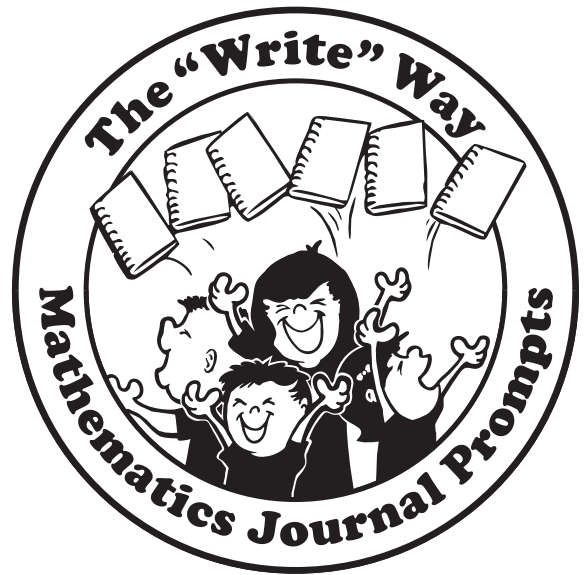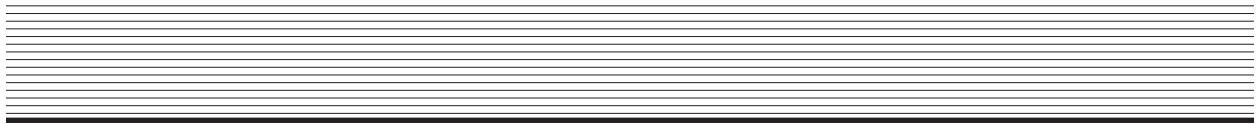# The "Write" Way
Mathematics Journal Prompts

& More

FOR GRADES 3-4

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providing quality educational programs and
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Mathematics Journal Prompts and More

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Introduction

The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) includes a separate process standard for Communication for grades 3 through 5. This standard advocates using multiple activities that give students opportunities to demonstrate their understandings. These tasks should allow students to express themselves in a variety of ways such as with words, concrete models, pictures or drawings, or abstract symbols. Each of these forms lets students communicate with others about their thinking. At the same time, students are developing a deeper understanding of the mathematics because they have to be clear and explicit when they are writing to an audience that cannot see their facial expressions and gestures, or ask clarifying questions. Thus students understand more and retain their understanding longer.

Communication can include at least five types of activities: (1) speaking, (2) listening critically, (3) reading, (4) modeling or constructing, and (5) writing. Speaking or talking about mathematics should be encouraged and used daily in mathematics classes. Students can talk in pairs or small groups as well as present their ideas to the whole class. If students are talking about mathematics, others should be critically listening so that any student should be able to comment on the ideas presented by another. Students should also be reading in a mathematics class. The reading could be centered on the textbook but can be expanded to include students' writing or trade books. Modeling or constructing means that students can also communicate to others by using concrete models (Miras™), drawing pictures or diagrams, or using technology such as graphing calculators. Finally, communication also includes writing such as journal writing and extended or expanded problem-solving tasks.

The following sections describe different types of journal writing, extended problem-solving tasks, and the use of these two forms of writing in assessment.

Journal Writing

Journal writing can be structured to give teachers cohesive and comparable information about students and their thinking while challenging them through contextual situations. A structure for journal writing includes prompts that focus on (1) mathematical content, (2) mathematical processes, and (3) student attitude or affect. Journal prompts give situations or questions to which students respond. Responses may include words, pictures or drawings, or symbols. Students are encouraged to support their ideas and to clearly explain what they mean. They can give specific examples as part of their explanations or use counterexamples.

Content prompts relate or connect topics within and outside of mathematics, targeting important or meaningful concepts and skills. They can also provide situations that focus on areas where students often have misunderstandings or misconceptions. The responses to the prompts give teachers (and students) insight into how a student has interpreted a mathematical idea.

Process prompts promote the awareness of how students solve or approach problems or algorithms. The responses to these prompts can give insight into students' preferences for problem-solving strategies or algorithms and into how they learn or remember. As students become aware of how they learn and solve problems, they grow more confident in approaching new or novel problems.

Attitudinal or affective prompts focus on students' feelings about themselves as mathematicians and students of mathematics. Students' responses allow teachers to assess how positive attitudes about mathematics and mathematicians are developing in the classroom environment.

Extended Problem-solving Tasks

Extended or expanded problem-solving tasks provide opportunities for students to explore and solve problems that require novel solution approaches. For this purpose, problem solving is defined as confronting a problem that does not have an obvious solution or solution path. In most cases, a non-routine solution method (or combination of methods) is required such as making a list, drawing a diagram, working backwards, guessing-and-testing, or creating a table.

Extended problem-solving tasks require more time and thought to solve than routine problems. Students draw on their previous knowledge and experiences to reason through the problem. Because their thought processes will be more complex, writing an expanded solution is an important part of communicating their methods or processes to others. Writing a response to an extended problem-solving task also helps students create a solution process as they clarify what the problem is asking, what information is given in the problem, and what solution methods would be appropriate.

Many students believe problem solving to be a linear process. That is, they read a problem, think of a solution method, solve the problem, and check their answer. Problem solving is more complex. It often requires re-reading a problem or abandoning one solution method for another.

Assessment Tasks Requiring Writing

Assessing student understanding can be done in a variety of ways including journal writing, homework problems, problem-solving write-ups, quizzes, and tests. Any assessment should encompass at least three types of tasks:

(1) problem solving, (2) conceptual understanding, and (3) skill acquisition.

Of the three types of assessment tasks, skill acquisition is most often assessed. These tasks would include solving equations and inequalities or using formulas by primarily symbol manipulation. Students often apply an algorithm that may or may not convey their understanding.

Items that are designed to assess students' conceptual understanding or ability to problem solve can provide a rich means by which students demonstrate their thinking and interpretations of concepts through expanded responses. The inclusion of these types of items link assessment with classroom practice. If students are required in mathematics classes to explain their thinking in class discussions or on their homework papers, it is important that assessments also include similar tasks. Likewise, if state assessments include self-constructed response items, students will develop skill in responding to such items when these types of tasks are regularly included on a daily basis as well as on assessments.

Description of Materials and Teaching Suggestions

There are over 150 journal prompts here for teachers to choose from. Some of them may be used more than once to ascertain how students are growing in their understandings of and beliefs about mathematics. The prompts may also be used as assessment tasks for expanded response or self-constructed response items. As they use these prompts teacher may generate other ideas that can be used to create their own journal prompts.

Journal Prompts

✎ Each mathematical content prompt includes a short description of what to expect or consider in students' responses. The mathematical content response should reflect a student's progress in understanding the concept or skill.

✎ The content prompts were selected to match recommendations from the National Council of Teachers of Mathematics (*Principles and Standards for School Mathematics, 2000*) rather than to fit a specific text. Locate the topic students are studying and then select the prompt that best fits the class. The wording may be changed if students will not understand the terminology or vocabulary used.

✎ Process prompts give students opportunities to show how they do a mathematical process or algorithm (step-by-step procedure). When teachers use these prompts, they gain insight into how students have interpreted or altered procedures. In some cases, they may find student-invented algorithms that are appropriate to share with the class.

✎ Affective or attitudinal prompt responses show students' beliefs. If used more than once, teachers should be able to detect changes in beliefs or feelings about mathematics as a discipline as the year or semester progresses. Similarly, students' views of themselves as mathematicians or as students of mathematics can be assessed.

Teachers can use writing prompts daily or intermittently, depending on the class, and prompts can be repeated from time to time. In prompts that use students' names, they may change the names to represent their students. If teachers use journal prompts daily, it is recommended that students respond to at least three content prompts, no more than one process prompt, and no more than one attitudinal prompt in a week. The more frequently writing is used as a regular part of mathematics class, the better students become at responding. More specific directions for implementing journal writing are included at the end of this section.

. . . and more

✎ There are 12 extended problem-solving tasks. Each task requires more time to solve than one class period. Students often provide the best solutions if they are given 10 days in which to solve it. The teacher may decide to use one of these every 3 weeks or so.

✎ In some cases, teachers may assign the tasks for the entire class to work individually. These tasks also give teachers and students the opportunity to use pair or group problem solving. Regardless, it is important that students write their responses in a way that a reader can see the flow of their thinking and understand the solution method or path that they used.

✎ It is recommended that students do extended problem solving on a regular basis. This practice supports their development of problem-solving strategies and boosts their confidence to solve complex problems.

✎ For each problem-solving task a solution has been given. However, there are multiple methods to solve each problem. Teachers should be open to creative ways that students may approach these problems.

✎ There are 10 assessment items included here. These items represent a conceptual approach to a particular mathematical topic. There should be no more than one of these items on a chapter test. If used independent of the chapter quizzes or tests, however, it is possible to use more than one. Additionally, any of the content or process journal prompts can be used on formative or summative assessments in a similar manner.

✎ It is important to recognize that students should have experience with talking and writing mathematics daily in order for the use of these assessment items to be representative of student understanding. If a teacher uses a lecture-based approach, some of these items may need alteration to allow for optimal responses from students.

Evaluating Students' Responses

Students tend to give the writing more thought if it is to be scored. One method used was to score the responses on content rather than on grammar and spelling. An essay grading method was used. By reading two or three papers to get a feel for students' responses these first papers formed the baseline for scoring other papers. This method, however, did not provide students a guideline for writing in advance.

When teachers use rubrics and metrics, however, scoring guidelines can be provided in advance. A rubric establishes qualitative levels that define what characteristics a response has or what criteria it meets. The qualitative levels are not used as points. On the other hand, a metric is a point system. The levels of a metric have specific criteria associated with each point value. The same criteria may be used for either a rubric or a metric system. The difference is in whether or not points or qualitative levels are established for the response.

There are many ways to create a rubric or metric. Teachers can develop one alone, or students can work with the teacher to develop a rubric or metric specific to their class or to the task. Sample rubrics and metrics developed with students' participation, follow on the next page.

Although grammar and punctuation are not usually scored, students should communicate their ideas in ways that are understandable. Some students may use drawings, tables, charts, or other means to convey their ideas. They should be encouraged to use whatever ways they need to make their ideas clear.

As writing tasks are used, excerpts can be taken from students' papers to illustrate qualities that you consider important. Both high- and low-quality responses can be used to show students the comparison with rubric or metric criteria. Of course, authorship of whatever responses are selected should be kept anonymous.

Rubrics or metrics to score the problem-solving tasks as extended types or as assessment items can be developed for each individual task or created as a general guide for student performance. The rubrics or metrics given above can also be adapted to serve as a generalized set of criteria to guide students' solution approaches. If rubrics or metrics are being used by the entire mathematics department, it may be appropriate to have department-wide discussions to agree upon criteria. This will provide a means by which to motivate consistent and cohesive student work across grades, courses, and teachers.

With any form of writing and any type of rubric or metric, students can self-evaluate their responses or conduct peer evaluations. This allows you to see if students truly understand the criteria outlined in the rubric or metric. This activity also requires students to think at a much higher level as they analyze critically others' work.

General Metric

4 points The student's work includes—
- ✎ completed prompt or an answer to the question posed
- ✎ support for statements made by using either examples or counterexamples
- ✎ ideas clearly communicated to the reader
- ✎ legible writing, drawings, pictures, charts or tables, and diagrams
- ✎ accurate mathematics or information

3 points Omission of one criterion from level 4

2 points Omission of two criteria from level 4

1 point Omission of three criteria from level 4

0 points Omission of more than three criteria from level 4

Three-level Rubric (or Metric)

The student's work shows a response that—

Exceeds standard
- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader

Meets standard
- ✎ addresses the question raised in the prompt
- ✎ has some correct or accurate mathematics
- ✎ is legible
- ✎ does not support or justify some of the statements made
- ✎ makes sense to the reader

Below standard
- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not support or justify statements made

Five-level Rubric (or Metric)

4   The student's work shows a response that—
- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader.

3   The student's work shows a response that—
- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ does not have fully justified or supported statements
- ✎ makes sense to the reader

2   The student's work shows a response that—
- ✎ addresses the question raised in the prompt
- ✎ has some incorrect or inaccurate mathematics
- ✎ is legible
- ✎ does not have justified or supported statements
- ✎ is somewhat clear to the reader

1   The student's work shows a response that—
- ✎ addresses the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is partially legible
- ✎ does not have justified or supported statements
- ✎ does not make sense to the reader

0   The student's work shows a response that—
- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not make sense to the reader

Implementing Journal Writing in Your Classroom

✎ To begin using writing in your classroom, you will need to make sure your students understand your expectations for writing. The following offers one method for helping students learn what is meant by *writing in mathematics.*

✎ Share with students the rubric or metric you will be using. You may opt to create a rubric or metric with your students rather than creating one yourself. Make copies of the rubric or metric for students to keep in their notebooks. Post one copy in the classroom for easy reference.

✎ Give students a practice prompt to write. If it is used as a warm-up, allow about 6 minutes for them to respond. The practice prompt can be any type, but the rubric or metric may work better with a content prompt. Select one that you feel all students in your class can attempt.

✎ Have students compare what they wrote with their partner or table mates. They should check the rubric or metric. Have students focus on 3 things they could do to improve their writing to the next higher level. If their writing already includes all the indicators for the top level, ask them to write another question that this prompt made them think about.

✎ Allow 3 minutes for students to correct or revise their work. They should strive to reach the top two levels of the rubric or metric.

✎ Collect the work. Score it with your rubric or metric. However, to allow students time to learn to meet your expectations, you may not want to record the score yet.

✎ For the next several days, whether you assign prompts as a warm-up or for homework, allow students time to revise or correct their work. You should stress that they should strive to reach the top two levels. Repeat this phase as often as needed to help students understand your expectations for their writing.

✎ If possible, show students samples of other students' writing. Use this sample to illustrate what you mean by your criteria in the rubric or metric. A sample of student writing in the middle grades follows.

Sample of Student Work

Prompt: Kolio said, "Subtraction is commutative." Do you agree with Kolio? Why or why not?

No, I don't agree with Kolio. He must be thinking about multiplication or addition because subtraction is not commutative. If it was commutative, it would mean that you can put the numbers you're subtracting in any order and you can't. If you put 7 ¬- 2 you get 5. But if you put 2 ¬- 7 you get something that is less than 0 and that's not the same.

In multiplication you can put the numbers in any order and still get the same answer. Like if you multiplied 3 x 2 or 2 x 3, it doesn't matter. You still get 6.

Kolio must be confused about subtraction. He might think that you can put the numbers in any order but he has to remember ORDER MATTERS.

Journal Prompts

In this section 168 prompts are provided in categories described in the introduction and description of materials. As a teacher you can give prompts as a homework assignment in conjunction with a textbook assignment. Or, you can use them as a warm-up for the first 5–7 minutes of the class period. Student responses can be collected and graded or students can share their ideas within a whole class discussion. Their ideas can be recorded on chart paper, on transparencies, or on the board. The chart paper provides a permanent record or archive of student responses so that they can be revisited later in the unit of study or in an associated unit. An archive for responses allows the progress of students' understandings, or thinking about mathematical concepts or skills, to be monitored.

You can also use the prompts in a problem-solving context where students create a solution in a pair, group, or individually. If pairs or groups work on the problems, they can share their ideas before the whole class. Individual student responses can be shared by having other students read them and analyze the response.

Prompts are organized as follows:

Content Prompts

Number Sense, Properties and Operations

1. **Terrell wrote, "$\frac{2}{11}$ is close to 0." "No, it's not," said Martin. "It's close to $\frac{1}{2}$." Who do you agree with? Why?**

   Students should agree with Terrell. The use of benchmarks is important in students' development of conceptual understandings of fractions. Other fractions or benchmarks can be substituted for those used in this prompt.

2. **Tim said, "The zero in 103 is not really important. After all, zero just means nothing." Jon thought it was important, but he wasn't sure. What do you think? Explain your thinking.**

   The zero is important. Without it, the number would represent a lesser quantity than 103. Students should provide some explanation of the zero as a placeholder.

3. **Kenya rounded a number to 800. Find a number that Kenya could have used. Explain why it could be rounded to 800. Find at least two more numbers that could be rounded to 800.**

   Some students may round from the tens position and others from the hundreds position. Students may give numbers that involve fractions or decimals. For example, 790 and 802 could both be rounded to 800 reasonably, as could 800.7 or $775\frac{1}{2}$. There are multiple solutions.

4. **Michelle asked her teacher, "What do we use a number line for?" What do you think Michelle's teacher told her? Be specific.**

   Responses should indicate different uses of number lines. Watch for appropriate use as models for operations. Use students' comments as part of a class discussion on operations. Some students may comment on using a number line for measurement or to determine relationships of numbers.

5. **"I think subtraction is commutative," said Randy. "I don't agree," said Brenna. Who do you agree with? Use examples to support your response.**

   Students should agree with Brenna. Subtraction is not commutative. Students' responses may include examples that show that changing the order of the minuend and the subtrahend gives differences that are not the same. Given the developmental level, students may not use integers. It may also be appropriate if students draw pictures of the concrete models or use the number line to support their response.

6. **Lee found a number line like the one shown. Where on the number line would you find a number larger than 27? How do you know?**

27

Numbers larger than 27 are found to the right of 27. Students may give examples to illustrate the correctness of their response.

7. **Aaron asked, "Are decimals the same thing as whole numbers?" What would you tell Aaron? Support your answer.**

All whole numbers are decimals. However, not all decimal representations are whole numbers. For example, 3 can be represented as 3.0 but 3.2 is not a whole number. It is slightly more than 3.

8. **Maria wondered what whole numbers are. What would you tell her? Describe whole numbers with as much detail as possible.**

Whole numbers are the counting numbers and 0. Thus, the set of whole numbers is 0, 1, 2, 3, 4, 5, . . .

9. **Sara asked, "What's the largest number you've ever used?" What would you tell Sara? Be specific. Describe how you used the number.**

Answers will vary. Students' responses often include everyday contexts that require large numbers.

10. **Aiya said, "I added 4 and 6 and got 10." Matt said, "That's correct. Did you notice that you added two even numbers and got an even number in the sum?" Will Matt's idea always be true? Why or why not?**

Matt is correct. The sum of two even numbers will always be even. Students can use a variety of methods to prove why this is true.

11. **Amy wrote, "3 × 2 = 5 + 1." Larry asked, "What does the '=' mean in your equation?" What should Amy tell Larry?**

Responses should indicate that both 3 × 2 and 5 + 1 represent the same amount or quantity. Encourage students to generalize or make statements about the use of equal signs to show equivalent amounts.

12. **Seth said, "I ate one half of a pizza, which was three slices." Jena said, "I ate four slices of pizza, and it was one half of a pizza." Who is telling the truth? Prove it.**

Both could be telling the truth. Seth's pizza could have been cut into six pieces while Jena's was cut into eight pieces.

13. **Sara drew a number line.**

She asked, "Trey, can you find a fraction between $\frac{1}{4}$ and $\frac{5}{6}$?" "I can do that," said Trey. What fraction do you think Trey found? Justify or support your answer.

There are infinite fractions that could be found. For example, $\frac{1}{2}, \frac{4}{5}, \frac{2}{3}, \frac{5}{8}$, and $\frac{8}{15}$ are all possibilities. Students should present some justification or support for their responses.

14. **"I wrote an equation but I can't remember what the word problem was," said Faith. Write a word problem that can be solved by 153 ÷ 3 = 51. Describe why it can be solved with this division problem.**

Answers will vary. Be sure to check the problem to determine if division is the correct operation.

15. **Amanda solved a division problem.**

$$79 ÷ 24 = 3 \text{ R}7$$

Tino asked, "What does the remainder 7 mean?" What do you think Amanda told him?

Students may indicate that 7 represents the amount left over when you group the 79 into groups of 24. Or they may indicate that it is the amount left over when you put 79 into 24 groups.

16. **Booker asked Courtney, "Can you draw a picture that would convince me that 78 < 112?" What picture could Courtney draw? Why? Explain why it shows 78 < 112.**

Answers will vary. Some students use pictorial representations of base-ten blocks while others may explain the relationship in words.

17. **Darrell made a chart to show equivalent fractions. He wrote,
$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=$ . . . Draw pictures or write a description that
would explain why Darrell is correct.**

Students' descriptions can be used to stimulate class discussion. You can
distribute students' responses and ask other students to verify their correctness.

18. **Tana asked, "If I add two fractions, like $\frac{1}{3}$ and $\frac{3}{4}$, will the sum
be more than or less than 1?" "I think the sum is more than
1," said Lori. Why do you think Lori thought that? Explain
your thinking.**

Lori is correct. Students may explain it in multiple ways. One way to approach
it is to think that $\frac{1}{3}$ is more than $\frac{1}{4}$, the sum is more than 1. Other descriptions
may be used to substantiate students' responses.

19. **Corey said $\frac{1}{2} \div \frac{2}{4}$ was solved by dividing 4 by 2 and 9 by 3. Her
quotient is $\frac{2}{3}$. Do you agree or disagree with her method?
Why?**

This prompt will provide some interesting looks at fractional division. New
algorithms may be developed.

20. **"I have counted all my money," said Steven. "I have seven
quarters, nine dimes, eight nickels, and 12 pennies." How can
Steven trade his money so that he has the least number of
bills and coins? Explain your answer.**

Steven can trade his money into three one-dollar bills, one dime, one nickel, and
two pennies. He has $3.17 total. Students should show the process they used to
arrive at their answer. Many ways are possible such as adding up all amounts first
and then finding the bills and coins needed. Or students may "trade as they go,"
showing how the trades are done.

21. **Mr. I. M. Money has decided to give $1,000.00 to a student who
can spend it wisely. Write a letter to Mr. Money and explain
how you would spend the money. Be sure to explain why you
would spend it that way.**

Answers will vary. Make sure that responses are reasonable. This activity can be
expanded to language arts class.

22. **Write a word problem that can be solved by multiplying two
numbers. Show the solution.**

Answers will vary. Students should write a problem that can be solved by
multiplying. Be sure to check that multiplication is the appropriate operation.

23. **Pat asked, "Why do fractions have to have common denominators before they can be added?" What do you think? Support your answer with specific examples.**

Responses should indicate that you cannot identify the sum (or the quantity) without having some common basis for comparison. The common denominator provides that common basis. To extend this prompt, you can ask students about using a common denominator for subtraction with fractions.

24. **Micah dropped spaghetti sauce on his math paper. What do you think is under the sauce he dropped? Why?**

$$132 - \blacksquare = 14$$

Students should indicate that 118 must be under the spaghetti sauce. Their reasoning can vary. Intuitively, students may say that the number under the spaghetti sauce has to be 14 less than 132. Thus one solution method is to subtraction 14 from 132 to get 118. Other students may work backwards to find the subtrahend.

25. **Jennifer and Patrick were arguing about what a fraction is. Describe what a fraction is and give examples of fractions.**

Answers will vary. It will be interesting to note how many students, if any, indicate that a whole number is also a fraction. Pay particular attention to how students represent a fraction. Some may use only continuous models while others will include discrete models.

26. **Darron's calculator is broken. It doesn't always add correctly. He has difficulty deciding if a problem is done correctly. He added 43 + 201 + 605 + 113. His calculator showed a sum of 692. Do you agree? Why or why not?**

Students should disagree because an estimate of the sum is closer to 900 to 1,000. Some students may show you how to solve the problem or they may use an estimation strategy such as front-end or compatible numbers. Others may note that one addend is 605 and another is over 100. Thus the sum must be larger than 692.

27. **Beau asked, "How are addition and multiplication related?" Eli said, "I can show you how they are related." What do you think Eli showed Beau? Be specific.**

Answers will vary. Some students may comment on the repeated addition model of multiplication. Be sure they explain their answers.

28. **Tanya rounded a fraction to $\frac{1}{2}$. What fraction(s) could she have rounded? Why?**

Students may comment on the relations of the numerators and denominators in estimating fractions. They may list many fractions, including but not limited to $\frac{3}{5}, \frac{4}{9}, \frac{3}{6}, \frac{11}{20}, \frac{3}{5}$ and so on.

29. **Carina wrote this problem: $\frac{1}{2}+\frac{7}{15}$? Is the sum about 0, $\frac{1}{2}$, or 1? How did you decide?**

Students usually comment on the relations of the numerators and denominators in estimating each fraction. They may show how they estimated. Watch for students who solve and then estimate (or round) the sum. Encourage students to demonstrate some method for estimating the size of fractions using 0, $\frac{1}{2}$, and 1 as benchmarks without doing the addition. The sum is close to 1 because $\frac{7}{15}$ is close $\frac{1}{2}$. When added to $\frac{1}{2}$, the other addend, the sum is close to 1.

30. **June asked her teacher, "Can a fraction have a numerator that is larger than the denominator?" What do you think June's teacher said? Explain.**

Responses should discuss improper fractions and possibly the definition of a fraction. Examples should be included in the response. Or students should include an explanation in words that will support their comments.

31. **Ella asked "What does division mean?" What would you tell Ella? Use specific examples or drawings to justify your responses.**

Answers will vary. Responses should describe one of the models for division such as repeated subtraction, grouping, or partitioning.

32. **"What does the numerator of the fraction tell me?" asked Phil. Marcie said, "I can tell you what the numerator and the denominator means." What do you think Marcie said? Be specific. Use examples to support your response.**

Responses usually indicate something about part-to-whole relationships. Encourage both continuous and discrete models for fractions. If students use a continuous model, they may describe a whole that is cut into pieces, each piece the same size. If they use a discrete model, they may discuss a set of, say, blue and red blocks. The fraction could represent the number of blue blocks compared with the entire set.

33. **Brad asked, "What's the largest number of digits possible in the sum of a two-digit number and a three-digit number?" What would you tell Brad? Be sure to provide an argument that is convincing.**

Students should indicate that the maximum number of digits in the sum can be four. They may provide multiple ways of giving a rationale for their response. One way is to use 99 + 999 as a means to determine the maximum number of digits. This prompt can be changed in terms of the number of digits and the operation.

34. **Noni asked, "How are whole numbers related to fractions?" What would you say to Noni? Describe in detail.**

Whole numbers can be expressed as fractions. Students should describe the relationship with sufficient detail.