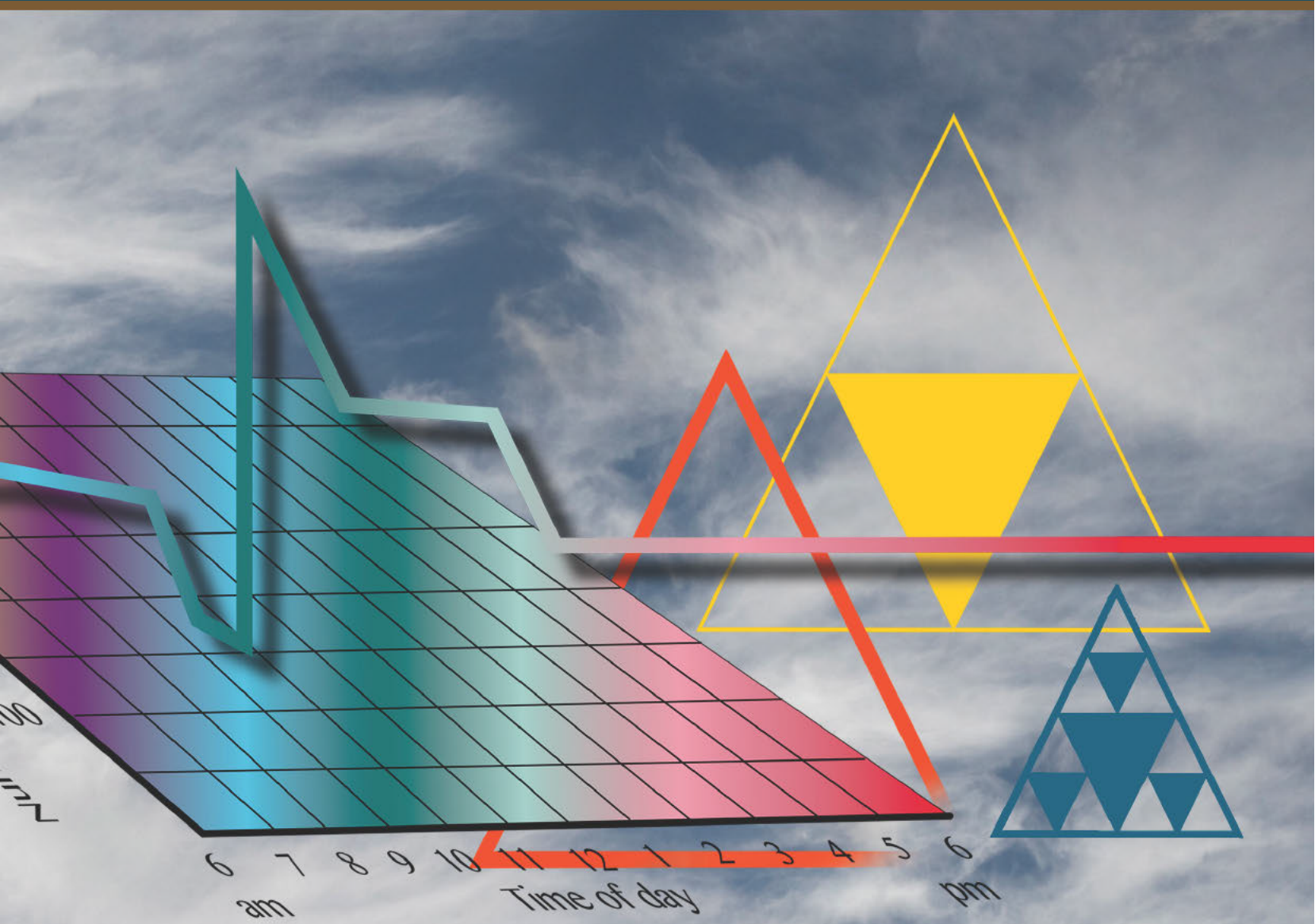
A Modeling Approach to Algebra

Second Edition



A Modeling Approach to Algebra

Second Edition

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Letter to Students

You are about to begin using a program for learning algebra called *A Modeling Approach to Algebra*. The program's objective is to support your development as a mathematically proficient student. An important focus of the program is for you to use modeling to solve problems for which you do not have ready solutions. You will be expected to apply what you know, become comfortable making assumptions, and try out ideas.

Mathematical modeling is a means by which you can simplify a problematic situation in order to study it and find solutions using a variety of representations. In this course, you will create models that allow you to represent many situations, solve problems, and apply new contexts. Along the way, you may have to decide if the model created is "the right one" or if it needs to be revised.

Besides solving problems, you will participate in mathematics activities in ways that may be different from what you have experienced before. You will share in the responsibility for your own learning by engaging in mathematical practices such as working collaboratively with other students to make sense of and solve problems, actively contributing to class discussions, using a variety of mathematical tools appropriately, and applying what you've learned from one problem to help you solve another.

A wise teacher once said, "Mathematics is not a spectator sport." These words are telling you not to hesitate to ask a question about something you don't understand, make a conjecture about possible results of an investigation, respectfully disagree with an idea that has been proposed, or try new methods for solving a problem.

And now it's time for you to get started. Enjoy the school year!

A Modeling Approach to Algebra

Unit 1
Relationships Between Quantities and Reasoning with Equations

LESSON 1.8 *Measuring Cups*

What volumes of water can you make?

In the 1995 blockbuster movie *Die Hard: With a Vengeance*, a maniacal criminal posed a measurement riddle. The challenge was for the hero, John McClane, to solve the riddle in order to disarm a bomb that was planted in a busy park before it exploded.

This is the riddle: You have only two jugs. The jugs hold 3 and 5 gallons, respectively. (There are no markings on the jugs.) With access to as much water as needed, make *exactly* 4 gallons. You are allowed three types of actions:

- Fill an empty container.
- Pour from 1 container into the other either until that container is full or until the container from which you are pouring is empty.
- Empty out a full container.

1. a. Can John McClane's problem be solved? If not, why? If so, how?

   b. Could the problem have been solved if the 2 jugs were used to make exactly 6 gallons? If not, why? If so, how?

   c. Could the problem have been solved if the two jugs were used to make exactly 1 gallon? If not, why? If so, how?

2. a. If you had 2 unmarked containers with capacities of 7 gallons and 11 gallons, respectively, can you make a quantity of exactly of 3 gallons?

   b. If you had unmarked containers with capacities of 7 gallons and 11 gallons, respectively, what quantities less than 11 gallons can you make? What quantities cannot be made?

3. For which of these pairs of containers can you make all quantities less than the capacity of the larger container?

   a. 3 and 7 c. 6 and 8 e. 6 and 15

   b. 3 and 8 d. 5 and 13 f. 11 and 17

4. Specify the capacities of 2 containers such that it is not possible to make some quantities less than the capacity of the larger container. How do you know?

Unit 2
Linear and Non-Linear Relationships

Which table and graph can represent a story?

Part I

1. Moe, Larry, and Curly all start on a trip at the same time and arrive at their destination at the same time.

   a. These tables describe their trips. Write a story to explain each trip based on the respective table.

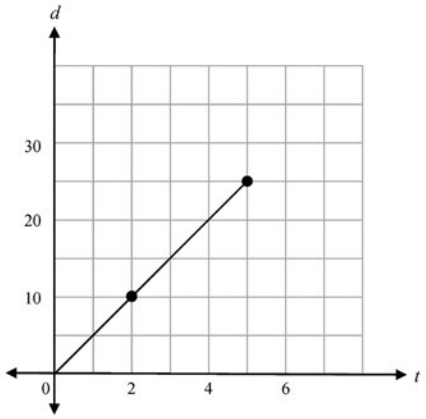
Moe's time (in hours)	Moe's distance from home (in miles)
0	0
1	5
2	10
3	15
4	20
5	25

Larry's time (in hours)	Larry's distance from home (in miles)
0	0
1	8
2	16
3	16
4	16
5	25

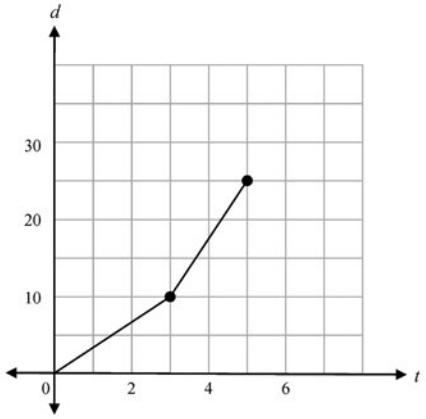
Curly's time (in hours)	Curly's distance from home (in miles)
0	0
1	3
2	6
3	9
4	17
5	25

b. Here are the graphs of the three trips. Match the tables with the graphs.

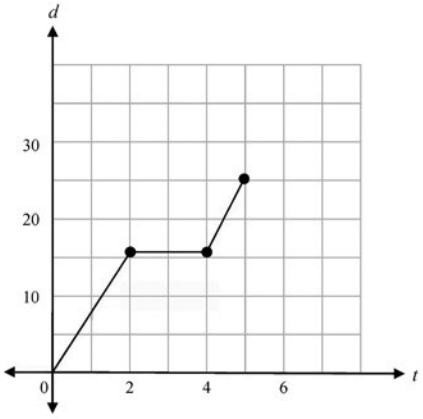
(A)



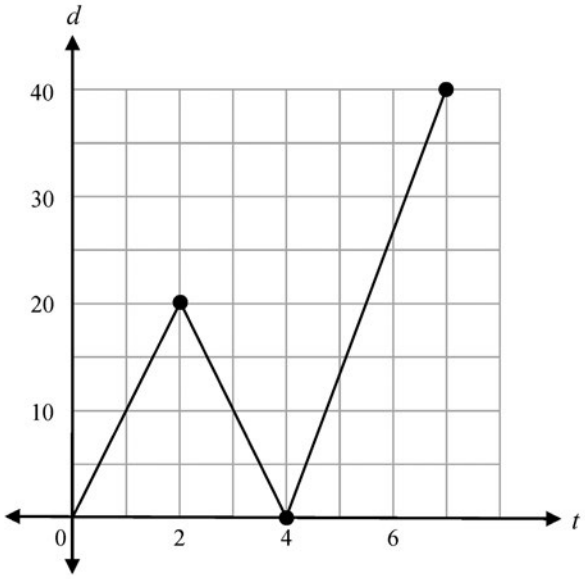
(B)



(C)



2. Here is a graph of Shemp's trip. Write a story to explain his trip.



Part II

Working with your group, match a story, a table, and a graph that represent the same relationship of elapsed time and volume of water. As you work on this task, think about the following:

1. How did you make your decision in matching each story and table?
2. How did you make your decision in matching each story and graph?

Unit 3
Data and Decision Making

Part I

Who is the Best Over Time?

In sports, athletes are ranked all the time. If you had to rank 2 players, how would you decide who was the better of the 2?

It is common for 2 players to be rivals and for their fans to discuss which of the players is "better." For 3 years in high school, 2 softball players, Kelli and Mari, were each considered superstars, mostly because of their hitting. Fans of each team tried to persuade the fans of the other team that their player was the better hitter. The table below gives the batting statistics for each player for the 3 years in high school. Who do you think is the better hitter and why?

	Year 1	Year 2	Year 3
Kelli	8 hits in 22 at bats	110 hits in 262 at bats	117 hits in 295 at bats
Mari	68 hits in 182 at bats	27 hits in 62 at bats	88 hits in 220 at bats

Part II

Critiquing the Critique

Did LeBron James fade in the fourth quarter of the game?

On June 12, 2012, the Miami Heat lost to the Oklahoma City Thunder in Game 1 of the National Basketball Association Finals. After the loss, LeBron James, one of the lead players on the Miami Heat, was criticized for letting the team down in the fourth quarter. ESPN tried to make the case in a post-game review by sharing statistics of the game. Below are the statistics reported by ESPN.

Not Finishing How He Started: LeBron James in Game 1

	First 3 Quarters	4th Quarter
Points	23	7
Field Goals	9 of 18	2 of 6
Free Throws	4 of 5	3 of 4
Rebounds	7	2

* Idea for this lesson came from Mr. Keith Ishihara. Source is "LeBron James faded in fourth quarter, ESPN graphic claims, in defiance of math", from http://sports.yahoo.com/blogs/nba-ball-dont-lie/lebron-james-faded-fourth-quarter-espn-graphic-claims 164638891--nba.html.

1. How convincing a case do you think ESPN made that James faded in the fourth quarter of game 1? Justify your thinking.

2. One piece of data not given by ESPN is that in the first 3 quarters James had 3 assists and in the fourth quarter he had 1 assist. How does this additional information affect your response to Problem 1?

Unit 4
Expressions and Equations

LESSON 4.9 *Family Stories**

What can you learn from your ethnic heritage using mathematical modeling?

Genealogical studies have always been popular with people from all over the world. Because the United States is a nation of immigrants, the stories of our ancestors and their journeys have always been a big part of the American story. Every family has stories that have been passed down from generation to generation. In modern times, scientific advances that show relationships and the accessibility of data made possible by the Internet have made tracing your genealogy easier, and for many, this has become a hobby that brings generations of a family together.

1. One pair of Lisa's great-great-great-grandparents were 100% Norwegian and none of her other ancestors were Norwegian. If no other descendant of Norwegian ethnicity enters Lisa's family, how many generations after Lisa will the first baby be born that will be less than 1% Norwegian? Use mathematical modeling to justify your response.

2. If no one else with Norwegian ethnicity enters Lisa's family, how many generations after Lisa will the first baby be born that is less than 0.1% Norwegian? Use mathematical modeling to justify your response.

* Adapted from School and University Partnership for Educational Renewal in Mathematics http://superm.math. hawaii.edu/sites/default/files/l

Unit 5
Quadratic Functions and Modeling

LESSON 5.11 *BINGO Revisited*

How do patterns involving squares compare?

Designs made with square grids are frequently found. We have seen examples in Lesson 0.2 BINGO Card, where the BINGO card was made up of a 5 by 5 grid, and Lesson 2.7 Grains of Rice, where the chessboard was made up of an 8 by 8 grid.

1. After your teacher shows you the beginning of a video dealing with stacking toilet paper rolls, answer the following:
   a. How many rolls of toilet paper are along each edge?
   b. The video says they are building a pyramid shape. How do you think toilet paper rolls will be used to make the shape?
   c. How many toilet paper rolls will be needed to build the shape?
2. How is the mathematics of stacking of toilet paper rolls like that of the BINGO Card lesson?
3. Compare squares on a chessboard with toilet paper rolls for a square pyramid that has 8 rolls along an edge.
   a. How many squares are there on a chessboard?
   b. How many toilet paper rolls would be used to build a pyramid with eight rolls of toilet paper used along each side of a square base?
4. Compare the number of rolls of toilet paper in a square pyramid with the number of squares in a square grid.
   a. How can you determine the number of toilet paper rolls needed to build a square pyramid with n rolls along each side of the square?
   b. How can you determine the number of squares on an n by n grid?
5. Beyond square pyramids.
   a. If you made a stack of toilet paper with a rectangular base that was 4 rolls wide and 10 rolls long, how many rolls of toilet paper would you use?
   b. If there is a 4 by 10 grid, how many squares would it have?

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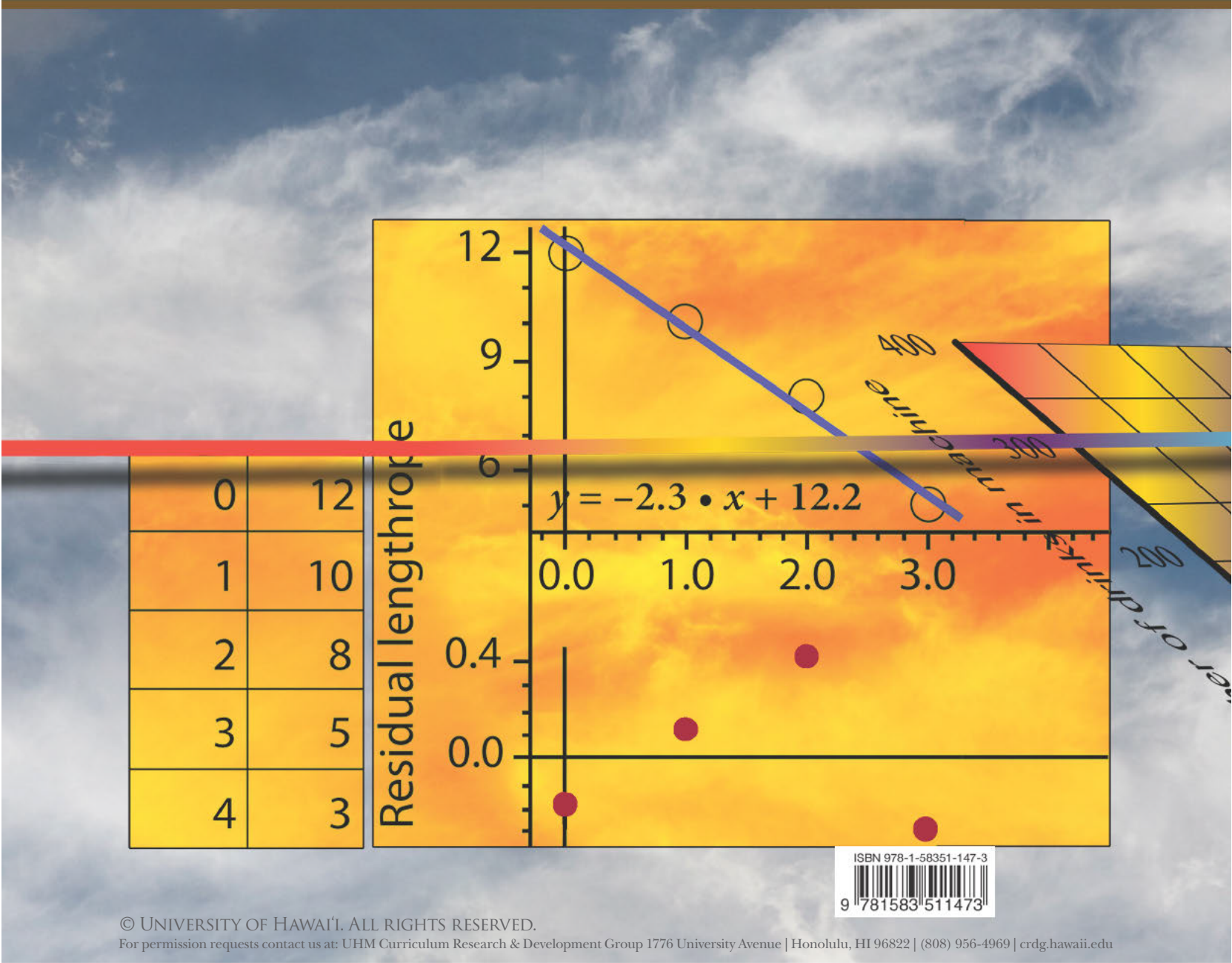

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