Reshaping Mathematics for Understanding

Getting Started

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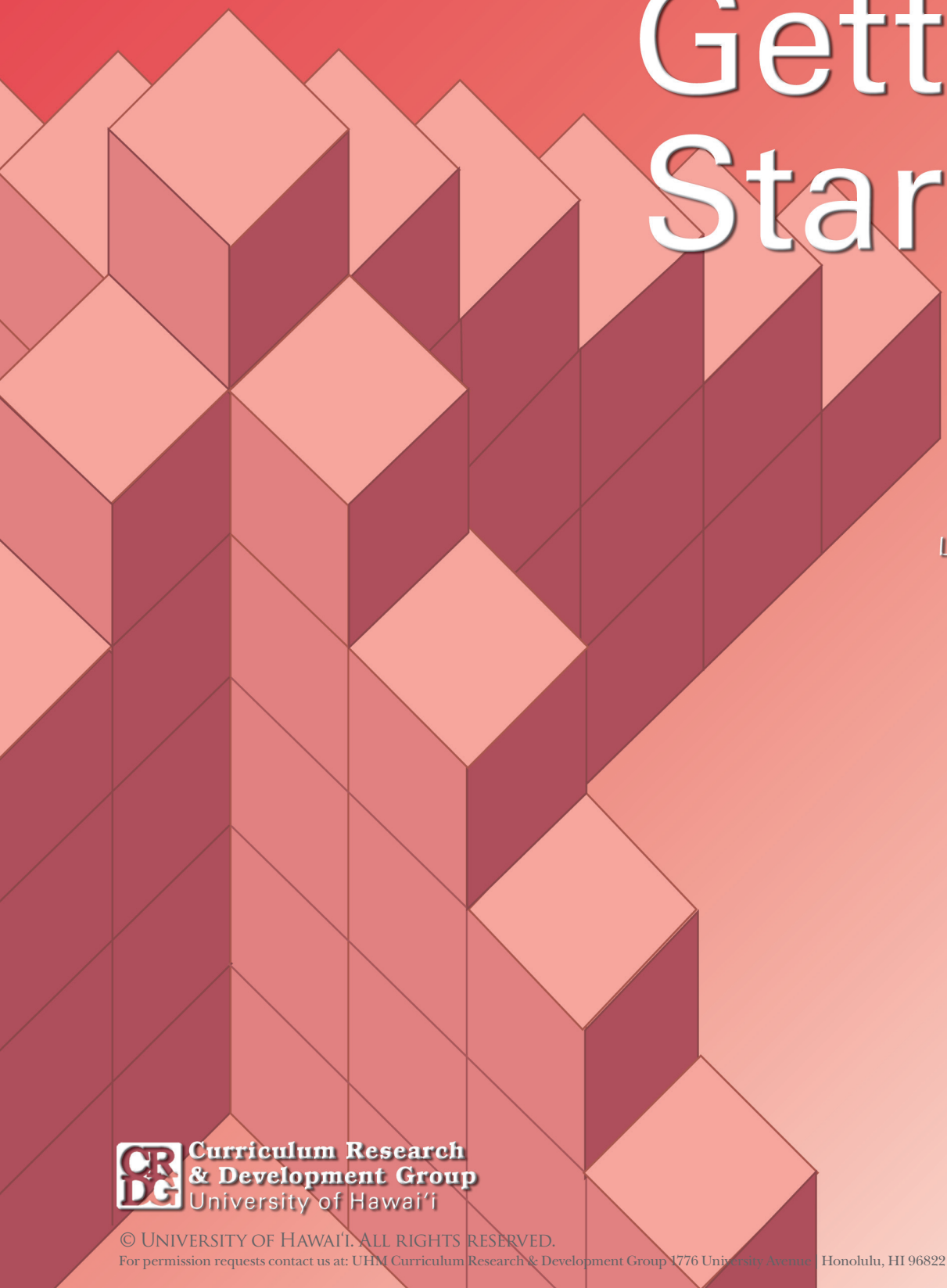
Linda Venenciano
Melanie Ishihara
Cynthia Beppu



Curriculum Research & Development Group
University of Hawai'i

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About the series . . .

Reshaping Mathematics for Understanding is a series of fourteen units suitable for sixth- through eighth-grade students that addresses important topics in middle-grades mathematics, including geometry, measurement, proportional reasoning, rational numbers, probability and statistics, and algebra. The entire series is designed to help students learn to think mathematically. It focuses on developing students' understanding of mathematical concepts and on their ability to draw connections among these concepts. It may serve either as the primary resource in the mathematics curriculum or as a complement to other material. The titles for the full series are listed below.

- Getting Started
- Motion Geometry
- Measurement
- Polygons
- Dilations
- Fractions
- Decimals
- Ratio and Proportion
- Area of Polygons
- Solids
- Probability and Statistics
- Integers
- Algebra Patterns and Relationships
- Number Theory

Access for Every Student

The problems and tasks in the *Reshaping Mathematics for Understanding* series are designed to enable every student to approach mathematics through familiar contexts. Building problem situations on students' past experience makes the study of mathematics more accessible and allows them to expand their thinking. Similarly, to promote genuine engagement, many problems have more than one solution path, giving students opportunities to choose the strategies they prefer using to solve problems. Additionally, to give students opportunities to interact with each concept at varying levels of abstractness and generality, the lessons present new ideas over several days. This design feature also allows students to learn at different rates.

Learning with Understanding

Students can learn only what they truly understand. To learn, they must understand the concepts that underlie operations; they must make connections among processes and concepts; and they must know when and how to apply concepts and operations to problems. Understanding mathematics means doing more than learning how to perform calculations to get correct answers. The topics in this series, developed through problems and lab explorations, encourage students to deepen their conceptual understanding through the practice of reasoning and problem solving.

Role of Visual/Spatial Thinking and Reasoning

Mathematics can and should be a lively course of study for students. It should engage them in active inquiry and give them many opportunities to explore problems whose solutions add to their understanding of the world. For many students, however, mathematics involves merely manipulating numbers. With this limited view, they do not learn how to use multiple contexts, tools, and strategies to solve problems or how to integrate mathematical concepts into broader contexts. Transformational (motion) geometry, which plays an important role in the sequence of the concepts in this series, emphasizes the use of visual contexts and spatial thinking. Lesson discussions further encourage students to use their understanding of spatial relations to make connections among concepts. The Motion Geometry and Dilations units, in particular, give students valuable experience with transformations and enhance the study of many related topics.

Unit Design

Each unit in the *Reshaping Mathematics for Understanding* series can be used individually, or units can be used in clusters. Throughout the series, references among units direct teachers and students to tasks that will help them connect their understanding of new concepts to related experiences.

The lessons in each unit are uniquely designed to enable students to progress through a sequence of tasks that maximize learning with understanding. Rather than present a topic in its entirety in one day, the lessons develop concepts over time. Most concepts begin with an open-ended problem that draws on students' previous experience and intuition and allows for multiple responses. The variety of solutions students propose to such a problem helps the teacher assess what background knowledge students bring to the topic. The problems and tasks that follow develop the concept through a sequence of approaches that provide several direct examples of the concept, raise questions for clarification, offer alternative viewpoints, and prompt students to summarize ideas**.**

The Lessons

There are two forms of lessons in each unit, problem sets and in-class labs. Problem sets consisting of three to five problems are designed to cover several concepts related to the unit topic. Problem sets should be assigned for homework and discussed in class the next day. When students find a problem too difficult to solve on their own, they should be instructed to write questions to help them solve it, and to ask the questions in the discussion. Students work on the labs in small groups in class and debrief afterward, giving them an opportunity to work collaboratively and to concentrate on one strand.

The class discussions are essential in helping students build conceptual understanding. Sharing their solutions and questions allows students to reflect on their thinking and to consider input from others. Both the teacher and students share the responsibility for making discussions productive. As they solve the problems and complete the lab tasks, students explain their thinking, offer alternative responses, and ask questions. Teachers facilitate, asking strategic questions to focus students' thinking on critical ideas. In guiding the discussion, teachers should ensure engagement in the learning process by encouraging students to monitor their learning and by providing a safe, open learning environment in which to share, discuss, and address misconceptions.

Assessment

Except for the Getting Started unit, designed to orient students to the series, all units include suggested assessment items. To check periodically that students understand and that they are participating in the class discussions, teachers can also create "instant" quizzes to give the day after a problem set discussion or lab debriefing. These quizzes should be unannounced and should take between five and ten minutes at the start of class. They have three purposes:

to emphasize the importance of student discussions;

to emphasize the value of information shared by students;

to highlight the importance of asking questions to clarify understanding.

In creating an instant quiz, teachers should focus on students' ideas from the discussion and write two or three short-answer questions as follows:

- a question that is content-based or refers to an understanding that has been established.

  Example: What did we decide it meant to measure the perimeter of a polygon?

- a question that refers to someone's method for solving a problem or an alternative point of view.

  Example: What method did Leslie use to find the area of the trapezoid?

- a question that highlights an undefined assumption or a statement that needed clarification.

  Example: What did Jose mean by *proportional*?

Materials

A list of the materials needed for each lesson appears at the front of every unit. Although students need no special materials or equipment to complete the homework problems, it would be helpful to have them available during discussions. Encouraging students to use a broad range of tools to explore and express their thinking promotes greater understanding.

About the unit . . .

The Getting Started unit introduces students to the types of problems and processes used throughout the *Reshaping Mathematics for Understanding* series. The unit may be introduced at the beginning of the school year or whenever it is appropriate to teach strategies that use problem solving to develop mathematics concepts. As students solve the problems in this unit and discuss their solutions, they practice behaviors that deepen their understanding and contribute to a positive learning environment.

Deepening Understanding

The problems in the Getting Started unit deepen students' understanding of mathematics by encouraging them to clarify concepts and challenge their own assumptions. Additionally, by providing opportunities to give and follow directions, create representations, and explain their thinking, the problems in this unit encourage students to recognize and accept multiple strategies and to be flexible in their thinking.

Creating a Positive Learning Environment

The Getting Started unit encourages students to ask questions, test ideas, and offer alternative points of view. These behaviors allow students to practice the skills they need for communicating effectively in a mathematics classroom and encourage them to share the responsibility for making the class successful. At the same time, the problems ask students to explain their thinking, ask productive questions, and challenge answers without being negative. Engaging in mathematics tasks that are specifically designed for students to practice desired behaviors and to establish social norms is the most effective means for building a positive classroom environment.

The Strands in Getting Started

Communication: To communicate effectively, students must explain, describe, clarify, interpret, and question. The problems in this unit are designed to generate active inquiry. Problems that are open to interpretation motivate students to explain their points of view. Encountering problems that they may solve in more than one way or that have more than one correct answer encourages them to exchange ideas and build understanding. Having to describe and explain their processes makes them more aware of their own thinking and gives them more control over when to apply problem-solving strategies. Communication also helps students learn other ways to solve problems. They often ask each other, "How did you know to use that method?" The thinking that underlies the strategy is often a mystery to them. A focused inquiry helps them to be more conscious of the thought processes they use to solve problems.

Problem Solving: While the entire unit takes a problem-solving approach, this strand introduces key processes essential in helping students develop the strategies they need to become effective problem solvers. First, they must understand that solving problems requires

them to make certain assumptions about the problem conditions. The problems in this strand lead them to recognize that their assumptions stem from their previous knowledge and experience. Becoming aware of their assumptions leads them to recognize a wider range of possible contexts for problems. Second, to solve these problems, students must use organized strategies. Problems that require them to identify, describe, and extend patterns give them valuable strategic experience in recognizing mathematical relationships. Lastly, to teach them that they must learn to test their solutions independently, many problems in this strand ask them to explain and justify their solutions.

Representation: The ability to represent and interpret representations of problem situations enhances students' problem-solving abilities. Representation helps students mathematically communicate how they understood the problem. In the Getting Started unit, students use diagrams, tables, and symbols to create and interpret mathematical situations.

Visual/Spatial Thinking: Visual/spatial thinking helps all students understand and solve problems. Students who have not attained fluency with numbers may see patterns and relationships more easily when they are represented geometrically. Exercising visual/spatial skills also builds students' conceptual understanding and prepares them for more abstract problem-solving tasks. The entire *RMU* series offers many opportunities for students to use visual/spatial thinking.

Introduction to Motion: Transformational-geometry activities are important in middle-grades mathematics. They provide a context for students to observe figures, reason about relationships among figures, and make connections to important middle-grades topics such as proportion. The problems in the Introduction to Motion strand of this unit ask students to build informal proofs about figures and their properties, a practice that develops the understanding they will need in high school mathematics courses.

Concept Development

The numbers in this matrix refer to the problem numbers in each lesson.

STRAND and Focus	Lab A	Lab B	PS 1	PS 2	PS 3	PS 4	PS 5	PS 6
Communication								
Giving/Following Directions	1–4	Tie a Bow						3
Description				2				
Problem Solving								
Making Assumptions			1		1			
Numerical Patterns					3		1	3
Problem Conditions				1				
Testing Solutions							1	
Representation								
Interpreting Diagrams				3		2		
Representing Motion								3
Visual/Spatial Thinking								
Networks				4		1		
Reconfiguration					2		2, 3	1, 2
Visual Patterns			2	1	4	2		2
Testing Perceptions			3	2, 3		3		
Introduction To Motion								
Using Motion in Problem Solving				2	4	3		
Congruence								2

Materials

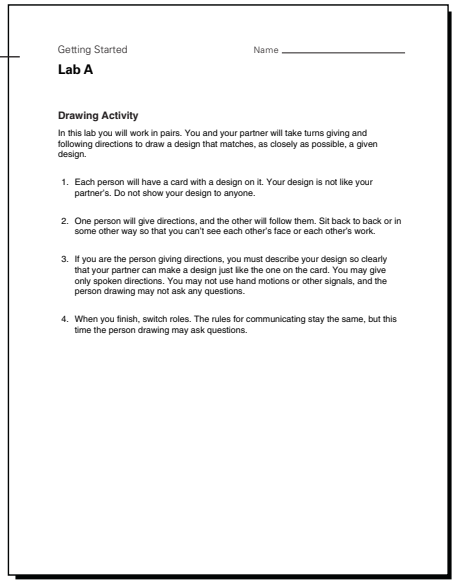
Special materials for Getting Started

Lab A	Drawing Activity Cards (see blackline masters at the end of the unit), 1 per student
Lab B	Shoe with ties or laces, 1 per student
Problem Set 1	Geometric compass, 1 per group Straight-edge*, 1 per student Square tiles, 20 per group
Problem Set 2	Straight-edge*, 1 per student Tracing paper
Problem Set 3	Counters, 20 per group Toothpicks, 20 per group Tracing paper
Problem Set 4	Geometric compass, 1 per group Cubes such as centimeter cubes or connecting cubes, 70 per group Straight-edge*, 1 per group Tracing paper
Problem Set 5	Toothpicks, 20 per group
Problem Set 6	Dimes and pennies, 3 of each per group Square tiles, 70 per group Toothpicks, 20 per group

*Straight-edges without unit markings are preferable to rulers. Blank index cards could also be used.

Lab A

Giving Clear Directions


Getting Started Name _____

Lab A

Drawing Activity

In this lab you will work in pairs. You and your partner will take turns giving and following directions to draw a design that matches, as closely as possible, a given design.

1. Each person will have a card with a design on it. Your design is not like your partner's. Do not show your design to anyone.
2. One person will give directions, and the other will follow them. Sit back to back or in some other way so that you can't see each other's face or each other's work.
3. If you are the person giving directions, you must describe your design so clearly that your partner can make a design just like the one on the card. You may give only spoken directions. You may not use hand motions or other signals, and the person drawing may not ask any questions.
4. When you finish, switch roles. The rules for communicating stay the same, but this time the person drawing may ask questions.

Lab A, Nos. 1–4

Strand: Communication

Focus: Giving/Following directions

Task: Rehearsing roles as givers and receivers of information

PROBLEM

Drawing Activity

In this lab you will work in pairs. You and your partner will take turns giving and following directions to draw a design that matches, as closely as possible, a given design.

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4. When you finish, switch roles. The rules for communicating stay the same, but this time the person drawing may ask questions.

Teacher's Insight

Students are important sources of information in mathematics class discussions. In productive discussions, many students participate, proposing ideas, questioning ideas, and assessing other viewpoints. In this activity, students practice giving and receiving information needed to complete a task. It is crucial that they be as clear as possible in giving directions and that they listen carefully to their partners. Even students who have worked in small groups may have little experience in collaborating to complete a task.

Have students read all the directions, making sure they understand the procedure. In the first round, students who are drawing may not ask any questions. In the second round they may. This change helps them grasp how helpful asking questions can be when someone is explaining a complex idea.

The debriefing gives students a chance to assess the effectiveness of their communication. Remind them to concentrate on what worked well and what they could improve. Help them assess their drawings to see how well they captured the details of the design.

ANSWER

Drawings from the second round should be more accurate than those from the first round.

★HOMEWORK

Write directions for tying a bow.

Discussion

How did the drawings turn out?

Look carefully at the details (position, shape, size, and orientation).

How did you decide if a drawing was good?

To direction-givers: What was hardest to communicate? What was easy?

To drawers: What was hardest to picture in your mind? What was easy?

How did you handle the challenges?

How did not being able to ask and answer questions make this task more challenging?

How does asking questions help you understand?