Journal Writing

Journal writing can be structured to give teachers cohesive and comparable information about students and their thinking while challenging them through contextual situations. A structure for journal writing includes prompts that focus on (1) mathematical content, (2) mathematical processes, and (3) student attitude or affect. Journal prompts give situations or questions to which students respond. Responses may include words, pictures or drawings, or symbols. Students are encouraged to support their ideas and to clearly explain what they mean. They can give specific examples as part of their explanations or use counterexamples.

Content prompts relate or connect topics within and outside of mathematics, targeting important or meaningful concepts and skills. They can also provide situations that focus on areas where students often have misunderstandings or misconceptions. The responses to the prompts give teachers (and students) insight into how a student has interpreted a mathematical idea.

Process prompts promote the awareness of how students solve or approach problems or algorithms. The responses to these prompts can give insight into students' preferences for problem-solving strategies or algorithms and into how they learn or remember. As students become aware of how they learn and solve problems, they grow more confident in approaching new or novel problems.

Attitudinal or affective prompts focus on students' feelings about themselves as mathematicians and students of mathematics. Students' responses allow teachers to assess how positive attitudes about mathematics and mathematicians are developing in the classroom environment.

Extended Problem-solving Tasks

Extended or expanded problem-solving tasks provide opportunities for students to explore and solve problems that require novel solution approaches. For this purpose, problem solving is defined as confronting a problem that does not have an obvious solution or solution path. In most cases, a non-routine solution method (or combination of methods) is required such as making a list, drawing a diagram, working backwards, guessing-and-testing, or creating a table.

Extended problem-solving tasks require more time and thought to solve than routine problems. Students draw on their previous knowledge and experiences to reason through the problem. Because their thought processes will be more complex, writing an expanded solution is an important part of communicating their methods or processes to others. Writing a response to an extended problem-solving task also helps students create a solution process as they clarify what the problem is asking, what information is given in the problem, and what solution methods would be appropriate.

Many students believe problem solving to be a linear process. That is, they read a problem, think of a solution method, solve the problem, and check their answer. Problem solving is more complex. It often requires re-reading a problem or abandoning one solution method for another.

Assessment Tasks Requiring Writing

Assessing student understanding can be done in a variety of ways including journal writing, homework problems, problem-solving write-ups, quizzes, and tests. Any assessment should encompass at least three types of tasks:

(1) problem solving, (2) conceptual understanding, and (3) skill acquisition.

Of the three types of assessment tasks, skill acquisition is most often assessed. These tasks would include solving equations and inequalities or using formulas by primarily symbol manipulation. Students often apply an algorithm that may or may not convey their understanding.

Items that are designed to assess students' conceptual understanding or ability to problem solve can provide a rich means by which students demonstrate their thinking and interpretations of concepts through expanded responses. The inclusion of these types of items link assessment with classroom practice. If students are required in mathematics classes to explain their thinking in class discussions or on their homework papers, it is important that assessments also include similar tasks. Likewise, if state assessments include self-constructed response items, students will develop skill in responding to such items when these types of tasks are regularly included on a daily basis as well as on assessments.

Description of Materials and Teaching Suggestions

There are over 150 journal prompts here for teachers to choose from. Some of them may be used more than once to ascertain how students are growing in their understandings of and beliefs about mathematics. The prompts may also be used as assessment tasks for expanded response or self-constructed response items. As they use these prompts teachers may generate other ideas that can be used to create their own journal prompts.

Journal Prompts

- ✎ Each mathematical content prompt includes a short description of what to expect or consider in students' responses. The mathematical content response should reflect a student's progress in understanding the concept or skill.
- ✎ The content prompts were selected to match recommendations from the National Council of Teachers of Mathematics (*Principles and Standards for School Mathematics, 2000*) rather than to fit a specific text. Locate the topic students are studying and then select the prompt that best fits the class. The wording may be changed if students will not understand the terminology or vocabulary used.
- ✎ Process prompts give students opportunities to show how they do a mathematical process or algorithm (step-by-step procedure). When teachers use these prompts, they gain insight into how students have interpreted or altered procedures. In some cases, they may find student-invented algorithms that are appropriate to share with the class.
- ✎ Affective or attitudinal prompt responses show students' beliefs. If used more than once, teachers should be able to detect changes in beliefs or feelings about mathematics as a discipline as the year or semester progresses. Similarly, students' views of themselves as mathematicians or as students of mathematics can be assessed.

  Teachers can use writing prompts daily or intermittently, depending on the class, and prompts can be repeated from time to time. In prompts that use students' names, they may change the names to represent their students. If teachers use journal prompts daily, it is recommended that students respond to at least three content prompts, no more than one process prompt, and no more than one attitudinal prompt in a week. The more frequently writing is used as a regular part of mathematics class, the better students become at responding. More specific directions for implementing journal writing are included at the end of this section.

. . . and more

- ✎ There are 12 extended problem-solving tasks. Each task requires more time to solve than one class period. Students often provide the best solutions if they are given 10 days in which to solve it. The teacher may decide to use one of these every 3 weeks or so.
- ✎ In some cases, teachers may assign the tasks for the entire class to work individually. These tasks also give teachers and students the opportunity to use pair or group problem solving. Regardless, it is important that students write their responses in a way that a reader can see the flow of their thinking and understand the solution method or path that they used.
- ✎ It is recommended that students do extended problem solving on a regular basis. This practice supports their development of problem-solving strategies and boosts their confidence to solve complex problems.
- ✎ For each problem-solving task a solution has been given. However, there are multiple methods to solve each problem. Teachers should be open to creative ways that students may approach these problems.
- ✎ There are 10 assessment items included here. These items represent a conceptual approach to a particular mathematical topic. There should be no more than one of these items on a chapter test. If used independent of the chapter quizzes or tests, however, it is possible to use more than one. Additionally, any of the content or process journal prompts can be used on formative or summative assessments in a similar manner.
- ✎ It is important to recognize that students should have experience with talking and writing mathematics daily in order for the use of these assessment items to be representative of student understanding. If a teacher uses a lecture-based approach, some of these items may need alteration to allow for optimal responses from students.

Evaluating Students' Responses

Students tend to give the writing more thought if it is to be scored. One method used was to score the responses on content rather than on grammar and spelling. An essay grading method was used. By reading two or three papers to get a feel for students' responses these first papers formed the baseline for scoring other papers. This method, however, did not provide students a guideline for writing in advance.

When teachers use rubrics and metrics, however, scoring guidelines can be provided in advance. A rubric establishes qualitative levels that define what characteristics a response has or what criteria it meets. The qualitative levels are not used as points. On the other hand, a metric is a point system. The levels of a metric have specific criteria associated with each point value. The same criteria may be used for either a rubric or a metric system. The difference is in whether or not points or qualitative levels are established for the response.

There are many ways to create a rubric or metric. Teachers can develop one alone, or students can work with the teacher to develop a rubric or metric specific to their class or to the task. Sample rubrics and metrics, developed with students' participation, follow on the next page.

Although grammar and punctuation are not usually scored, students should communicate their ideas in ways that are understandable. Some students may use drawings, tables, charts, or other means to convey their ideas. They should be encouraged to use whatever ways they need to make their ideas clear.

As writing tasks are used, excerpts can be taken from students' papers to illustrate qualities that you consider important. Both high- and low-quality responses can be used to show students the comparison with rubric or metric criteria. Of course, authorship of whatever responses are selected should be kept anonymous.

Rubrics or metrics to score the problem-solving tasks as extended types or as assessment items can be developed for each individual task or created as a general guide for student performance. The rubrics or metrics that follow can also be adapted to serve as a generalized set of criteria to guide students' solution approaches. If rubrics or metrics are being used by the entire mathematics department, it may be appropriate to have department-wide discussions to agree upon criteria. This will provide a means by which to motivate consistent and cohesive student work across grades, courses, and teachers.

With any form of writing and any type of rubric or metric, students can self-evaluate their responses or conduct peer evaluations. This allows you to see if students truly understand the criteria outlined in the rubric or metric. This activity also requires students to think at a much higher level as they analyze critically others' work .

General Metric

4 points **The student's work includes—**

- ✎ completed prompt or an answer to the question posed
- ✎ support for statements made by using either examples or counterexamples
- ✎ ideas clearly communicated to the reader
- ✎ legible writing, drawings, pictures, charts or tables, and diagrams
- ✎ accurate mathematics or information

3 points Omission of one criterion from level 4

2 points Omission of two criteria from level 4

1 point Omission of three criteria from level 4

0 points Omission of more than three criteria from level 4

Three-level Rubric (or Metric)

The student's work shows a response that—

Exceeds standard

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader

Meets standard

- ✎ addresses the question raised in the prompt
- ✎ has some correct or accurate mathematics
- ✎ is legible
- ✎ does not support or justify some of the statements made
- ✎ makes sense to the reader

Below standard

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not support or justify statements made

Five-level Rubric (or Metric)

4 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader.

3 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ does not have fully justified or supported statements
- ✎ makes sense to the reader

2 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has some incorrect or inaccurate mathematics
- ✎ is legible
- ✎ does not have justified or supported statements
- ✎ is somewhat clear to the reader

1 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is partially legible
- ✎ does not have justified or supported statements
- ✎ does not make sense to the reader

0 **The student's work shows a response that—**

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not make sense to the reader

Implementing Journal Writing in Your Classroom

- ✎ To begin using writing in your classroom, you will need to make sure your students understand your expectations for writing. The following offers one method for helping students learn what is meant by *writing in mathematics.*
- ✎ Share with students the rubric or metric you will be using. You may opt to create a rubric or metric with your students rather than creating one yourself. Make copies of the rubric or metric for students to keep in their notebooks. Post one copy in the classroom for easy reference.
- ✎ Give students a practice prompt to write. If it is used as a warm-up, allow about 6 minutes for them to respond. The practice prompt can be any type, but the rubric or metric may work better with a content prompt. Select one that you feel all students in your class can attempt.
- ✎ Have students compare what they wrote with their partner or table mates. They should check the rubric or metric. Have students focus on 3 things they could do to improve their writing to the next higher level. If their writing already includes all the indicators for the top level, ask them to write another question that this prompt made them think about.
- ✎ Allow 3 minutes for students to correct or revise their work. They should strive to reach the top two levels of the rubric or metric.
- ✎ Collect the work. Score it with your rubric or metric. However, to allow students time to learn to meet your expectations, you may not want to record the score yet.
- ✎ For the next several days, whether you assign prompts as a warm-up or for homework, allow students time to revise or correct their work. You should stress that they should strive to reach the top two levels. Repeat this phase as often as needed to help students understand your expectations for their writing.
- ✎ If possible, show students samples of other students' writing. Use this sample to illustrate what you mean by your criteria in the rubric or metric. A sample of student writing in the middle grades follows.

Sample of Student Work

Prompt: Corey said, "Division is commutative." Do you agree with Corey? Why or why not?

No, I don't agree with Corey. He must be thinking about multiplication or addition because division is not commutative. If it was commutative, it would mean that you can put the numbers you're dividing in any order and you can't. If you put 4 ÷ 2 you get 2. But if you put 2 ÷ 4 you get one half and that's not the same.

In multiplication you can put the numbers in any order and still get the same answer. Like if you multiplied 3 x 2 or 2 x 3, it doesn't matter. You still get 6.

I think Corey messed up and really meant to say that division is NOT commutative. He should be more careful about his work.

Journal Prompts

In this section 168 prompts are provided in categories described in the introduction and description of materials. As a teacher you can give prompts as a homework assignment in conjunction with a textbook assignment. Or, you can use them as a warm-up for the first 5–7 minutes of the class period. Student responses can be collected and graded or students can share their ideas within a whole class discussion. Their ideas can be recorded on chart paper, on transparencies, or on the board. The chart paper provides a permanent record or archive of student responses so that they can be revisited later in the unit of study or in an associated unit. An archive for responses allows the progress of students' understandings, or thinking about mathematical concepts or skills, to be monitored.

You can also use the prompts in a problem-solving context where students create a solution in a pair, group, or individually. If pairs or groups work on the problems, they can share their ideas before the whole class. Individual student responses can be shared by having other students read them and analyze the response.

Prompts are organized as follows:

Content Prompts

Number Sense, Properties and Operations with Real Numbers

1. **Sharona wrote, "$\frac{2}{5} + \frac{3}{7} = \frac{5}{12}$." Use an estimation strategy to show that her solution is either correct or incorrect. Be specific in your description of how you used your estimation strategy.**

   Her solution is incorrect. A benchmark estimation strategy may be used. The benchmarks are 0, $\frac{1}{2}$, and 1 could be mentioned but other estimation strategies are also possible.

2. **Jorge said, "I get the same product when I multiply 1.8 x 48 as well as when I multiply 18 x 4.8." "That makes sense," said Tori. Why would Tori say this? Verify that this is true.**

   Students may include a discussion about the equivalency of the two expressions. If one factor is increased by a factor of 10 and the other factor by a factor of 0.10, the product will remain the same. Some students may multiply to show that the product is the same but encourage them to focus on the generalization of the mathematics.

3. **Molly said, "Fractions were difficult. Decimals and percents should be easier." "But aren't fractions, decimals and percents related?" asked Bart. What would you tell Molly and Bart? Give specific examples to support your response.**

   They are all rational numbers. Students may suggest that each of these number types can represent part-whole relationships. Models for fractions can be applied to percents or decimals.

4. **"Which problem has the larger product?" asked Carrie. "$\frac{1}{2}$ x $\frac{3}{8}$, or $\frac{1}{8}$ x $\frac{3}{2}$?" Decide which one you believe has the larger product. Justify your answer <u>without</u> computing the product.**

   The products are equal. Students may note that with the multiplication algorithm, denominators are multiplied. Thus the change does not affect the product. Other students may focus on the quantities represented by the fractions and note the meaning of multiplication of fractions.

5. **"I think division is not commutative," said Mishi. "I agree," said Olivia, "but I can think of a time when order does not matter in division." What do you think Olivia meant? Can you find an instance when order does not matter? Explain.**

   Students should describe an instance when the dividend and the divisor are the same numbers. This is the only time when order does not matter. There should be mention that the divisor (and subsequently, the dividend) cannot be 0.

6. **Tara wrote, "The difference of two whole numbers is always a whole number." Kris read the statement. Then she said, "Is that always true?" What do you think? Justify your idea(s) with details.**

   It is not always true, only if the subtrahend is smaller than or equal to the minuend.

7. **"I asked Carenna how much pizza she had. She said she had half a pizza. But when I asked the teacher how many students were gone today, she said half!" exclaimed Minna. "How can she use half to describe both things?" What would you tell Minna? Use specific examples.**

   One-half can be used to describe situations in which there is a ratio of one to two. Even though the number is the same, the quantity it represents is relative or proportional to the whole.

8. **Darron's calculator is broken. It doesn't always put the decimal point in the right place. So he has difficulty deciding if an answer is correct. He multiplied 43.7892 x 2.01605. His calculator showed a product of 8.8281216. Do you agree? Why or why not? Support your answer.**

   Disagree because an estimate of the product is closer to 88. Some students may show you how to solve the problem with the multiplication algorithm Expect more justification than merely counting the decimal places.

9. **Shamika said, "Justin, I'm going to give you a number sentence. Your challenge is to write a word problem that uses percents and can be solved by the number sentence. The number sentence is 0.25 x _____ = 140.10." What do you think Justin wrote? Justify your answer.**

   Answers will vary. Be sure to check the problem to determine if multiplication is the correct operation and aligns with the context of the problem.

10. **Amy wrote the following on the board.**

    4 ☐ 4 ☐ 4 ☐ 4 = 24

    Use any operational symbols in the boxes to make the number sentence true. Justify your answer by describing your process.

    A possible solution is $4 + 4 + 4 \times 4 = 24$. Students should relate the solution to the order of operations.

11. Find four ways to write 64 as a product of two numbers. Justify your answer.

There are infinite multiplication problems that can be written. Some of the pairs of numbers that can be used are 1 and 64, 2 and 32, 4 and 16, 8 and 8, 0.50 and 128, and so on. Some type of justification should be included.

12. Norma said, "-2^2 is -4." Norm argued, "No, -2^2 is 4." Who do you agree with? Why?

Students should agree with Norma and support their answer. Some students may use order of operations to support their answers.

13. Lia drew a rectangle. She shaded an area that represented $\frac{1}{3}$ of the area. Maiki traced Lia's rectangle and shaded an area that represented $\frac{3}{9}$ of the area. Kerstin traced Lia's rectangle and shaded an area that represented $\frac{4}{12}$. Pete looked at their rectangles and the shaded areas. He said, "I notice something." What do you think Pete noticed? Be specific and support your answer.

The shaded portions should represent the same area. The fractional amounts are equivalent fractions.

14. Seraphina drew a number line.

She asked, "Trey, can you find a fraction between $\frac{1}{4}$ and $\frac{5}{6}$?" "I can do that," said Trey. "In fact, I can find five fractions." What fractions do you think Trey found? Justify or support your answer.

There are infinite fractions that could be found. For example, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{8}{15}$ are all possibilities. Students should present some justification or support for their responses.

15. Place parentheses in the expression to make a true statement. Justify your answer.

$$5 \cdot 2 + 3 \cdot 4 = 22$$

$(5 \cdot 2) + (3 \cdot 4) = 22$. Some justification should be given.

16. Gibson wanted to compute the sales tax on a CD that cost \$18 and a book that cost \$9. If the tax is 6%, show Gibson two ways he could compute the tax correctly. Describe both ways to him.

Students typically use the distributive property to find the two ways. The tax can be computed on the amounts separately and then added OR the tax can be computed on the total cost.

17. Find five pairs of numbers whose sum is 0. Write the addition sentences. What patterns do you notice?

Students should notice that the numbers are opposites or additive inverses. These patterns can introduce a discussion about these topics.

18. Ashe said, "Here's a challenge for you. Can you find two fractions with unlike denominators whose sum is $\frac{2}{3}$?" "I think I can," said Tyra. What fractions do you think Tyra found? Describe your thinking.

There are multiple answers to Ashe's challenge. For example, one pair might be $\frac{1}{2}$ and $\frac{1}{3}$.

19. Brendan asked, "I'm adding two fractions, $\frac{5}{6}$ and $\frac{3}{4}$. Is the sum more than or less than 1?" "I think the sum is more than 1," said Bryce. Why do you think Bryce thought that? Explain your thinking.

Bryce is correct. Students may explain it in multiple ways. One way to approach is to think that both fractions are close to 1. If you add them, then their sum should be more than 1.

20. Rheta said $\frac{4}{9} \div \frac{2}{3}$ was solved by dividing 4 by 2 and 9 by 3. Her quotient is $\frac{2}{3}$. Do you agree or disagree with her method? Why?

This prompt will provide some interesting looks at fractional division. New algorithms may be developed.

21. Tina found a prime number larger than 37 but smaller than 61. What number do you think she found? Prove that it is prime.

Prime numbers larger than 37 but smaller than 61 include 41, 43, 47, 53, or 59. Students should provide a discussion about the factors of their number or some other justification that is related to prime numbers.

22. Kealoha asked, "How can I estimate the product of 47.84110792483121 x 7.3?" "I know an easy way," said Jack. What do you think Jack told Kealoha? Describe Jack's process without computing. Be specific.

Students may use the front-end method of estimation. Their estimate may vary by the numbers that are chosen. For example, one student may use 47 x 7 and then indicate that the actual product is larger than this because 47 and 7 are both smaller than the actual numbers. Other students may use 50 and 7 or 50 and 7.3 with similar discussions.

23. Ms. Jackson told her class that addition and subtraction are opposite operations. What did she mean by that? Be specific in your explanation.

Addition and subtraction can be thought of as opposite operations because one can 'undo' the other. Students may present informal explanations that show this or others may present a formal proof.

24. "We're studying scientific notation," said Wendy. "Why are you studying scientific notation?" asked Tracy. "When will you ever use it?" What could Wendy say to Tracy? Support your answer with examples.

Students usually indicate that it is needed to describe very large or very small numbers. Some may relate it to calculator use or give science examples.

25. Marty dropped spaghetti sauce on his math paper. What do you think is under the sauce he dropped? Why?

14 ÷ 0 = 

Division by 0 is undefined and they should give a rationale as to why it is undefined.

26. Megan said, "I think prime numbers have some things in common with composite numbers." "Like what?" asked Hudson. What do you think Megan would tell Hudson? Be specific.

Answers will vary. Students' responses may focus on the idea that both have factors.

27. Lehua said, "There are many prime numbers. I know if we keep looking for prime numbers, we can find one that is even and larger than 111." Jonathan replied, "That can never be possible." Who do you agree with? Justify your choice.

Any even number after 2 has at least 2 as a factor. Therefore those numbers cannot be prime. Agree with Jonathan.

28. Sasha said, "One is the smallest prime number." Jeri said, "No, two is the smallest prime number." Who do you agree with? Support your choice.

One has only one factor, itself. Therefore it is not a prime number. Two is the smallest prime number. Agree with Jeri.

29. Tanya rounded a fraction to $\frac{1}{2}$. What fraction(s) could she have rounded? Why?

Students may comment on the relations of the numerators and denominators in estimating fractions. They may list many fractions, including but not limited to $\frac{3}{5}$, $\frac{4}{9}$, $\frac{3}{6}$, $\frac{11}{20}$, $\frac{3}{5}$ and so on.

30. Why is two the only even prime number? Explain, giving examples to support your response.

After two, all even numbers have two as a factor.

31. Jason said, "All numbers have an even number of factors." "No, they don't," said Brad. "Some special numbers have an odd number of factors." Why would Brad say that? What was he talking about?

Students should mention that square numbers have an odd number of factors. Other composite and prime numbers have an even number of factors.

32. Elly asked, "How do I determine if a number is prime?" What would you tell Elly? Use specific examples to justify your responses.

Students may use divisibility rules or ideas to explain how Elly can determine if a number is prime. Accept other methods that are appropriate. Be sure that students include at one specific example to support their response.

33. Is 112 a multiple of 14? Explain.

It is a multiple. Students should provide a credible explanation or show that $14 \cdot 8 = 112$.

34. "Prime numbers and composite numbers must have something in common," said Chris. "They do," said Jessica. "They also have some differences." Name at least three ways prime and composite numbers are alike and at least three ways they are different.

They both have factors but prime numbers do not have as many factors as composite numbers. There is only one even number that is prime but there are many even numbers that are composite. Other similarities or differences are possible.

35. The product of two numbers is –16. List all possible pairs of integers that could be factors. What do the pairs have in common?

Students usually notice that only one factor is negative. The pairs they should include are –1 and 16, 1 and –16, –2 and 8, 2 and –8, 4 and –4.

36. "I think there is some relationship between factors and multiples," said Chloe. "No," said Clayton. "They are two different things. There is no relationship." Who do you agree with? Why? Support your answer.

There is a relationship. The product of two factors is a multiple of both factors.

37. How could you explain to a 5th grader why the product of 13 and 27 is less than product of 17 and 23? Be sure to be explicit.

The factor 13 can be thought of as being added 27 times versus adding 17, 23 times. Another explanation might be to think that 13 is multiplied by 20 and by 7 while 17 is multiplied by 20 and 3. Thus 27 × 13 results in a smaller product. If this prompt is used, students' responses can be used to consider generalized results so that if any four distinct numbers are given, students can use their generalization to create two two-digit numbers that give either the smallest or the largest product possible.

38. Noni asked, "How are whole numbers related to integers?" What would you say to Noni? Describe in detail.

Integers are whole numbers and their opposites. Students should describe the relationship with sufficient detail.