: none"> ○ Model the use of a mathematical practice as you complete the task making key facets of the that practice explicit as you work <p>Have participants work for about 10 minutes. If time permits, have participants share a few of their modifications.</p> <div data-bbox="829 397 1213 690" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;">Approaches to modifying problems</p> <p>Try modifying a task by:</p> <ul style="list-style-type: none"> • Changing it to have more than one right answer • Reversing the task to work backwards • Making it into a multi-step problem • Making it into one where students have to find all solutions • Turning it into an analytic task, e.g.: <ul style="list-style-type: none"> – Analysis of an alternative method – A general claim – Evaluation of correctness or validity – If incorrect, error analysis </div> <div data-bbox="829 711 1213 1003" style="border: 1px solid black; padding: 5px;"> <p style="text-align: center;">Using modified tasks to make mathematical practices explicit</p> <ul style="list-style-type: none"> • Modify the task in different ways <ul style="list-style-type: none"> – What, specifically, did you change about the task? – How do you think each modification enhances the potential to engage students in mathematical reasoning or practices? • Select one of the modified tasks to use as the context for making a mathematical practice explicit <ul style="list-style-type: none"> – Model the use of a mathematical practice as you complete the task making key facets of that practice explicit as you work </div>	
<p>5. To conclude work on this activity, show <i>Video B</i> in which Dr. Ball shares different ways in which participants may have modified tasks and tried to be more explicit about mathematical practices within the design of tasks.</p>	<p><i>The ways to modify problems listed in the slide will likely include some of the ideas participants mention; this list might also be improved by adding other ideas from participants.</i></p>

Part 5: Wrap up (~5 minutes)

<u>Goals</u>	<u>Instructional sequence</u>	<u>Resources</u>
<ul style="list-style-type: none"> Participants will understand ways of connecting the session content to their classroom. 	<ol style="list-style-type: none"> Summarize the work of the session. Explain and distribute the Classroom Connection Activities. 	

Detailed description of activity	Comments & other resources
<p>1. Summarize the session by emphasizing that the session provided an opportunity to consolidate learning about reasoning, explanation, and mathematical practices and to extend the idea of “scaling” mathematical problems to tasks that are typically found in curriculum materials. Specifically, participants:</p> <ul style="list-style-type: none"> Explain how particular patterns in Pascal’s Triangle function and how they are produced Identified examples of the mathematical practices in action Planned instruction for reasoning and engagement in mathematical practices 	<p><i>The next session consolidates work on using video workshop to learn from and improve teaching through a final engagement in workshops.</i></p>
<p>2. Distribute the <i>handout</i> you customized with the final Classroom Connection Activities.</p> <p><u>Required:</u></p> <ul style="list-style-type: none"> In your classroom, spend 15-20 minutes working on the problem you selected that will be the basis for the last video workshop. Use your teaching as an opportunity to try asking questions about the task that engage the whole class in reasoning. Also try using this problem to make the mathematical practices explicit to students. Video record the entire activity and collect student work samples (approximately 6 samples that represent a range in student reasoning). Complete the reflection questions listed in the Classroom Connection Activities <i>handout</i>. Reflect on your learning from video workshop. Look across the records and reflections you have generated and take stock of what you have learned about your own teaching and about the process of “doing” video workshop. Answer the reflection questions listed on the CCA <i>handout</i>. <p><u>Optional:</u></p> <ul style="list-style-type: none"> Read the Analysis of Pascal’s Triangle Math Notes on novel approaches to the problem and connections between the problem and the mathematical practices. 	

Summary

In this session, you:

- Explained how particular patterns function and how they are produced
- Identified examples of the mathematical practices in action
- Planned instruction for reasoning and engagement in mathematical practices

List of Common Core State Standards Mathematical Practices

- 1) Make sense of problems and persevere in solving them.
- 2) Reason abstractly and quantitatively.
- 3) Construct viable arguments and critique the reasoning of others.
- 4) Model with mathematics.
- 5) Use appropriate tools strategically.
- 6) Attend to precision.
- 7) Look for and make use of structure.
- 8) Look for and express regularity in repeated reasoning.