Handout: The Mathematical Practices[1]

1) Make sense of problems and persevere in solving them.

- Explaining the meaning of a problem by analyzing givens, constraints, relationships, and goals
- Making conjectures about the form and meaning of solutions and planning solution pathways rather than simply jumping into solution attempts
- Trying simpler forms of the original problem in order to gain sight into its solution
- Explaining correspondences between representations of the problem
- Monitoring and evaluating their progress, changing course if necessary
- Routinely asking "Does this make sense?"

2) Reason abstractly and quantitatively.

- Making sense of quantities and their relationships in problem situations
- *Decontextualizing*—representing a given situation symbolically and manipulating the represented symbols—and *contextualizing*, pausing as needed during the manipulation process to probe into the referents for the symbols involved
- Creating coherent representations of the problem at hand
- Considering the units involved; attending to the meaning of the quantities

3) Construct viable arguments and critique the reasoning of others.

- Understanding and using stated assumptions, definitions, and previously established results in constructing arguments
- Making conjectures and justifying conclusions by building a logical progression of statements to explore the truth of conjectures
- Comparing the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explaining what the flaw is
- Constructing arguments using concrete referents such as objects, drawings, diagrams, and actions
- Listening or reading the arguments of others, deciding whether they make sense, and asking useful questions to clarify or improve the arguments

[1] Definitions based largely on Bell, M., Bell, J., Bretzlauf, J., Dillard, A., Hartfield, R., Isaacs, A., et al. (2007). *Everyday mathematics: The University of Chicago school mathematics project, Student reference book.* Chicago, IL: Wright Group/McGraw-Hill.

4) Model with mathematics.

- Applying mathematics to solve problems arising in everyday life, society, and the workplace (e.g., writing an addition equation to describe a situation)
- Making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later
- Identifying important quantities in a practical situation and mapping relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Interpreting the mathematical results of a model in the context of the situation and reflecting on whether the results make sense, possibly improving the model if it has not served its purpose

5) Use appropriate tools strategically.

- Considering the available tools when solving a mathematical problem including pencil and paper, a concrete model, a ruler, a protractor, a calculator, or a spreadsheet
- Making sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations

6) Attend to precision.

- Communicating precisely, including using clear definitions and stating the meaning of symbols (e.g., using the equal sign consistently and appropriately)
- Specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem
- Calculating accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context

7) Look for and make use of structure.

- Discerning a pattern or structure (e.g., noticing that the number of permutations of three digits is the same as the number of possible orders for three children standing in a straight line)
- Shifting perspectives to see the overall nature of an object or its internal mechanisms

8) Look for and express regularity in repeated reasoning.

- Noticing if calculations are repeated, and looking both for general methods and for shortcuts (e.g., noticing when dividing 25 by 11 that they are repeating the same calculations over and over again, and concluding they have a repeating decimal)
- Maintaining oversight of the process, while attending to the details
- Evaluating the reasonableness of intermediate results