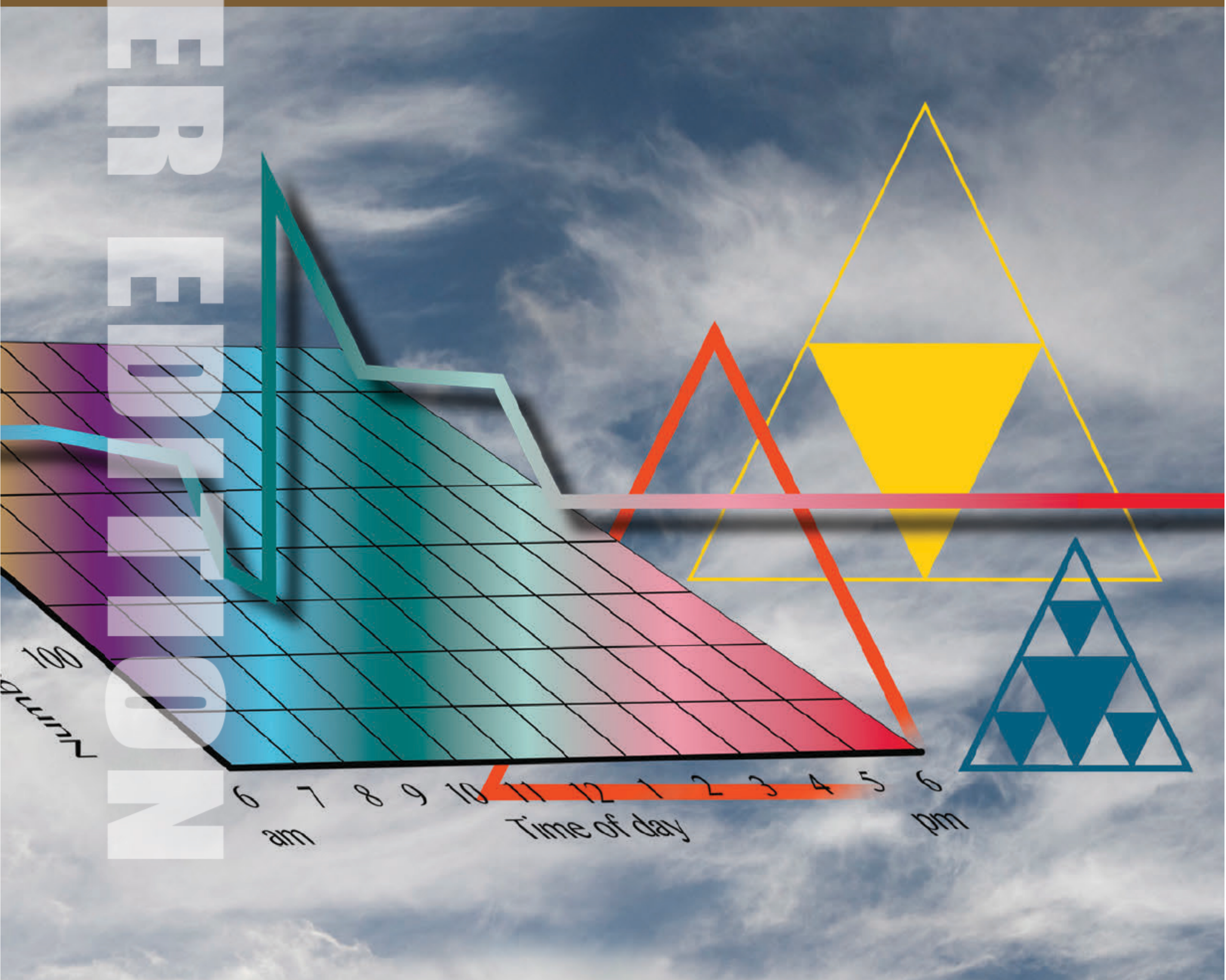
TEACHER EDITION

# A Modeling Approach to Algebra

Sec nd Edition



# A Modeling Approach to Algebra

*Second Edition*

**Teacher Edition**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# A Modeling Approach to Algebra

*A Modeling Approach to Algebra* (AMAA) follows the premise that learning algebra requires more than memorizing formulas and finding answers. Research on how students solve problems has suggested five features of curricula that affect student learning:

1. opportunities to speak, read, write, and model mathematical ideas
2. connections with prior knowledge or experiences
3. problem-solving tasks to introduce new ideas
4. time to develop concepts, generalizations, and skills
5. challenges for all students

To optimize these, this curriculum is designed to engage high school students in mathematical investigations that foster discussions. As students discuss their strategies for conducting investigations and solving problems, they gradually internalize algebraic ideas and develop an understanding of algebraic techniques. The lessons emphasize using models; promote open-ended inquiry; and provide time for developing concepts, generalizations, and skills.

## Curriculum Overview

The content of this curriculum targets specific high school standards from the section Traditional Pathway: High School Algebra I in the Common Core State Standards for Mathematics while also emphasizing the eight Standards for Mathematical Practice, which are at the foundation of the mathematics standards. The complete Common Core State Standards, which are referenced throughout this document as CCSSM, 2010, can be found at www.corestandards.org/the-standards. *A Modeling Approach to Algebra* is intended to complement a traditional algebra course through a focus on standards that provide students with opportunities to use modeling in solving problems (see pages ix–xi). The investigations are motivated by practical questions from applied contexts as well as other interesting questions arising from pure mathematics. Suggestions for including current events, provided in PublishView™, promote students' awareness of the mathematics data that exist in everyday experiences. Three longer-term projects give students the opportunity to apply what they have learned in Units 3–5. In addition, the curriculum includes examples of exercises that prompt students to build facility using conceptual processes and develop stronger skills needed for success in the investigations and in Algebra I and beyond. The current events component and exercises can be found in the digital version of the materials.

Technology enables students to interact with dynamic representations of concepts for classroom instruction. Prepared documents for teachers to use in TI-Nspire Teacher Software or for students to use with TI-Nspire Student Software or on handhelds enhance and extend algebraic concepts throughout the curriculum. The use of technology focuses on graphical representations for data, encourages making conjectures and validating conclusions, and emphasizes relationships between quantities through multiple representations.

The investigative, problem-based approach changes the roles of teachers and students. The pedagogy is student-centered, with students and teacher sharing ideas in the mathematical community of the classroom. Students explain their thinking, question their own and others' ideas, and analyze suggested strategies. The teacher orchestrates the discussion with thought-provoking questions, selects examples of student work to be shared that further the learning opportunities, and provides suggestions for techniques of mathematical inquiry and discussion when students need guidance.

## Use of the Materials

The investigations that make up *A Modeling Approach to Algebra* introduce and develop concepts through carefully constructed problems. The lesson pacing should give students an opportunity to conduct the investigations and to discuss results. Time should also be allotted for students to understand the problems, know what is expected of them, collaborate to develop productive strategies for solving the problems, and make and test conjectures about the mathematics. The debriefing discussions should focus on ideas students share with the class. Students should be encouraged to offer alternate solutions and solution methods, question others' methods and results, and reflect on their own understanding.

Each unit is introduced by a brief preview of the themes explored and a pacing guide for the unit. The suggested timing is an estimation of the minimum number of days an investigation should take if the class is working thoroughly on the problems and the discussions are probing thinking and understanding deeply. Some classes need more time than is suggested, especially as students are becoming familiar with conducting investigations. You may choose to skip the extensions at the end of some of the lessons.

The teacher edition contains Teacher Notes and Annotated Student Pages for each lesson.

**Teacher Notes.** The Teacher Notes support lesson planning by providing information that addresses content, pedagogy, and pedagogical content knowledge. Sections of the Teacher Notes contain a summary of the content and objectives for the investigation, highlight opportunities to model with mathematics, and anticipate student thinking and possible responses—including common misunderstandings. There is a section listing additional materials when needed and a section on technology use. Throughout the Unit 0 Teacher Notes, opportunities for students to engage in mathematical practices are indicated. These are important elements of the Common Core State Standards for mathematics.

**Annotated Student Pages.** The Annotated Student Pages comprise the material in the student book, notes for managing the investigation, and discussion questions. The questions, highlighted by shading, do not have to be asked verbatim but indicate what mathematics topics and ideas should be considered deeply by teacher and students. As students get used to the instructional approach, they will raise these issues themselves or pose the questions spontaneously. They should be encouraged to pose questions that extend a problem or probe its mathematical content. However, many students are not accustomed to asking questions beyond, "How do you do this problem?" One suggestion that may help is to ask, "What questions do you have?" instead of, "Do you have any questions?" The former signals that there likely are questions. The latter makes it easy to avoid asking a question if one is shy about revealing an uncertainty.

## Digital Versions

The entire *A Modeling Approach to Algebra* curriculum is provided in a digital format using the TI-Nspire Teacher Software PublishView™ feature. The AMAA digital version contains all the Student Pages, Teacher Notes, and Annotated Student Pages as well as interactive TI-Nspire documents to use with the whole class or, if available, with TI-Nspire handhelds, or the TI-Nspire app. Current events, assessment materials, and exercises can only be found in the digital version of the teacher materials.

All of the TI-Nspire documents are linked, making it easy to use them for planning and during instruction. It is important to open the AMAA Publish View READ ME.tnsp document first to learn how to navigate using PublishView™. After reading through this document, close it and begin with the document AMAA BEGIN HERE.tnsp to start navigating through the curriculum.

The TI-Nspire documents found in the digital version of AMAA are an important component of the AMAA curriculum. These electronic files contain TI-Nspire documents and web links that are integral to developing student understanding of algebraic concepts found in a number of the AMAA lessons and projects. For example, for students who may not readily see the connections among the data they gather, a table organizing that data, and the associated equation, the TI-Nspire documents enable the class to toggle back and forth between representations and trace components of them that help to highlight the connections. In some lessons, the mathematics can only be fully developed by using the TI-Nspire documents. Therefore, the digital version of the files is an important component of the AMAA curriculum and should be used in conjunction with the printed AMAA materials.

## Assessing and Evaluating Students' Work

Changes in teaching approaches, or in what is taught, require changes in student assessment and evaluation. Students should have multiple opportunities to demonstrate their growth and understanding by working through a variety of tasks, which can be used to monitor their progress either formatively or summatively. A bank of problems is available for this purpose in the digital materials for Units 0–2, including information on the relevant content and suggestions for complete responses for each problem. For Units 3–5, projects are provided from which students' mathematical thinking and use of modeling can be assessed. Students should work on these projects both inside and outside the classroom over an extended period of time.

These types of assessments call for changes in how we evaluate student work. Consider these possibilities.

One important technique is scoring with a rubric. Rubrics focus on students' thinking and processes rather than only on answers. Charles, Lester, and O'Daffer presented a focused holistic scoring point system in their 1987 book *How to Evaluate Progress in Problem Solving*. This type of rubric focuses on the total solution, evaluating work with reference to criteria established for scoring quality levels from 0 to 4.

An example of a focused holistic scoring scale follows. This rubric can be applied to any task you want to evaluate, or it can apply to the whole investigation.

## Sample Rubric

| | |
|---|---|
| **Level 4** | The student<br>• chooses appropriate strategies.<br>• carries out the strategies correctly.<br>• gives complete and correct answers.<br>• explains the solution methods in words and uses mathematical symbols appropriately. |
| **Level 3** | The student<br>• chooses correct solution methods but does not carry them out correctly.<br>OR<br>• gives correct answers but the solution method is flawed.<br>• explains the solution method in words and uses mathematical symbols appropriately. |
| **Level 2** | The student<br>• gives correct answers but no work is shown and no verbal explanation is given.<br>OR<br>• gives incorrect answers.<br>• chooses a correct strategy but does not carry it out correctly or complete the solution. |
| **Level 1** | The student<br>• starts a solution method but does not complete the investigation.<br>• gives incorrect answers but explains the solution method. |
| **Level 0** | The student<br>• has a blank paper.<br>• copies the problem but does not solve it.<br>• gives incorrect answers with no work shown or explanation given. |

**Investigation reports.** Many of the investigations specify that students are to write a report of their findings and make a recommendation about the problem situation. Some teachers find it helpful to have students use the headings from the modeling cycle in the High School Modeling standards of the Common Core State Standards to organize their reports. These reports can be scored easily by using a rubric. The general criteria in the rubric above can be adapted slightly to fit the writing task.

Although grammar and punctuation are not usually scored in evaluating students' writing, it is important that students communicate their ideas in ways that are understandable. Because of the focus on modeling and representations, students are likely to use drawings, tables, charts, or other means to convey their ideas. Encourage them to be conscious of making clear what they mean.

Some teachers involve students in setting up the criteria used in the report rubric. Early in the year, after the first writing assignment, you can pull excerpts from papers to illustrate the rubric's criteria. Students rarely write lengthy responses in their first exposure to writing in a mathematics class. To help them learn to write, show comparisons between high- and low-quality responses (anonymously, of course). Students should also have copies of the rubric for reference.

## A further word about AMAA and the CCSSM

In *A Modeling Approach to Algebra*, mathematical modeling and the modeling cycle are basic to productive investigation experiences. Modeling serves as the framework for this curriculum through the CCSSM standards it addresses, the investigations that lend themselves to modeling, a student-centered instructional approach, use of technology and other tools, and the approach to assessing student work.

### The Standards for Mathematical Practice

The curriculum overview (see page iii) highlights the emphasis on the Standards for Mathematical Practice (SMP) throughout the investigations. The MP notations in the Teacher Notes of Units 0 and 1 illustrate how we have thought about the SMP in creating the student and teacher materials. Yet there is not just one way to think about students' opportunities for engaging in the SMP as they work on the investigations and discuss their findings. We encourage you to continue thinking about these opportunities as you plan daily.

### Use of the Modeling Cycle

The modeling cycle (see further discussion on page 2) helps to identify opportunities for students to engage in mathematical practices as they work through the investigations. When formulating their initial steps in investigating problem solutions, students must analyze the conditions of the context, determine how elements are related, and identify what the investigation is seeking to discover (MP1). They make assumptions and conjectures (MP3), decide on how best to represent the problem (MP2), and ask themselves whether their strategies make sense when interpreting results (MP1). Finally, in reporting on findings from the investigation, students communicate what they have learned, using mathematical terms clearly and appropriately (MP5) to explain and justify the validity of their findings (MP3).

**Problem-initiated study of mathematics**

You will notice that there are several different types of investigations and problems in AMAA. Some of the investigations provide opportunities for students to discern patterns and structure (MP7). For example, in Lesson 1.4 A New Look at Equations, students take a broader view of expressions and equations to recognize groups of terms within an equation as a single object. In Lesson 5.7 L-Shaped Patios, they explore the structure of the difference of squares through the context of tiling a patio area. Some investigations yield data from which patterns emerge leading to formula models. Lesson 1.3 Toothpick Polygons, Lesson 2.9 Painted Cubes, and Lesson 4.2 Quilting Patterns are examples of such investigations. With other problems, particularly those in which the contexts yield "messy" data, such as in Lesson 3.3 Open the Elevators or Lesson 3.11 Bungee Jumping, students may need to "interpret their mathematical results in the context of the situation and reflect on whether the results make sense" (CCSSM, page 7) (MP4). All of the problems can be solved through multiple approaches. This gives students opportunities to justify their approach (MP3) and consider connections among the various approaches used (MP1).

**Emphasis on communication**

The emphasis on communication (see pages 1–2) addresses multiple forms, including speaking, writing, representing, and listening. During class discussions, students have the opportunity to construct arguments for their approach to a problem, justify their results, and question other students (MP3). The Annotated Student Pages suggest questions to prompt these discussions, and you may add some of your own to the respective PublishView™ documents. Students are also expected to write reports on the investigations they conduct. As mentioned above, the modeling cycle may serve as a guide for students to organize their reports. The reports represent a student's progress through the investigation, including assumptions and conjectures made (MP3, MP4), how terms were defined and used (MP6), tools used (MP5), and conclusions justified (MP3).

**Infusion of technology**

Much of AMAA is built around modeling tasks that can be dynamically explored with spreadsheets, graphs, or specific links to Internet explorations (MP4), as in Lesson 0.6 The Path of a Billiard Ball. The technology can broaden students' opportunities to engage in the MPs by giving them access to visual representations and making conjectures in activities such as Lessons 4.3 Exploring Exponential versus Power and 4.8 The Shape of a Bottle (MP5). Students explore consequences and compare predictions with data in Lessons 3.8 Bouncing Ball and 3.11 Bungee Jumping (MP5). Students also use the technology to visually explain correspondences among equations, tables, and graphs to make sense of the problems such as in Lessons 2.8 Waiting for Rock Concert Tickets and 2.11 Going Viral (MP1).

Technology also provides a tool to help students justify their conclusions, communicate them to others, and respond to the arguments of others such as in Lesson 5.6 Things That Go Round (MP3, MP5). As students work with symbolic representations they use technology to manipulate the representing symbols (MP2) to calculate accurately and efficiently as in Lesson 5.9 The Changing Value of Money (MP6). Technology can help students look for and express regularity in repeated reasoning as in Lesson 2.7 Grains of Rice (MP8) and discern a pattern or structure as in Lesson 2.4 Rods and Spools (MP7).

### Content Standards Related to Modeling

The tables that follow represent connections between the mathematics in AMAA Units 1–5 and the high school content standards designated for modeling. Students who complete each unit will experience the mathematics described by the respective set of standards.

### Common Core State Standards for Mathematics in AMAA

| | Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 |
|---|---|---|---|---|---|
| N.Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | x | x | x | | |
| N.Q.2: Define appropriate quantities for the purpose of descriptive modeling. | x | | x | x | x |
| N.Q.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | x | | x | x | x |
| A.SSE.1: Interpret expressions that represent a quantity in terms of its context. | x | x | | x | x |
| A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.<br>a. Factor a quadratic expression to reveal the zeros of the function it defines.<br>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.<br>c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. | | | | x | x |
| A.CED.1: Create equations and inequalities in one variable and use them to solve problems. | x | x | | x | x |

| | | | | | |
|---|---|---|---|---|---|
| A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | x | x | x | x | x |
| A.CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | | x | x | | x |
| A.CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$.* | x | | | | |
| A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | x | | x | x | |
| F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | x | x | x | x | x |
| F.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.* | x | x | x | x | x |
| F.IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.<br>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.<br>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | x | | | x |
| F.BF.1: Write a function that describes a relationship between two quantities.<br>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.<br>b. Combine standard function types using arithmetic operations. | | x | x | x | x |

| | | | | | |
|---|---|---|---|---|---|
| F.BF.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | | x | x | | |
| F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.<br>a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.<br>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.<br>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | x | x | x | |
| F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | | x | x | x |
| F.LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | x | | x | |
| F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context. | x | x | x | | x |
| G.MG.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | | | x | x |
| G.MG.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | | | | x | x |
| S.ID.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related<br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.<br>b. Informally assess the fit of a function by plotting and analyzing residuals.<br>c. Fit a linear function for a scatter plot that suggests a linear association. | x | | x | x | |
| S.ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | | x | x | |

# Unit 1
# Relationships Between Quantities and Reasoning with Equations

Suggested timing: 2 days

## Content and objectives

Students create equations to explain the relationship between the number of squares/hexagons and the number of toothpicks in a constructed arrangement. The task begins at a level that is highly accessible and progresses to encourage students to develop a generalized rule for solving the problem for larger quantities. The different ways of describing the same arrangement of toothpicks should lead to a discussion of equivalent expressions.

## Opportunity to model with mathematics

Students are encouraged to begin by using concrete items, move to a diagrammatic representation, and then verbalize a rule. The kinesthetic experience of building the squares with toothpicks may provide students with insight into patterns of the arrangement. This progression focuses on developing flexibility between the forms and provides greater meaning to the algebraic equations they create.

It is important for students to share how they determined the number of toothpicks by relating their thinking to the physical patterns they see in the toothpick arrangement. The models students describe should match the way in which they thought about the toothpick arrangement.

A possible extension is to generalize the experience with squares and hexagons to any line of regular polygons. Students could find the formula for the number of toothpicks needed for a chain of polygons with $n$ sides and answer a question such as, "If you have 73 toothpicks, what regular polygon would make a chain that uses them up exactly?" Similarly, students could use the model to critically consider the question, "If you have 80 toothpicks, what regular polygon could make a chain that uses them up exactly?" Grappling with these types of questions on finding patterns leads to the highest level of generalization.

## Insights to student thinking/possible responses

Although one can easily count the number of toothpicks in the diagram one-by-one, students are asked to find other strategies. Students might describe these possible models for the chain of squares:

- The top and bottom rows each have 5 toothpicks and the middle has 6 or (5 + 1).
- There is 1 vertical toothpick to start, then 5 squares are formed with backward C shapes, each built with 3 toothpicks.

See the Annotated Student Page for models representing different strategies for counting. The examples are considered models because they include sketches with loops strategically placed to indicate connections among the sketch, the verbal description, and the expression.

Students can carry out the counting task later to validate the model they created. Students may need to discuss what is meant by *exactly.*

## Materials

toothpicks
paper on which line segments can be drawn to model the toothpicks

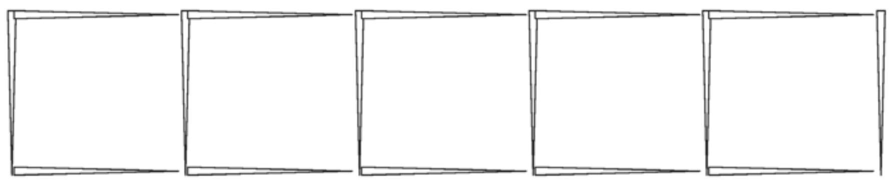
## Technology

On page 1.2 in the Toothpick Polygons.tns document, students can enter the number of toothpicks for each square being built. On page 1.3, a graph of the relationship can be shown. If students predict how to determine the number of toothpicks for any number of squares with an equation, they can use Menu > Analyze > Plot Function, then enter the equation. They can then observe whether the graph of the equation goes through all points.

Pages 2.1 and 2.2 can be used for exploring the number of toothpicks in the hexagon problem. Pages 3.1 and 3.2 can be used for exploring the problem dealing with the perimeter of a row of hexagons.

Without counting, how many toothpicks are in the arrangement of polygons?

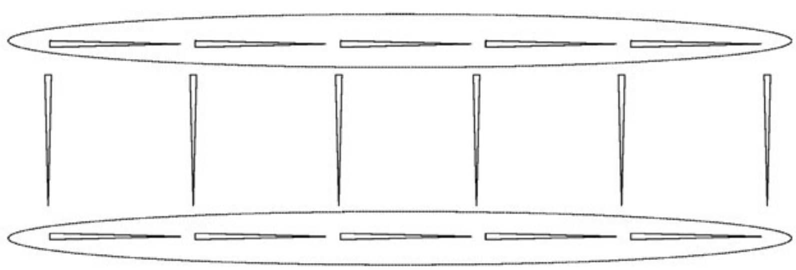
1. a. Without counting toothpicks one-by-one, find a way to determine how many toothpicks are needed to make this arrangement of 5 squares.



   b. Draw a sketch that models how you determined your answer.

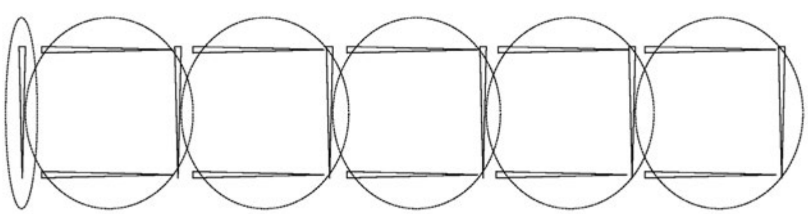
   c. Find other ways to determine the number of toothpicks needed. Describe your methods and model them with diagrams. How are the various models similar or different?

**Have students share their sketches. These are examples of models students may share:**



5 + 5 + (5 + 1)

**5 + 5 + (5 + 1), because 5 across the top, 5 across the bottom, vertical has 1 more**



1 + 5 • 3

**1 + 5 • 3, because 1 toothpick to start and 3 toothpicks form each square 5 times**

**Solicit several different ways of counting from the class.**

**What is the same about all of these ways of counting?**

2. a. Without actually building it, describe a method to determine how many toothpicks you would need to build a row of 16 squares.

   b. Ashley has built a row of squares on her desk. She can tell you how many squares there are, but she did not keep track of how many toothpicks she used. Write directions for figuring out how many toothpicks she used.

**As the number of squares increases by 1, what happens to the total number of toothpicks?**

**Use the Toothpick Polygons.tns document and ask students how the increase of 3 is shown on the graph on page 1.3.**

**How does this graph compare to the graph from Trapezoid Trains? Paper Stacks? Why does this pattern of increase tell us that the relationship will be linear?**

**Students may generate the equation $y = 3x + 1$.**

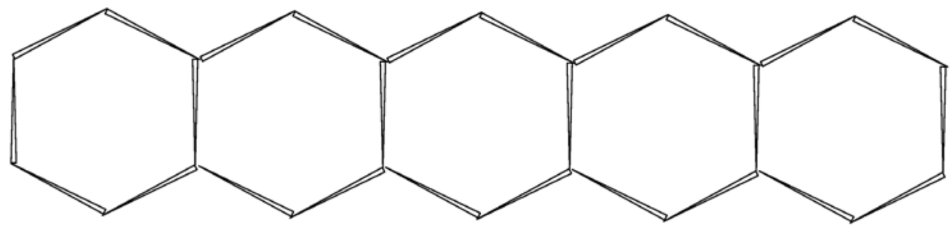
**What do the 3 and 1 in the equation have to do with the toothpick arrangement?**

3. a. Make up a new problem by filling in the blanks below. Then solve your problem. ____________________________has __________toothpicks. What is the largest number of squares in a row that could be built with this number of toothpicks? Justify your answer.

   b. Is it possible to use exactly 420 toothpicks to make a row of squares in this fashion? Explain your answer.

**Here is an opportunity to initiate discussion on domain (number of squares) and range (number of toothpicks based on the corresponding number of squares).**

4. Nikki decided to use toothpicks to make a configuration of hexagons in a row.



   a. Without counting, find a way to determine how many toothpicks are needed to make this arrangement of 5 hexagons.

   b. Draw a sketch that models how you determined your answer.

   c. Find other ways to determine the number of toothpicks needed. Describe your methods and model them with diagrams. How are the various models similar or different?

**How is this pattern different from the pattern of squares?**

   d. Without actually building it, describe a method to determine how many toothpicks you would need to build a row of 16 hexagons.

   e. Triste has built a row of hexagons on her desk. She can tell you how many hexagons there are, but she does not know how many toothpicks she used. Write directions for figuring out how many toothpicks she used.

   f. Evelyn has 820 toothpicks. What is the largest number of hexagons in a row that could be built with this number of toothpicks? Justify your answer.

   g. Is it possible to use exactly 420 toothpicks to make a row of hexagons in this fashion? Explain your answer.

5. a. What is the perimeter of the row of 5 hexagons?

   b. How would you find the perimeter of *any* row of hexagons?

**Pages 2.1 and 2.2 on the Toothpick Polygons.tns document can be used for exploring the number of toothpicks in the hexagon problem. Pages 3.1 and 3.2 can be used for exploring the problem dealing with the perimeter of a row of hexagons.**

# Unit 2
# Linear and Non-Linear Relationships

Suggested timing: 2 days

## Content and objectives

In this investigation, students build on previous experiences with sequences to explore growth patterns that are neither linear nor exponential. The work students do in this investigation involves patterns they have encountered earlier: square and triangular numbers, the latter in Lesson 0.3 Skeleton Tower. Besides providing students the opportunity to understand the structures and to develop equations to model their construction, the investigation sets the stage for later work with quadratic relationships.

In each sequence, there are two functions to consider, the function for the rods and the function for the spools. Students will be able to consider the sequences both recursively and explicitly and reason flexibly depending on the problem posed. For example, when given the number of rods (or spools) students should be able to find the stage number of an arrangement or know if an arrangement could have a particular number of rods or spools.

## Opportunities to model with mathematics

In looking at the sequences of spools and rods for each of the triangular and square arrangements, students might see numerous relationships. For example, they might see a pattern in how the values in the sequence increase. While there are questions that ask about finding the number of rods or spools for any stage number of arrangements, the solutions students obtain should come from looking at patterns and relationships between numbers. Entering the data on the spreadsheet of the Rods and Spools.tns document is a means of creating a model to study a pattern.

For example, the number of the spools in the arrangements in Problem 2 {1, 4, 9, 16, 25, …} should lead students to recognize square numbers. In addition, the number of rods {0, 4, 10, 18, 28…} might be seen as adding $n - 2$ to the corresponding number of spool values for each respective $n$ arrangement. Hence, the number of rods for any value of $n$ is $n^2 + (n - 2)$. Interestingly, a similar relationship exists between the number of rods and spools for the triangular arrangement. That is, if you add $n - 2$ to the number of spools you will get the number of rods.

Students will have further experiences with patterns that explore quadratic relationships in later units.

## Insights to student thinking/possible responses

Students most likely will be able to continue building the stages of arrangements using manipulatives such as toothpicks and modeling clay or by drawing diagrams. Some students will benefit from describing the pattern of growth recursively before they attempt to find an equation to model it. For students who are challenged with problems asking them to use an inverse function, looking back at Lesson 1.9 Formulas for Success helps to build a connection to a previous experience. Students may need to revisit the concept of square root to solve some of the problems.

Students may first see the number of rods and spools as a recursive pattern. The pattern for the spools in the triangular arrangement is the sum of consecutive integers sequence that students saw in Lesson 0.3 Skeleton Tower. This may provide another model for students to develop a relationship between the stage number and the number of spools. However, it is not necessary for students to come up with this equation.

## Materials

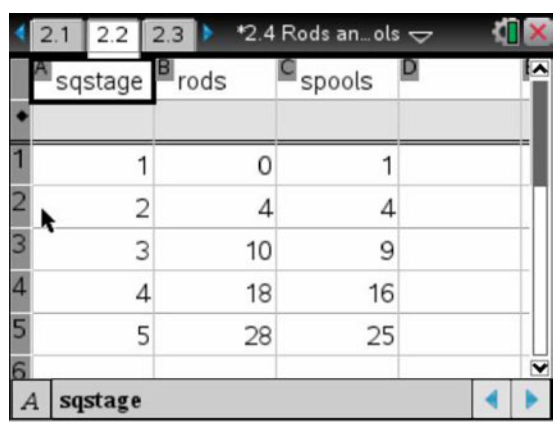
simple manipulatives to represent rods and spools such as toothpicks or straws
clay

## Technology

The Rods and Spools.tns document contains a spreadsheet and associated graphs for both the triangular and square arrangements.

The data for spools and rods for the triangular arrangements can be entered on page 1.2. On page 1.3 a graph of the data on page 1.2 is provided. After entering the data for rods and spools from the investigation of the triangular arrangements, students can be asked to discuss patterns they notice both in the table and in the graphs.

On page 2.2 the data for rods and spools for the square arrangements can be entered, and on page 2.3 the data can be graphed. Similar questions can be posed when investigating the square arrangements. For example, the pair of graphs in the triangular arrangement looks similar to the pair of graphs in the square arrangement.




| | A sqstage | B rods | C spools | D |
|---|---|---|---|---|
| 1 | 1 | 0 | 1 | |
| 2 | 2 | 4 | 4 | |
| 3 | 3 | 10 | 9 | |
| 4 | 4 | 18 | 16 | |
| 5 | 5 | 28 | 25 | |

If an equation is determined, use Menu > Analyze > Plot Function, and enter the equation to see if it goes through the points of the graph.

If a regression equation was desired for any of the patterns, each would be quadratic. However, if students suggest exponential or linear, it is worth testing to see how closely the regression graph matches the points from the data and in predicting additional values. Regression equations can be found using Menu > Analyze > Regression and then choosing the type of regression desired.

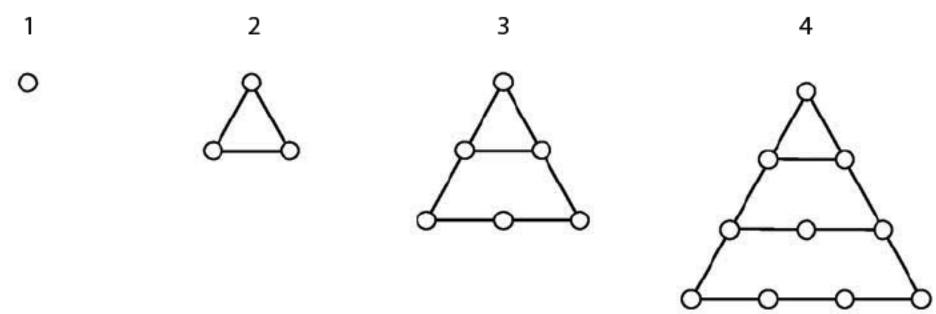
# LESSON 2.4 *Rods and Spools, Annotated Student Page*

How many rods and spools are there in different stages of the pattern?

Theo and his sister liked to make structures with their Rods and Spools connector kit.

Theo made a triangular structure with the rods and spools and his sister made a square structure.

1. Theo built up his triangular rods and spools structure in stages. Here are his first 4 stages.




   a. Sketch the fifth and sixth triangular rod and spool stages.

   b. How many rods and how many spools are in the triangular structure at each stage? What patterns do you see?

**How did you decide what to draw for the fifth and sixth rods and spools stages of Theo's structures? What patterns did you find in the numbers of spools and numbers of rods for each stage structure? How did you represent the patterns you found?**

**This should be the focus of the discussion. If no one suggests a table, prompt students to create one for their data and discuss the advantages of representing the data in this manner.**

   c. How could you find the number of rods and the number of spools in any stage of the triangular structure?

**Direct students to apply their method to finding the number of rods and spools in the tenth triangular stage and explain or verify why their answers are correct.**

**Use the Rods and Spools.tns document to explore both the spreadsheet and associated graphs for the rods and spools arrangements. On page 1.2 the data for spools and rods for the triangular arrangements can be entered. On page 1.3 a graph of the data on page 1.2 is provided.**

**The Rods and Spools.tns document can also be used to generate an equation for these patterns by either entering a student-generated equation using Menu > Analyze > Plot Function to see if it goes through the points of the graph or generating a regression equation with Menu > Analyze > Regression and then choosing quadratic.**

**However, if students suggest exponential or linear, it is worth testing to see how closely the regression graph matches the points from the data and how closely it predicts additional values.**

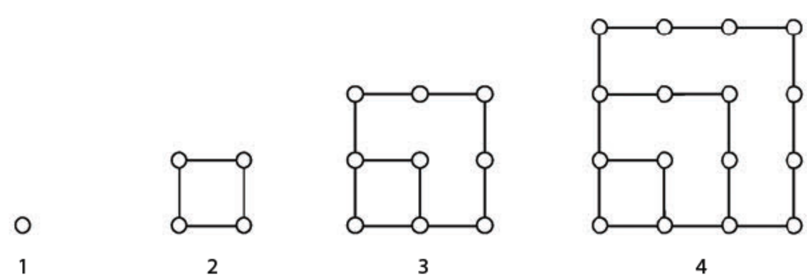
**How are the graphs related to the equations of the patterns of rods and spools?**

**Use this opportunity to highlight the idea that the regression equation or student-generated equation is the graph of a continuous function. The graphs of the data in the original problem are only points as the data are discrete as defined by the context. This provides an opportunity to start exploring and continue development of the idea of domain and range.**

**If *function* was not defined or discussed with students in Lesson 2.2 Number Sequences or earlier this would be a good opportunity to do so. A function is defined as a relation given by a set of ordered pairs ($x$, $y$) for which each value $x$ is associated with exactly one value of $y$. As students find a generalized formula or rule that describes how to determine the value $y$ for a selected value of $x$, they should know what they are describing is a function.**

**If you know the numbers of rods and spools in a particular stage, how could you find what number the stage is?**
**Theo remembered he had used 36 spools and 42 rods to build one of the structures, but he could not remember which stage it was.**
**How could Theo figure out which number stage this structure was?**
**What information is needed in order to determine which stage it was?**

2. Theo's sister also built her square structure in stages. Here are diagrams of her first 4 stages.




a. Sketch the fifth and sixth square rod and spool stages.

b. How many rods and how many spools are in the square structure at each stage? What patterns do you see?

**How did you decide what to draw for the fifth and sixth rod and spool stages of the series of square structures?**
**What patterns did you find in the numbers of spools and numbers of rods for each stage structure?**
**How did you represent the patterns you found?**

**If no one suggests a table, prompt students to create one for their data and discuss the advantages of representing the data in this manner.**

c. How could you find the number of rods and the number of spools in any stage of the square structure?

**Have a similar discussion regarding function related to the square structure as suggested in Problem 1.c.**

**Use the <u>Rods and Spools.tns</u> document to enter the data for spools and rods for the square arrangements on page 2.2, and on page 2.3 the data can be graphed. As with the triangular arrangement, a student-generated equation can be entered using Menu > Analyze > Plot Function to see if it goes through the points of the graph, or a regression equation can be found by using Menu > Analyze > Regression and then choosing quadratic. As with the triangular arrangement, if students suggest exponential or linear, it is worth testing to see how closely the regression graph matches the points from the data and how closely it predicts additional values.**

## Extension

**Theo's sister had 64 spools. What is the largest stage square structure she can make? How many rods would she need for that structure?**
**Theo decided he wanted to make square structures. He made a stage 4 square structure.**
**Can his sister take it apart and using exactly the same number of rods and spools, make a triangular structure of some stage?**

# Unit 3
# Data and Decision Making

# Unit Project *Leaky Faucets, Teacher Notes*

Suggested timing: 15 days (includes in- and out-of-class time)

**Introduction of Project: Following Lesson 3.7**
**1st progress check: Following Lesson 3.9**
**2nd progress check: Following Lesson 3.11**

## Content and objectives

This group project gives students an opportunity to apply what they have learned thus far in this course. It is recommended that students have 15 days outside of class to work on the project, as well as in-class days for progress checks.

The premise of this investigation is leaky faucets, but students may think of other interesting examples of water waste as well. Whatever the situation, they must identify sources of data and determine relevance to the context chosen. They may use proportions and estimation in their initial work, but depending on the direction taken by the group, other concepts will surface.

## Opportunity to model with mathematics

Although this project is open-ended, students are expected to create a model(s) to describe different aspects of the problem using data they collect. They may create an experience to simulate their water-wasting problem. They may decide to use a regression model and explain their reasoning for selecting that model, as well as analyze residuals to support the implications of their findings.

The basic modeling cycle (CCSSM, p.72) of Formulate, Compute, Interpret, Validate, and Report will be helpful as students work on this modeling project.

## Insights to student thinking/possible responses

Leaky Faucets may initially seem an overwhelming task for the students due to the complexities that the students may foresee. It is important to provide students with opportunities to discuss the elements that they need to consider in completing this project.

The modeling cycle will be useful in planning the progress checks. When introduced to this investigation, students should focus on the Formulate stage of the modeling cycle. The other components of the cycle can be used to focus project work during the two suggested progress checks. Progress checks may not take an entire class period. They should provide students the opportunity to reflect and discuss their progress and next steps.

Assessment criteria are included in the assessment resources for *A Modeling Approach to Algebra.*

## Materials

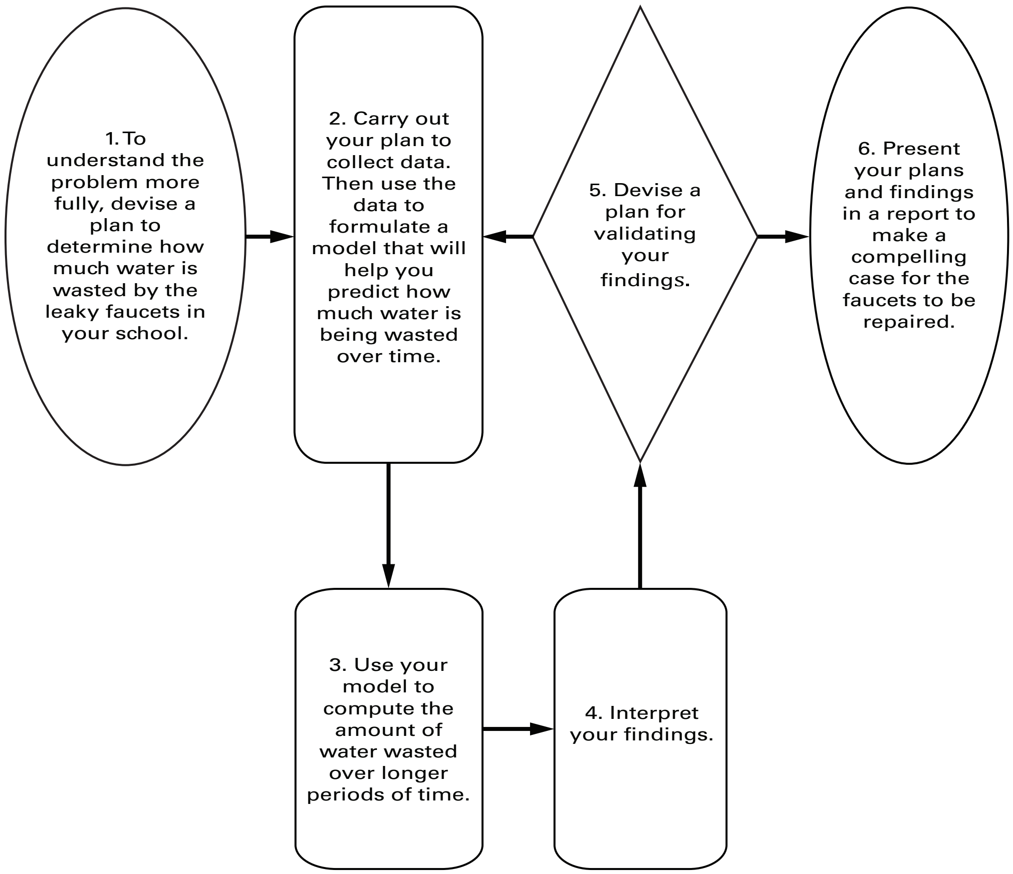
None

## Technology

Students should configure tables, spreadsheets, and/or graphs for optimal display of their data, including explanations of how and why technology supported their work.

# Unit Project *Leaky Faucets, Annotated Student Page*

How much water is wasted by leaky faucets in your school?




## Introduction of Project: Following Lesson 3.7

**Have students review the modeling cycle introduced at the beginning of the course. Discuss the components of the cycle.**
**Think about the prior lessons from this course.**

**What is an example of devising a plan? Formulating a model? Using a model to compute values for the dependent variable? Interpreting your findings? Validating your findings? Reporting your findings?**

**Following this review, have students discuss how the modeling cycle frames the investigation for this project.**
**Provide time for the groups to devise a plan for collecting data. Have them agree to a timeline to carry out their plan.**

What question are you trying to answer?
What assumptions are you making?
What are you defining as a "waste of water"?
What data do you need to collect?
How will you collect your data so that it is reliable?
What computations will be needed? And what will you learn from the results of the computations?
What other questions come to mind, and how might these affect your work on this project?
Are there other examples of wasting water you are concerned about?

If students identify interesting examples, you may decide not to limit the investigation to leaky faucets and have some of the groups work on other water-wasting events.

### 1st Progress Check: After Lesson 3.9

Students are expected to have collected preliminary data to do an initial analysis.

What data have you collected?
What analyses have you done with your data?
What trends do your data suggest?
In interpreting the results of your computations, do the numbers make sense in light of the original question?
How else might your data be represented?
What other data might be helpful to collect?

### 2nd Progress Check: After Lesson 3.11

Students are expected to have designed their model and used the model to make estimates of how much water is wasted over longer periods of time.

What models can you use to represent your data?
What mathematical generalizations can you make?
What predictions can you make based on your generalizations?
How confident are you of these predictions? Explain.
How are your results similar to or different from others'?
How will you use your findings to plan for your report?
What will you include in your report?

Set aside a day at the end of the project to have students share their reports.

# Unit 4
# Expressions and Equations

Suggested timing: 2 days

## Content and objectives

The problems in both parts of this investigation provide an opportunity to work with linear and quadratic expressions and equations. While no problem asks students to find perimeters or areas, the connection between the problems and geometric measurement provide a context for students to understand linear and quadratic relationships. Both Parts I and II build from specific cases to general cases where equations are used to model the problems.

## Opportunities to model with mathematics

There are opportunities to model these problems in multiple ways. The given diagrams are important to students' understanding of the problems and making progress in the investigations. Students can start by describing and recording the make-up of successive quilt patterns. The combination of the verbal and organized recording of how the elements of the quilt designs change should help students derive the expressions and equations that model the problem.

## Insights to student thinking/possible responses

For the problems in Part I, students will need to be clear about the area relationships of the quilt pieces. They will need time to explore how adding increments of 10 cm to the lengths of the sides of the overall design affects the number of pieces of each type. For example, when the lengths are doubled (from 20 cm to 40 cm) the area quadruples. Students may initially think all the numbers of quilt pieces are multiplied by four. Their sketches should help them correct this thinking.

The number of each type of piece in the patterns can be viewed several ways, and students may use different words to describe the same relationship. In Part I, when using a recording scheme as found on page 1.2 of the Quilting Pattern.tns document, some students may be able to describe the relationship recursively, such as, "If you add all the pieces (squares, big triangles, and small triangles) for one row, you get the number of squares for the next row." Other students will see that the number of whole squares on a design equals one number squared plus the square of one less than that number [$x^2 + (x - 1)^2$]. Careful questioning can help them realize that $x$ is determined by the number of 10 cm increments of length. Other students might rotate the design 45° and see that the rows of whole squares are increasing consecutive odd numbers up to a point and then become decreasing consecutive odd numbers. All of these ways of counting are related to the model.

Similarly, there are multiple ways to view the number of squares and triangle pieces in the designs in Part II. Taking the time to discuss the various ways students view the patterns will help them understand the equations that model the problem.

* Adapted from Mathematics Assessment Project http://map.mathshell.org/materials/index.php

## Materials

Dot paper may help students make their drawings. Tracing paper might be helpful also.

## Technology

The Quilting Pattern.tns document should be used for each part. On page 1.2, enter data from Part I related to the size of the square quilt (where every 10 cm along its size is 1 unit) and the numbers of small triangles, large triangles, and squares. On page 1.3, investigate the relationships among the variables by exploring graphs, which illustrate differences among quadratic, linear, and constant relationships. If desired, one can compute the regression equation for the number of squares and see how it matches with any equations suggested by the students from an examination of the figures.

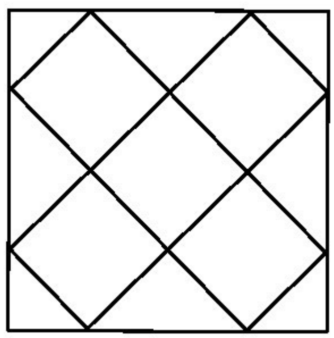
The Quilting Pattern.tns document should be used in a similar manner for Part II. On page 2.2, enter the data from Part II, including the size of the design and the number of individual triangles and squares. On page 2.3, investigate the relationships among the variables by exploring the graphs formed. If desired, one can compute the regression equation for the number of squares and see how it matches with any equations suggested by the students from an examination of the figures.

The Quilting Pattern.tns document should be sent to students' TI-Nspire handhelds if they have the devices.

What mathematical patterns can be found in quilts?

## Part I

1. This is a 20 cm by 20 cm design used in quilting. It is made up of whole square pieces, triangle pieces that are half of a whole square piece, and triangle pieces that are one-fourth of a whole square piece. As many whole squares as possible are used; the smaller triangle pieces are always used in the corners; and the larger triangle pieces are used along the edges. Congruent squares, congruent large triangles, and congruent small triangles are used no matter what size the quilt is.



   a. How many whole squares are used in the pattern?
   b. How many larger triangles are used in the pattern?
   c. How many smaller triangles are used in the pattern?

**Check that students understand what types of pieces are used in the design patterns. Students will most likely count the pieces.**

2. Michal wants to use pattern pieces in the same size of each type to create a similarly designed quilt 40 cm by 40 cm. Sketch the design.
   a. How many whole squares are used in the pattern?
   b. How many larger triangles are used in the pattern?
   c. How many smaller triangles are used in the pattern?

**Monitor as students sketch the 40 cm by 40 cm design. (Dot paper may help them with their sketches.) What methods are they using to count the pieces?**

**Have students describe how they went about completing the sketch and what they noticed along the way. Have several students share how they answered the questions.**

**How did you count the number of whole squares?**
**How did you count the number of large triangles?**
**How did you count the number of small triangles?**

**Rotating their sketch 45° might help some students see patterns more readily.**
**To help students work with Problem 3, it would be a good idea to give them time to explore how adding increments of 10 cm to each side length of the design is related to the numbers of squares, large triangles, and small triangles. If students need more structured experience here, suggest that they share the responsibility by investigating numbers of quilting pieces needed for one of the following side lengths: 10 cm, 30 cm, or 50 cm.**

**Students will need to use the data they have gathered to create a generalized model for the problem. Depending on your students' needs, you can also stop here and collect the data from the students' investigations. Using an organized representation—perhaps a table—should help them.**

**What would be a good way to organize the data from the different size designs?**
**How can we use that data to create a formula for counting the various types of pieces?**

3. If the design is only used to make a square quilt that is a multiple of 10 cm on a side,
   a. How many whole squares are used in this pattern?
   b. How many larger triangles are used in the pattern?
   c. How many smaller triangles are used in the pattern?

**There are many tools students can use to organize the data they gather to address this problem. The Quilting Pattern.tns document table with information on the size of the quilt (in terms of side length), and the numbers of squares, large triangles, and small triangles will provide the support many students may need. Give students time to make sense of what the values in the table represent before going on to other representations in the document.**

**Questions such as the following can help students make sense of the data:**

**Why does the number of small triangles remain the same?**
**As the side length increases by 10 cm, how does the number of large triangles change each time? Explain why.**

**The Quilting Pattern.tns document should be used for each part. On page 1.2, enter data from Part I related to the size of the square quilt (where every 10 cm along its side is 1 unit) and the numbers of small triangles, large triangles, and squares. On page 1.3, investigating relationships among the variables by graphing illustrates differences among quadratic, linear, and constant relationships. If desired, one can compute the regression equation for the number of squares and see how it matches with any equations suggested by the students from an examination of the figures.**

**The Quilting Pattern.tns document should be sent to students' TI-Nspire handhelds if they have the devices.**

**How did you use your experience with Problems 1 and 2 to solve Problem 3?**
**What model did you use to solve the problems? Describe how the model helped.**

**Students should discuss the differences among the relationships represented by the three parts of Problem 3.**

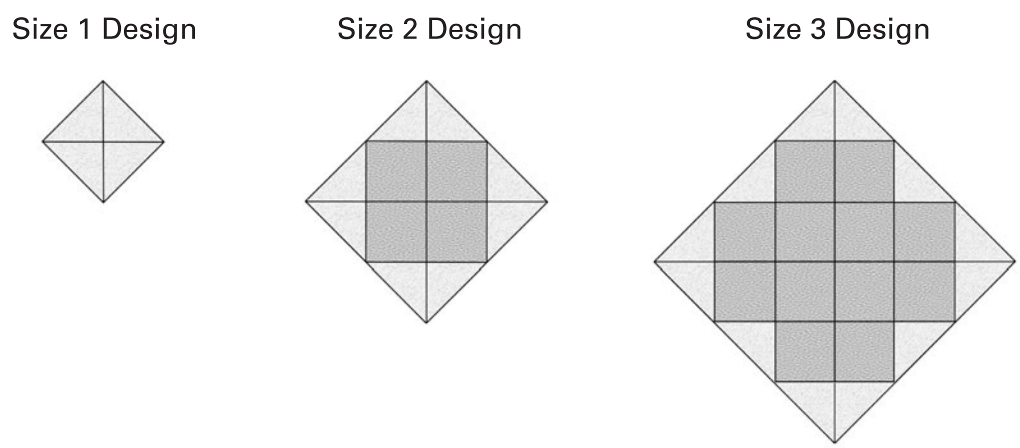
**If you used formulas, how many did you need to create?**
**Were any of the formulas easier to create than others?**
**How does the relationship represented by each formula connect to the diagram?**

**Suggest that students look back at the table for Lesson 2.9 Painted Cubes.**

**How does the number of small triangles relate to the data in the Painted Cubes table?**
**How does the number of larger triangles relate to the data in the Painted Cubes table?**

## Part II

1. A quilting store has a special design that can be used in a picture frame or wall hanging. In their display case they show the following three sizes but will make this design in larger sizes as well.




The triangles used to make the designs are all congruent and the squares used to make the designs are all congruent. The length of the design side is measured in units equal to the longest side of the right triangle. For example, the Size 3 design measures three units on a side.

a. How many triangle pieces and square pieces are needed for the Size 6 design?

b. How many triangle pieces and square pieces are needed for any size design?

**Here again, sketching the designs will help students see the patterns in the relationship between the type of piece and the dimensions of the overall design.**
**Discuss solutions to Problem 1 before students work on Problems 2 and 3.**

**How did you count the types of pieces in the designs as the dimensions increased?**
**What patterns did you notice?**
**How did that pattern help you solve Problem 1.b?**

**Monitor students as they work on Problems 2 and 3 so that you can strategically choose a variety of methods to share. Have students explain their methods for solving Problems 2 and 3.**

**Check to see if any student sees a connection between counting the squares and counting blocks in Lesson 0.3 Skeleton Tower problem. For example, it can be seen that the diagonals of the design partition the squares into four congruent sets, each resembling a "wing" of the Skeleton Tower.**

2. Lem was working at the quilting store and someone brought her 220 square pieces but did not tell her the size quilt she was to make. When Lem asks for triangle pieces, how many should she request?
3. Another day, Lem was given 23 triangle pieces and was to make the largest quilt of this special design possible without asking for more triangle pieces. How many square pieces does Lem need to make the design?

**Does Lem have any pieces left over? If yes, how many more pieces would she need to make the next largest design?**

**The <u>Quilting Pattern.tns</u> document should be used in a similar manner for Part II. On page 2.2, enter the data from Part II, including the size of the design and the numbers of individual triangles and squares. On page 2.3, investigate the relationships among the variables by exploring the graphs formed. If desired, one can compute the regression equation for the number of squares and see how it matches with any equations suggested by the students from an examination of the figures.**

# Unit 5
# Quadratic Functions and Modeling

## LESSON 5.4 *Area Maximized, Teacher Notes*

Suggested timing: 1 day

### Content and objectives

Area Maximized gives students another opportunity to explore and gather data related to creating a rectangle with maximum area. This is similar to the exploration involving volume in Lesson 2.13 Design-a-Box Contest. Students will graph the relationships between various quantities, both linear and non-linear, based on the data obtained. A discussion comparing, contrasting, and analyzing these representations should help the students develop understanding of maximum area, especially how it can be inferred from a graph. Also, students should see how a change in the width of the rectangle describes a covariate change in length. This can be seen in both a table and a graph.

### Opportunities to model with mathematics

After deciding on how to model the relationship to be explored, students should create a table to record data obtained from the various sizes of rectangles they generate. By plotting these points, students are able to create graphs and make statements about how the graphs help in solving the problems. If students limit their answers to whole numbers, have them grapple with ideas on what other values might be possible, given the non-specifity of the problem. Exploring the graph of the relationship between width (or length) and area should help with this discussion. Using the Area Maximized.tns document, students should model the relationships with equations and enter those equations to check if the equations match the data obtained.

### Insights to student thinking/possible responses

Students may think that all rectangles formed will have the same area because all are made using the 160 feet of fencing and the side of the auditorium. They may also think that a decrease (increase) of 10 feet on the width of the rectangle makes the increase (decrease) of the length also 10 feet. Students can compare the graphs of length versus area and width versus area.

Students may also have prior experiences with determining maximum area when one has a fixed amount of fencing to use for the entire perimeter. In that case, the solution is a square and students may think the answer to this problem might also be a square. If this comes up, you can ask students, “Why doesn’t forming a square apply in this situation?”

### Materials

Some students may benefit from having lengths of string available to experiment with a scaled version of the problem.

## Technology

The Area Maximized.tns document provides a vehicle for a classroom demonstration to enter and analyze data to investigate aspects of this situation. On page 1.2, data for length and width of the rectangle can be entered. Students may describe patterns in the data such as, “If 10 is added to the width of the rectangle, the length decreases by 20.” On page 1.3, the relationship between the length and width can be graphed. Students should be able to explain why the relationship is linear and how the equation represents that relationship. On page 1.4, students can graph the relationship between width and area of the rectangle. Students should note that for widths 0 and 80, there will be no area, and that there are two width values, such as 20 and 60, which give the same area values. Students should be able to determine the equation that matches the points plotted on page 1.4, but, if desired, the regression quadratic can also be found. They will probably correctly guess that the maximum value falls halfway between 0 and 80, or 40, and can determine the largest area for that width.

The Area Maximized.tns document should be sent to students’ TI-Nspire handhelds if they have the devices so they can enter their own data and do the investigations.

## LESSON 5.4 *Area Maximized, Annotated Student Page*

How do the dimensions of a rectangle affect its area?

As part of their sustainable agriculture project, the Math Club at Kohm High School petitioned the school administration to let them plant a garden. The Math Club was allowed to use the land beside the auditorium for their project and needed to build a fence for their garden. The club was given materials to build 160 feet of fence.

If they used the auditorium wall for one side of their garden, what size rectangle should they make to have the largest area?

**If *w* represents the width of the rectangle and is perpendicular to the auditorium wall, what is the largest value possible for *w*? Explain your reasoning.**
**How did you determine your solution? Did you consider whether the dimensions were whole numbers or not?**
**As the width increases by 5 feet, what changes happen to the length?**
**Why is the graph of (width, length) linear?**
**How would you describe the graph of the ordered pairs (width, area)?**

**The Area Maximized.tns document provides a vehicle for classroom demonstration to enter and analyze data to investigate aspects of this situation. On page 1.2, data for length and width of the rectangle can be entered. Students may describe patterns in the data such as, "If 10 is added to the width of the rectangle, the length decreases by 20." On page 1.3, the relationship between the length and width can be graphed. Students should be able to explain why the relationship is linear and how the equation represents that relationship. On page 1.4, students can graph the relationship between width and area of the rectangle. Students should note that for widths 0 and 80, there will be no area, and that there are two width values, such as 20 and 60, which give the same area values. Students should be able to determine the equation that matches the points plotted on page 1.4, but, if desired, the regression quadratic can also be found. They will probably correctly guess that the maximum value falls halfway between 0 and 80, or 40, and can determine the largest area for that width.**

**The Area Maximized.tns document should be sent to students' TI-Nspire handhelds if they have the devices so they can enter their own data and do the investigations.**

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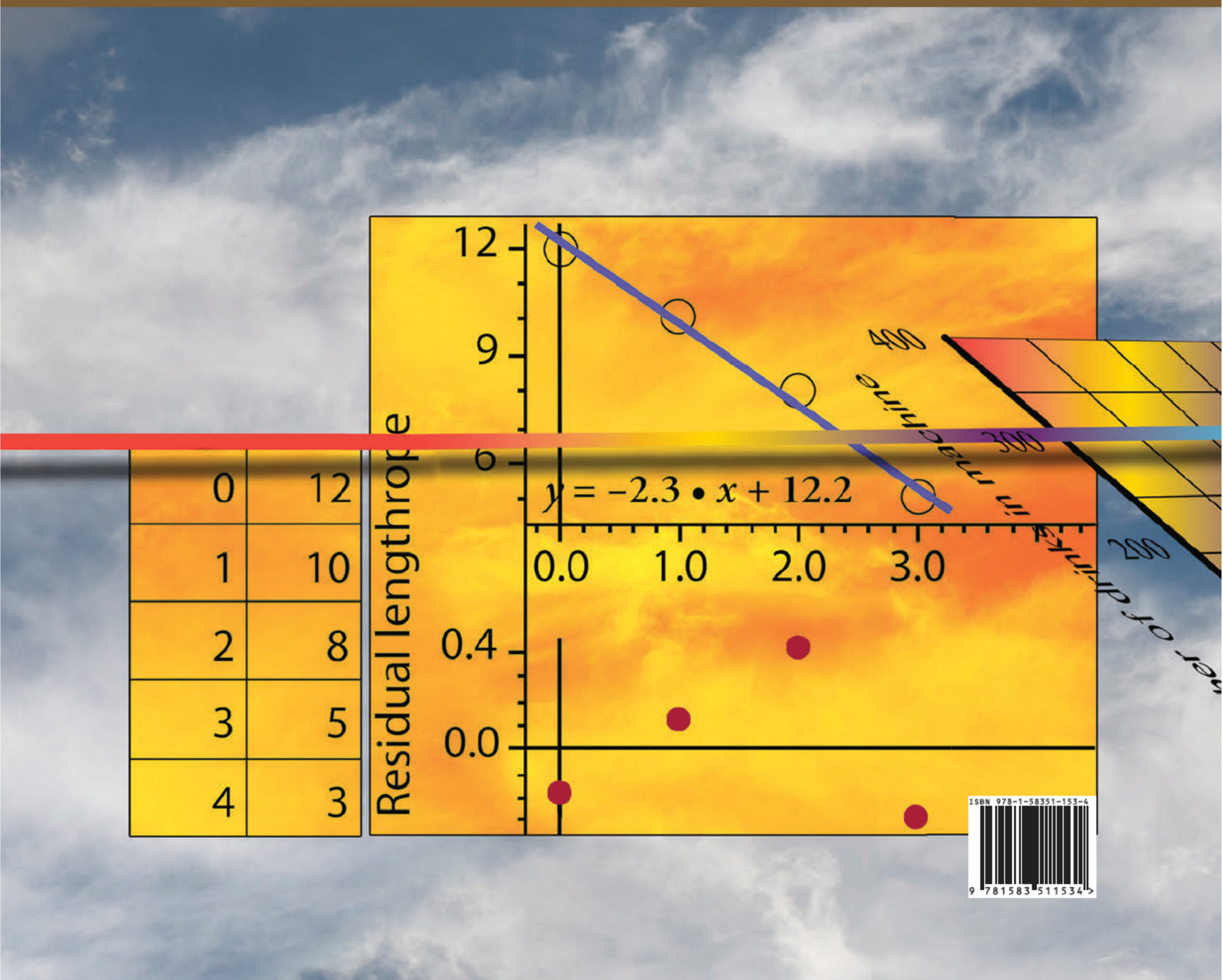


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