
Assessment Tasks Requiring Writing

Assessing student understanding can be done in a variety of ways including journal writing, homework problems, problem-solving write-ups, quizzes, and tests. Any assessment should encompass at least three types of tasks:

(1) problem solving, (2) conceptual understanding, and (3) skill acquisition.

Of the three types of assessment tasks, skill acquisition is most often assessed. These tasks would include solving equations and inequalities or using formulas by primarily symbol manipulation. Students often apply an algorithm that may or may not convey their understanding.

Items that are designed to assess students' conceptual understanding or ability to problem solve can provide a rich means by which students demonstrate their thinking and interpretations of concepts through expanded responses. The inclusion of these types of items link assessment with classroom practice. If students are required in mathematics classes to explain their thinking in class discussions or on their homework papers, it is important that assessments also include similar tasks. Likewise, if state assessments include self-constructed response items, students will develop skill in responding to such items when these types of tasks are regularly included on a daily basis as well as on assessments.

Description of Materials and Teaching Suggestions

There are over 150 journal prompts here for teachers to choose from. Some of them may be used more than once to ascertain how students are growing in their understandings of and beliefs about mathematics. The prompts may also be used as assessment tasks for expanded response or self-constructed response items. As they use these prompts teachers may generate other ideas that can be used to create their own journal prompts.

Journal Prompts

- ✎ Each mathematical content prompt includes a short description of what to expect or consider in students' responses. The mathematical content response should reflect a student's progress in understanding the concept or skill.
- ✎ The content prompts were selected to match recommendations from the National Council of Teachers of Mathematics (*Principles and Standards for School Mathematics, 2000*) rather than to fit a specific text. Locate the topic students are studying and then select the prompt that best fits the class. The wording may be changed if students will not understand the terminology or vocabulary used.
- ✎ Process prompts give students opportunities to show how they do a mathematical process or algorithm (step-by-step procedure). When teachers use these prompts, they gain insight into how students have interpreted or altered procedures. In some cases, they may find student-invented algorithms that are appropriate to share with the class.
- ✎ Affective or attitudinal prompt responses show students' beliefs. If used more than once, teachers should be able to detect changes in beliefs or feelings about mathematics as a discipline as the year or semester progresses. Similarly, students' views of themselves as mathematicians or as students of mathematics can be assessed.

Teachers can use writing prompts daily or intermittently, depending on the class, and prompts can be repeated from time to time. In prompts that use students' names, they may change the names to represent their students. If teachers use journal prompts daily, it is recommended that students respond to at least three content prompts, no more than one process prompt, and no more than one attitudinal prompt in a week. The more frequently writing is used as a regular part of mathematics class, the better students become at responding. More specific directions for implementing journal writing are included at the end of this section.

... and more

- ✎ There are 12 extended problem-solving tasks. Each task requires more time to solve than one class period. Students often provide the best solutions if they are given 10 days in which to solve it. The teacher may decide to use one of these every 3 weeks or so.
- ✎ In some cases, teachers may assign the tasks for the entire class to work individually. These tasks also give teachers and students the opportunity to use pair or group problem solving. Regardless, it is important that students write their responses in a way that a reader can see the flow of their thinking and understand the solution method or path that they used.
- ✎ It is recommended that students do extended problem solving on a regular basis. This practice supports their development of problem-solving strategies and boosts their confidence to solve complex problems.
- ✎ For each problem-solving task a solution has been given. However, there are multiple methods to solve each problem. Teachers should be open to creative ways that students may approach these problems.
- ✎ There are 10 assessment items included here. These items represent a conceptual approach to a particular mathematical topic. There should be no more than one of these items on a chapter test. If used independent of the chapter quizzes or tests, however, it is possible to use more than one. Additionally, any of the content or process journal prompts can be used on formative or summative assessments in a similar manner.
- ✎ It is important to recognize that students should have experience with talking and writing mathematics daily in order for the use of these assessment items to be representative of student understanding. If a teacher uses a lecture-based approach, some of these items may need alteration to allow for optimal responses from students.

Evaluating Students' Responses

Students tend to give the writing more thought if it is to be scored. One method used was to score the responses on content rather than on grammar and spelling. An essay grading method was used. By reading two or three papers to get a feel for students' responses these first papers formed the baseline for scoring other papers. This method, however, did not provide students a guideline for writing in advance.

When teachers use rubrics and metrics, however, scoring guidelines can be provided in advance. A rubric establishes qualitative levels that define what characteristics a response has or what criteria it meets. The qualitative levels are not used as points. On the other hand, a metric is a point system. The levels of a metric have specific criteria associated with each point value. The same criteria may be used for either a rubric or a metric system. The difference is in whether or not points or qualitative levels are established for the response.

There are many ways to create a rubric or metric. Teachers can develop one alone, or students can work with the teacher to develop a rubric or metric specific to their class or to the task. Sample rubrics and metrics, developed with students' participation, follow on the next page.

Although grammar and punctuation are not usually scored, students should communicate their ideas in ways that are understandable. Some students may use drawings, tables, charts, or other means to convey their ideas. They should be encouraged to use whatever ways they need to make their ideas clear.

As writing tasks are used, excerpts can be taken from students' papers to illustrate qualities that you consider important. Both high- and low-quality responses can be used to show students the comparison with rubric or metric criteria. Of course, authorship of whatever responses are selected should be kept anonymous.

Rubrics or metrics to score the problem-solving tasks as extended types or as assessment items can be developed for each individual task or created as a general guide for student performance. The rubrics or metrics that follow can also be adapted to serve as a generalized set of criteria to guide students' solution approaches. If rubrics or metrics are being used by the entire mathematics department, it may be appropriate to have department-wide discussions to agree upon criteria. This will provide a means by which to motivate consistent and cohesive student work across grades, courses, and teachers.

With any form of writing and any type of rubric or metric, students can self-evaluate their responses or conduct peer evaluations. This allows you to see if students truly understand the criteria outlined in the rubric or metric. This activity also requires students to think at a much higher level as they analyze critically others' work .

General Metric

4 points The student's work includes—

- ✎ completed prompt or an answer to the question posed
- ✎ support for statements made by using either examples or counterexamples
- ✎ ideas clearly communicated to the reader
- ✎ legible writing, drawings, pictures, charts or tables, and diagrams
- ✎ accurate mathematics or information

3 points Omission of one criterion from level 4

2 points Omission of two criteria from level 4

1 point Omission of three criteria from level 4

0 points Omission of more than three criteria from level 4

Three-level Rubric (or Metric)

The student's work shows a response that—

Exceeds standard

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader

Meets standard

- ✎ addresses the question raised in the prompt
- ✎ has some correct or accurate mathematics
- ✎ is legible
- ✎ does not support or justify some of the statements made
- ✎ makes sense to the reader

Below standard

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not support or justify statements made

Five-level Rubric (or Metric)

4 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader.

3 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ does not have fully justified or supported statements
- ✎ makes sense to the reader

2 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has some incorrect or inaccurate mathematics
- ✎ is legible
- ✎ does not have justified or supported statements
- ✎ is somewhat clear to the reader

1 The student's work shows a response that—

- ✎ addresses the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is partially legible
- ✎ does not have justified or supported statements
- ✎ does not make sense to the reader

0 The student's work shows a response that—

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not make sense to the reader

Implementing Journal Writing in Your Classroom

- ✍ To begin using writing in your classroom, you will need to make sure your students understand your expectations for writing. The following offers one method for helping students learn what is meant by *writing in mathematics*.
- ✍ Share with students the rubric or metric you will be using. You may opt to create a rubric or metric with your students rather than creating one yourself. Make copies of the rubric or metric for students to keep in their notebooks. Post one copy in the classroom for easy reference.
- ✍ Give students a practice prompt to write. If it is used as a warm-up, allow about 6 minutes for them to respond. The practice prompt can be any type, but the rubric or metric may work better with a content prompt. Select one that you feel all students in your class can attempt.
- ✍ Have students compare what they wrote with their partner or table mates. They should check the rubric or metric. Have students focus on 3 things they could do to improve their writing to the next higher level. If their writing already includes all the indicators for the top level, ask them to write another question that this prompt made them think about.
- ✍ Allow 3 minutes for students to correct or revise their work. They should strive to reach the top two levels of the rubric or metric.
- ✍ Collect the work. Score it with your rubric or metric. However, to allow students time to learn to meet your expectations, you may not want to record the score yet.
- ✍ For the next several days, whether you assign prompts as a warm-up or for homework, allow students time to revise or correct their work. You should stress that they should strive to reach the top two levels. Repeat this phase as often as needed to help students understand your expectations for their writing.
- ✍ If possible, show students samples of other students' writing. Use this sample to illustrate what you mean by your criteria in the rubric or metric. A sample of student writing in the middle grades follows.

Sample of Student Work

Prompt: Corey said, “Division is commutative.” Do you agree with Corey? Why or why not?

○	<p>No, I don't agree with Corey. He must be thinking about multiplication or addition because division is not commutative. If it was commutative, it would mean that you can put the numbers you're dividing in any order and you can't. If you put $4 \div 2$ you get 2. But if you put $2 \div 4$ you get one half and that's not the same.</p> <p>In multiplication you can put the numbers in any order and still get the same answer. Like if you multiplied 3×2 or 2×3, it doesn't matter. You still get 6.</p> <p>I think Corey messed up and really meant to say that division is NOT commutative. He should be more careful about his work.</p>
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Content prompts

Number Sense, Properties and Operations with Real Numbers

- 1. Temeka wrote, “ $\frac{1}{5} + \frac{5}{6} = \frac{6}{11}$.” Use an estimation strategy to show that her solution is either correct or incorrect. Be specific in your description of how you used your estimation strategy.**

Her solution is incorrect. A benchmark estimation strategy may be used. The benchmarks are 0, $\frac{1}{2}$, and 1 could be mentioned but other estimation strategies are also possible.

- 2. Kim said, “The zero in 13.01 is not really important. After all, zero just means nothing.” Jon thought it was important, but he wasn’t sure. What do you think? Explain your thinking.**

The zero is important. Without it, the number would represent a greater quantity than 13.01. Students should provide some explanation of the zero as a placeholder.

- 3. Kim said, “Fractions were difficult. Decimals should be easier.” “But aren’t fractions and decimals related?” asked Kris. What would you tell Kim and Kris? Give specific examples to support your response.**

They are both rational numbers. Students may suggest that each of these number types can represent part-whole relationships. Models for fractions can be applied to decimals.

- 4. Keisha rounded a number to 85. Find a number that Keisha could have used and explain why it could be rounded to 85. Then find at least two more numbers that could be rounded to 85. What do these numbers have in common?**

Some students may round from tenths or other decimal places (e.g., 84.6 or 85.21). Other students may use whole numbers such as 83 or 86. All responses should include an explanation about why the numbers they selected could be rounded to 85. A description of their similarities should also be included.

- 5. “I think subtraction is not commutative,” said Alvina. “I agree,” said Olivia, “but I can think of a time when order does not matter in subtraction.” What do you think Olivia meant? Can you find an instance when order does not matter? Explain.**

Students should describe an instance when the minuend and the subtrahend are the same numbers. This is the only time when order does not matter. The journal prompt can be changed to represent division rather than subtraction. In the case of division, the response is the same but students should note that the divisor and dividend cannot both be zero nor can the divisor be zero.

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6. **Hugh found a number line like the one shown. Where on the number line would you find a number smaller than 247? How do you know?**



Numbers smaller than 247 are found to the left of 247. Students should support their answer in some way.

7. **“I asked Maggie how much pizza she had. She said she had half a pizza. But when I asked the teacher how many students were gone today, she said half!” exclaimed Stephanie. “How can she use half to describe both things?” What would you tell Stephanie? Use specific examples.**

One-half can be used to describe situations in which there is a ratio of one to two. Even though the number is the same, the quantity it represents is relative or proportional to the whole.

8. **Carlos wondered if addition and subtraction were related. “If they are related,” said Carlos, “then that relationship could help me solve addition and subtraction problems.” Do you think they are related? Why or why not? If they are related, how could that help you solve addition and subtraction problems?**

Addition and subtraction are related because they are inverse operations or opposite operations. Some students may say they are not related because they do not see relationships in the computational algorithms or similarities in the concrete models. Encourage students, for example, to think about how addition can be used to solve subtraction problems.

9. **Samantha asked, “Why do we need numbers larger than 1,000,000?” What do you think Samantha’s teacher told her? Be specific.**

Answers will vary. Students' responses often include everyday contexts that require large numbers.

10. **Tasha said, “If I add two even numbers, I get a sum that is even.” Twaina said, “If I add two odd numbers, I get a sum that is even.” Do you agree with either of these girls? Justify why you agree with her.**

They are both correct. Students should support their position with specific examples or a word description.

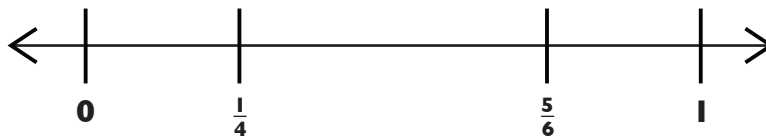
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- 11. Darrell made a chart to show equivalent fractions. He wrote “ $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} \dots$.” What patterns do you notice? What would be the next three fractions in his pattern?**

Patterns will vary. Some students may notice that the numerator increases by 1 and denominator increases by 2. Others may compare each fraction to $\frac{1}{2}$ and indicate that each fraction is a multiple. Some may note that the denominators are even numbers. His next fractions are $\frac{5}{10}, \frac{6}{12}, \frac{7}{14}$.

- 12. Seth said, “I ate one half of a pizza, which was three slices.” Jena said, “I ate four slices of pizza, and it was one half of a pizza.” Who is telling the truth? Prove it.**

Both could be telling the truth. Seth’s pizza could have been cut into six pieces while Jena’s was cut into eight pieces.

- 13. Sara drew a number line.**



She asked, “Trey, can you find a fraction between $\frac{1}{4}$ and $\frac{5}{6}$?” “I can do that,” said Trey. “In fact, I can find three fractions.” What fractions do you think Trey found? Justify or support your answer.

There are infinite fractions that could be found. For example, $\frac{1}{2}, \frac{4}{5}, \frac{2}{3}, \frac{5}{8}$, and $\frac{8}{15}$ are all possibilities. Students should present some justification or support for their responses.

- 14. “I wrote an equation but I can’t remember what the word problem was,” said Faith. Write a word problem that can be solved by $37 + 70 = 107$.**

Answers will vary. Be sure to check the problem to determine if addition is the correct operation.

- 15. Amanda solved a division problem.**

$$607 \div 24 = 25 \text{ R}7$$

Tino asked, “What does the remainder 7 mean?” What do you think Amanda told him?

Students may indicate that 7 represents the amount left over when you group the 607 into groups of 24. Or they may indicate that it is the amount left over when you put 607 into 24 groups.

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- 16. Find five pairs of numbers whose difference is 59. Write the subtraction sentences. What patterns do you notice?**

Check the subtraction sentences for accuracy. Students may say that all of the numbers are 59 units apart on a number line. Other descriptions are also possible.

- 17. Courtney said to her friend Ted, “We multiply two fractions in our homework. Can you think of a word problem that could be solved by multiplying two fractions?” What do you think Ted said? Be specific. If Ted knows a word problem, write the word problem he might have given Courtney.**

Students may describe various scenarios. Check their descriptions or word problems for appropriate links to multiplying two fractions.

- 18. Brendan asked, “I’m adding two fractions, $\frac{5}{6}$ and $\frac{7}{8}$. Is the sum more than or less than 1?” “I think the sum is more than 1,” said Bryce. Why do you think Bryce thought that? Explain your thinking.**

Bryce is correct. Students may explain it in multiple ways. One way is to think that both fractions are close to 1. If you add them, then their sum should be more than 1.

- 19. Rheta said $\frac{4}{9} \div \frac{2}{3}$ was solved by dividing 4 by 2 and 9 by 3. Her quotient is $\frac{2}{3}$. Do you agree or disagree with her method? Why?**

This prompt will provide some interesting looks at fractional division. New algorithms may be developed.

- 20. Tina asked, “How are ratios similar to proportions?” What would you tell Tina? Be specific.**

A ratio compares two quantities. A proportion indicates that two ratios are equal. You can extend this journal prompt by asking students to compare or contrast ratios with fractions.

- 21. Darron’s calculator is broken. It doesn’t always put the decimal point in the right place so he has difficulty deciding if a problem is correct. He added $43.7 + 2.01 + 6.05 + 1$. His calculator showed a sum of 527.6. Do you agree with the calculator? Why or why not?**

Students should disagree because an estimate of the sum is closer to 52. Some students may show you how to solve the problem with an exact answer. If you notice that students have used number sense or estimation skills, you may want them to share those with the whole class.

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- 22. Jennifer and Patty are arguing about what a rational number is. Describe, using examples, what you think it is.**

Answers will vary. It will be interesting to note how many students, if any, indicate that a whole number is also a rational number. If you have studied integers, students should also include integers in their descriptions.

- 23. Kenyatta asked, “Why do fractions have to have common denominators before they can be added or subtracted?” What do you think? Support your answer with specific examples.**

Responses should indicate that you cannot identify the sum (or the quantity) without having some common basis for comparison. The common denominator provides that common basis. To extend this prompt, you can ask students about using a common denominator for multiplication or division with fractions.

- 24. Marty dropped spaghetti sauce on his math paper. What do you think is under the sauce he dropped? Why?**

$$\frac{1}{4} \div \text{[sauce]} = 4$$

Students should indicate that $\frac{1}{16}$ must be the divisor. Their reasoning can vary. Intuitively, students may say that there are four $\frac{1}{16}$'s in $\frac{1}{4}$ rather than doing any computation. Encourage this type of thinking.

- 25. Megan said, “Composite numbers are different from prime numbers.” “How?” asked River. What do you think Megan would tell River? Be specific.**

Answers will vary. Students' responses may focus on the number or types of factors each has. Encourage students to give examples of composite and prime numbers. If area models have been used to illustrate these types of numbers, you can ask students to compare the models of each type of number.

- 26. Lehua said, “There are many prime numbers. I know if we keep looking for prime numbers, we can find one that is even and larger than 29.” Jonathan replied, “That can never be possible.” Who do you agree with? Justify your choice.**

Any even number after 2 has at least 2 as a factor. Therefore, those numbers cannot be prime. Agree with Jonathan.

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- 27. Sasha said, “One is the smallest prime number.” Jeri said, “No, two is the smallest prime number.” Who do you agree with? Support your choice.**

One has only one factor, itself. Therefore it is not a prime number. Two is the smallest prime number. Agree with Jeri.

- 28. Tanya rounded a fraction to $\frac{1}{2}$. What fraction(s) could she have rounded? Why?**

Students may comment on the relations of the numerators and denominators in estimating fractions. They may list many fractions, including but not limited to $\frac{3}{5}$, $\frac{4}{9}$, $\frac{3}{6}$, $\frac{11}{20}$, $\frac{3}{5}$ and so on.

- 29. Why is two the only even prime number? Explain, giving examples to support your response.**

After two, all even numbers have two as a factor.

- 30. Jason asked, “What does it mean if two numbers have a common multiple?” What can you tell Jason? Give examples that will support your answer.**

Students should include some description of what a common multiple means. They may comment on the process of finding a common multiple as part of their support.

- 31. Elly asked, “How do I determine if a number is prime?” What would you tell Elly? Use specific examples to justify your responses.**

Students may indicate that factoring would help. Accept other methods that are appropriate such as using divisibility rules. Be sure that students include at one specific example to support their response.

- 32. Is 102 a multiple of 14? Explain.**

It is a multiple. Students should provide a credible explanation or show that $14 \cdot 6 = 102$. Students may focus on the process but you may want to refocus their discussion on the definition of a multiple.

- 33. “Prime numbers and composite numbers must have something in common,” said Chris. “They do,” said Jessica. “They also have some differences.” Name at least three ways prime and composite numbers are alike and at least three ways they are different.**

They both have factors but prime numbers do not have as many factors as composite numbers. There is only one even number that is prime but there are many even numbers that are composite. Other similarities or differences are possible.

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- 34. Wen said, “Tony said that one is not a prime number. But I thought it was. Is Tony right?” What would you tell Wen? Be specific.**

Students should indicate that Tony is correct. One has only one factor, itself. It does not fit the definition of prime numbers.

- 35. “I think there is some relationship between factors and multiples,” said Chloe. “No,” said Clayton. “They are two different things. There is no relationship.” Who do you agree with? Why? Support your answer.**

There is a relationship. The product of two factors is a multiple of both factors.

- 36. Brad asked, “What’s the largest number of digits possible in the product of a two-digit number times a three-digit number?” What would you tell Brad? Be sure to provide an argument that is convincing.**

Students should indicate that the maximum number of digits in the product can be five. They may provide multiple ways of giving a rationale for their response. One way is to use 99×999 as a means to determine the maximum number of digits.

- 37. Noni asked, “How are whole numbers related to integers?” What would you say to Noni? Describe in detail.**

Integers are whole numbers and their opposites. Students should describe the relationship with sufficient detail.