The "Write" Way Mathematics Journal Prompts and More

FOR ALGEBRA II

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Introduction

The Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) includes a separate process standard for Communication for grades 9 through 12. This standard advocates using multiple activities that give students opportunities to demonstrate their understandings. These tasks should allow students to express themselves in a variety of ways such as with words, concrete models, pictures or drawings, or abstract symbols. Each of these forms lets students communicate with others about their thinking. At the same time, students are developing a deeper understanding of the mathematics because they have to be clear and explicit when they are writing to an audience that cannot see their facial expressions and gestures, or ask clarifying questions. Thus students understand more and retain their understanding longer.

Communication can include at least five types of activities: (1) speaking, (2) listening critically, (3) reading, (4) modeling or constructing, and (5) writing. Speaking or talking about mathematics should be encouraged and used daily in mathematics classes. Students can talk in pairs or small groups as well as present their ideas to the whole class. If students are talking about mathematics, others should be critically listening so that any student should be able to comment on the ideas presented by another. Students should also be reading in a mathematics class. The reading could be centered on the textbook but can be expanded to include students' writing or trade books. Modeling or constructing means that students can also communicate to others by using concrete models (such as Algebra Tiles™), drawing pictures or diagrams, or using technology such as graphing calculators. Finally, communication also includes writing such as journal writing and extended or expanded problem-solving tasks.

The following sections describe different types of journal writing, extended problem-solving tasks, and the use of these two forms of writing in assessment.

Journal Writing

Journal writing can be structured to give teachers cohesive and comparable information about students and their thinking while challenging them through contextual situations. A structure for journal writing includes prompts that focus on (1) mathematical content, (2) mathematical processes, and (3) student attitude or affect. Journal prompts give situations or questions to which students respond. Responses may include words, pictures or drawings, or symbols. Students are encouraged to support their ideas and to clearly explain what they mean. They can give specific examples as part of their explanations or use counterexamples.

Content prompts relate or connect topics within and outside of mathematics, targeting important or meaningful concepts and skills. They can also provide situations that focus on areas where students often have misunderstandings or misconceptions. The responses to the prompts give teachers (and students) insight into how a student has interpreted a mathematical idea.

Process prompts promote the awareness of how students solve or approach problems or algorithms. The responses to these prompts can give insight into students' preferences for problem-solving strategies or algorithms and into how they learn or remember. As students become aware of how they learn and solve problems, they grow more confident in approaching new or novel problems.

Attitudinal or affective prompts focus on students' feelings about themselves as mathematicians and students of mathematics. Students' responses allow teachers to assess how positive attitudes about mathematics and mathematicians are developing in the classroom environment.

Extended Problem-solving Tasks

Extended or expanded problem-solving tasks provide opportunities for students to explore and solve problems that require novel solution approaches. For this purpose, problem solving is defined as confronting a problem that does not have an obvious solution or solution path. In most cases, a non-routine solution method (or combination of methods) is required such as making a list, drawing a diagram, working backwards, guessing-and-testing, or creating a table.

Extended problem-solving tasks require more time and thought to solve than routine problems. Students draw on their previous knowledge and experiences to reason through the problem. Because their thought processes will be more complex, writing an expanded solution is an important part of communicating their methods or processes to others. Writing a response to an extended problem-solving task also helps students create a solution process as they clarify what the problem is asking, what information is given in the problem, and what solution methods would be appropriate.

Many students believe problem solving to be a linear process. That is, they read a problem, think of a solution method, solve the problem, and check their answer. Problem solving is more complex. It often requires re-reading a problem or abandoning one solution method for another.

Assessment Tasks Requiring Writing

Assessing student understanding can be done in a variety of ways including journal writing, homework problems, problem-solving write-ups, quizzes, and tests. Any assessment should encompass at least three types of tasks:

(1) problem solving, (2) conceptual understanding, and (3) skill acquisition.

Of the three types of assessment tasks, skill acquisition is most often assessed. These tasks would include solving equations and inequalities or using formulas by primarily symbol manipulation. Students often apply an algorithm that may or may not convey their understanding.

Items that are designed to assess students' conceptual understanding or ability to problem solve can provide a rich means by which students demonstrate their thinking and interpretations of concepts through expanded responses. The inclusion of these types of items link assessment with classroom practice. If students are required in mathematics classes to explain their thinking in class discussions or on their homework papers, it is important that assessments also include similar tasks. Likewise, if state assessments include self-constructed response items, students will develop skill in responding to such items when these types of tasks are regularly included on a daily basis as well as on assessments.

Description of Materials and Teaching Suggestions

There are over 150 journal prompts here for teachers to choose from. Some of them may be used more than once to ascertain how students are growing in their understandings of and beliefs about mathematics. The prompts may also be used as assessment tasks for expanded response or self-constructed response items. As they use these prompts teachers may generate other ideas that can be used to create their own journal prompts.

Journal Prompts

- ✎ Each mathematical content prompt includes a short description of what to expect or consider in students' responses. The mathematical content response should reflect a student's progress in understanding the concept or skill.
- ✎ The content prompts were selected to match recommendations from the National Council of Teachers of Mathematics (*Principles and Standards for School Mathematics, 2000*) rather than to fit a specific text. Locate the topic students are studying and then select the prompt that best fits the class. The wording may be changed if students will not understand the terminology or vocabulary used.
- ✎ Process prompts give students opportunities to show how they do a mathematical process or algorithm (step-by-step procedure). When teachers use these prompts, they gain insight into how students have interpreted or altered procedures. In some cases, they may find student-invented algorithms that are appropriate to share with the class.
- ✎ Affective or attitudinal prompt responses show students' beliefs. If used more than once, teachers should be able to detect changes in beliefs or feelings about mathematics as a discipline as the year or semester progresses. Similarly, students' views of themselves as mathematicians or as students of mathematics can be assessed.

Teachers can use writing prompts daily or intermittently, depending on the class, and prompts can be repeated from time to time. In prompts that use students' names, they may change the names to represent their students. If teachers use journal prompts daily, it is recommended that students respond to at least three content prompts, no more than one process prompt, and no more than one attitudinal prompt in a week. The more frequently writing is used as a regular part of mathematics class, the better students become at responding. More specific directions for implementing journal writing are included at the end of this section.

. . . and more

- ✎ There are 12 extended problem-solving tasks. Each task requires more time to solve than one class period. Students often provide the best solutions if they are given 10 days in which to solve it. The teacher may decide to use one of these every 3 weeks or so.
- ✎ In some cases, teachers may assign the tasks for the entire class to work individually. These tasks also give teachers and students the opportunity to use pair or group problem solving. Regardless, it is important that students write their responses in a way that a reader can see the flow of their thinking and understand the solution method or path that they used.
- ✎ It is recommended that students do extended problem solving on a regular basis. This practice supports their development of problem-solving strategies and boosts their confidence to solve complex problems.
- ✎ For each problem-solving task a solution has been given. However, there are multiple methods to solve each problem. Teachers should be open to creative ways that students may approach these problems.
- ✎ There are 10 assessment items included here. These items represent a conceptual approach to a particular mathematical topic. There should be no more than one of these items on a chapter test. If used independent of the chapter quizzes or tests, however, it is possible to use more than one. Additionally, any of the content or process journal prompts can be used on formative or summative assessments in a similar manner.
- ✎ It is important to recognize that students should have experience with talking and writing mathematics daily in order for the use of these assessment items to be representative of student understanding. If a teacher uses a lecture-based approach, some of these items may need alteration to allow for optimal responses from students.

Evaluating Students' Responses

Students tend to give the writing more thought if it is to be scored. One method used was to score the responses on content rather than on grammar and spelling. An essay grading method was used. By reading two or three papers to get a feel for students' responses these first papers formed the baseline for scoring other papers. This method, however, did not provide students a guideline for writing in advance.

When teachers use rubrics and metrics, however, scoring guidelines can be provided in advance. A rubric establishes qualitative levels that define what characteristics a response has or what criteria it meets. The qualitative levels are not used as points. On the other hand, a metric is a point system. The levels of a metric have specific criteria associated with each point value. The same criteria may be used for either a rubric or a metric system. The difference is in whether or not points or qualitative levels are established for the response.

There are many ways to create a rubric or metric. Teachers can develop one alone, or students can work with the teacher to develop a rubric or metric specific to their class or to the task. Sample rubrics and metrics, developed with students' participation, follow on the next page.

Although grammar and punctuation are not usually scored, students should communicate their ideas in ways that are understandable. Some students may use drawings, tables, charts, or other means to convey their ideas. They should be encouraged to use whatever ways they need to make their ideas clear.

As writing tasks are used, excerpts can be taken from students' papers to illustrate qualities that you consider important. Both high- and low-quality responses can be used to show students the comparison with rubric or metric criteria. Of course, authorship of whatever responses are selected should be kept anonymous.

Rubrics or metrics to score the problem-solving tasks as extended types or as assessment items can be developed for each individual task or created as a general guide for student performance. The rubrics or metrics that follow can also be adapted to serve as a generalized set of criteria to guide students' solution approaches. If rubrics or metrics are being used by the entire mathematics department, it may be appropriate to have department-wide discussions to agree upon criteria. This will provide a means by which to motivate consistent and cohesive student work across grades, courses, and teachers.

With any form of writing and any type of rubric or metric, students can self-evaluate their responses or conduct peer evaluations. This allows you to see if students truly understand the criteria outlined in the rubric or metric. This activity also requires students to think at a much higher level as they analyze critically others' work .

General Metric

4 points **The student's work includes—**

- ✎ completed prompt or an answer to the question posed
- ✎ support for statements made by using either examples or counterexamples
- ✎ ideas clearly communicated to the reader
- ✎ legible writing, drawings, pictures, charts or tables, and diagrams
- ✎ accurate mathematics or information

3 points Omission of one criterion from level 4

2 points Omission of two criteria from level 4

1 point Omission of three criteria from level 4

0 points Omission of more than three criteria from level 4

Three-level Rubric (or Metric)

The student's work shows a response that—

Exceeds standard

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader

Meets standard

- ✎ addresses the question raised in the prompt
- ✎ has some correct or accurate mathematics
- ✎ is legible
- ✎ does not support or justify some of the statements made
- ✎ makes sense to the reader

Below standard

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not support or justify statements made

Five-level Rubric (or Metric)

4 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ has support or justification for any statements made
- ✎ makes sense to the reader.

3 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has correct or accurate mathematics
- ✎ is legible
- ✎ does not have fully justified or supported statements
- ✎ makes sense to the reader

2 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has some incorrect or inaccurate mathematics
- ✎ is legible
- ✎ does not have justified or supported statements
- ✎ is somewhat clear to the reader

1 **The student's work shows a response that—**

- ✎ addresses the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is partially legible
- ✎ does not have justified or supported statements
- ✎ does not make sense to the reader

0 **The student's work shows a response that—**

- ✎ does not address the question raised in the prompt
- ✎ has incorrect or inaccurate mathematics
- ✎ is not legible
- ✎ does not make sense to the reader

Implementing Journal Writing in Your Classroom

- ✎ To begin using writing in your classroom, you will need to make sure your students understand your expectations for writing. The following offers one method for helping students learn what is meant by *writing in mathematics.*
- ✎ Share with students the rubric or metric you will be using. You may opt to create a rubric or metric with your students rather than creating one yourself. Make copies of the rubric or metric for students to keep in their notebooks. Post one copy in the classroom for easy reference.
- ✎ Give students a practice prompt to write. If it is used as a warm-up, allow about 6 minutes for them to respond. The practice prompt can be any type, but the rubric or metric may work better with a content prompt. Select one that you feel all students in your class can attempt.
- ✎ Have students compare what they wrote with their partner or table mates. They should check the rubric or metric. Have students focus on 3 things they could do to improve their writing to the next higher level. If their writing already includes all the indicators for the top level, ask them to write another question that this prompt made them think about.
- ✎ Allow 3 minutes for students to correct or revise their work. They should strive to reach the top two levels of the rubric or metric.
- ✎ Collect the work. Score it with your rubric or metric. However, to allow students time to learn to meet your expectations, you may not want to record the score yet.
- ✎ For the next several days, whether you assign prompts as a warm-up or for homework, allow students time to revise or correct their work. You should stress that they should strive to reach the top two levels. Repeat this phase as often as needed to help students understand your expectations for their writing.
- ✎ If possible, show students samples of other students' writing. Use this sample to illustrate what you mean by your criteria in the rubric or metric. A sample of student writing in the middle grades follows.

Sample of Student Work

Prompt: Cameron said, "When I evaluate a trig function of Θ, an angle in standard position, I know that some angles do not have a sine or cosine function defined." Do you agree with Cameron? Why or why not?

No, I don't agree with Cameron. He must be thinking about tangent, secant, cotangent or cosecant functions because sine and cosine are defined for any angle. I know that's true because if (x,y) is any point on the terminal side of Θ, (except the origin), then r the radius of a circle that has a center at the origin isn't 0. It's important that r isn't 0 because the sin Θ is y divided by r. If r is 0, then this would be undefined. It's sort of true for the cosine too of the same angle. Cosine Θ is x value divided by r. It's the same reason why Cos Θ can't be 0.

I think, tho, that if x is 0, then tangent and secant would be undefined and if y is 0, then cotangent and cosecant are undefined.

So I think Cameron messed up and really meant to say that sine and cosine are defined for any angle Θ.

Journal Prompts

In this section 168 prompts are provided in categories described in the introduction and description of materials. As a teacher you can give prompts as a homework assignment in conjunction with a textbook assignment. Or, you can use them as a warm-up for the first 5–7 minutes of the class period. Student responses can be collected and graded or students can share their ideas within a whole class discussion. Their ideas can be recorded on chart paper, on transparencies, or on the board. The chart paper provides a permanent record or archive of student responses so that they can be revisited later in the unit of study or in an associated unit. An archive for responses allows the progress of students' understandings, or thinking about mathematical concepts or skills, to be monitored.

You can also use the prompts in a problem-solving context where students create a solution in a pair, group, or individually. If pairs or groups work on the problems, they can share their ideas before the whole class. Individual student responses can be shared by having other students read them and analyze the response.

Prompts are organized as follows:

Content prompts

Number Sense, Properties and Operations with Real Numbers

1. **Jorge said, "If a is any real number, is –*a* a negative number?" "That makes sense," said Thad. Do you agree with Thad? Verify that this is true.**

   This is not true. If a is a positive number, then –*a* is a negative number. However, if a is a negative number, then –*a* is a positive number.

2. **"I can write the reciprocal of any number," said Camille. Decide if you believe Camille. Justify your answer.**

   Camille cannot write a reciprocal for 0. It would result in an undefined number.

3. **"I think division is really like multiplication," said Alvina. "I don't agree," said Kelli. Who do you agree with? Explain.**

   Division can be thought of as multiplying by the reciprocal. Since division by 0 is not possible, this definition can be applied to all division problems.

4. **Carmen said, "If I add or multiply two real numbers, the sum or product will always be a real number." Margo thought about Carmen's statement. Then she said, "Is that always true?" What do you think? Justify your idea(s) with details.**

   It is always true, because the real number system is closed under addition and multiplication.

5. **"I wonder," said Katelin, "if the difference of two whole numbers is always a whole number." What would you tell Katelin? Justify or support your answer.**

   This is only true if the whole-number subtrahend is less than the whole-number minuend. If the subtrahend is larger, then the difference will be an integer.

6. **"I found out something last night," said Sammi. "A number raised to an odd power is always odd." Do you agree with Sammi? Why or why not? Support your answer.**

   If two odd numbers are multiplied, the resulting product is odd. By using the third power, you would first multiply the number times itself. In the case of an odd number, this would give an odd number. The next multiplication, for the third power, will still yield an odd product. If the base or factor is even, multiplying it times itself would give an even number. Thus the next multiplication would also give an even number. Therefore Sammi's statement is only true if the base or number is odd. Students should disagree with Sammi.

7. **"I can take any number and raise it to a third power. The result will always be a positive number," said Anisha. Is that true? Why or why not? Support your answer.**

This is not true. If the base or number is 0 or an integer, the third power will not be a positive number.

8. **Jeremy wrote the following on the board.**

$$4^2 \square 4 \square 4 \square 4 = 76$$

Use any operational symbols in the boxes to make the number sentence true. Justify your answer by describing your process.

A possible solution is $(4^2 + 4) \times 4 - 4 = 76$. Students should relate the solution to the order of operations.

9. **"My teacher told us that if we want to find the fourth root of 8, it's the same thing as raising 8 to the one-fourth power," said Darrell. "Why is that?" asked Chelsea. What do you think Darrell told Chelsea? Support your answer.**

Students may offer multiple explanations and may be dependent upon the level of your class discussions. Some students may use an inverse method of explaining.

10. **Erica said, "-2^4 is –16." Eric argued, "No, -2^4 is 16." Who do you agree with? Why?**

Students should agree with Erica and support their answer. Some students may use order of operations to support their answers.

11. **Rex said the opposite of a number is the same as its additive inverse. Randy said they are not the same. With whom do you agree and why?**

Agree with Rex. The sum of a number and its additive inverse is 0, which is also true for opposites.

12. **When Steve simplified $(-3)^4$, he got 81. Sharon got –12. Do you agree with either solution? Why or why not?**

Agree with Steve. Sharon computed -3×4 rather than using –3 as a factor four times.

13. Place parentheses in the expression to make a true statement. Justify your answer.

$$5 \cdot 2 + 3 \cdot 4 = 22$$

$(5 \cdot 2) + (3 \cdot 4) = 22$. Some justification should be given.

14. Joni asked, "How is working with logarithms similar to working with exponents? They seem to be related." What would you say to Joni? Be specific and use examples as appropriate.

Students may indicate that some of the properties of logarithms are similar to properties of exponents. For example, if a, u, and v are positive numbers and $a \neq 1$, then $\log_a \frac{u}{v} = \log_a u - \log_a v$. This is similar to $\frac{a^u}{a^v} = a^{u-v}$. Students should note that, in essence, logarithms and exponential forms can be equivalent.

15. "How can I tell if two radicals are like radicals?" asked Terrance. "It's easy," said Beau. What do you think Beau means? Describe how to tell if two radicals are like radicals.

Students should indicate that like radicals have the same index and the same radicand. To extend the prompt, you can ask students to compare like radicals to like algebraic terms.

16. April said, "$4^{-2} \times 4^0$ equals 4^0 because you multiply exponents." Leigh said, "I think it equals 4^{-2}." Do you agree with either one? Why or why not?

Students should agree with Leigh. They may give a variety of reasons, but most frequently they note that 4^0 is 1, or they use the law of exponents related to multiplying with like bases.

17. Colin asked, "How is working with matrices like working with tables?" "There are a lot of relationships between the two," said Patrick. What do you think Patrick means? Give examples to support your answer.

Students may use multiple descriptions of matrices and tables to show the relationships. The use of rows and columns as well as entries in the matrix could be noted. Some students may use a table and a matrix to demonstrate the relationships.

18. Rhoda said "Matrices have properties like real numbers." "They do?" said Clark. "Like what?" What properties would Rhoda describe to Clark? Show how the properties apply to real numbers and to matrices.

Students may discussion the Associative Property, the Commutative Property, and the Distributive Properties related to scalar operation or matrices' operations. Other properties such as Multiplicative Identity could be included, depending on your class discussions.

19. "With real numbers, two numbers are inverses if their sum is 0," said Dean. "Two matrices, 2 x 2, are inverses if their product is the identity matrix," said Cassie. "Is that always true?" asked Ben. What do you think Dean or Cassie said? Support your answer.

Two 2×2 matrices are inverses of each other if their product, in both orders, is the identity matrix. However, in a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $ad - bc$ cannot equal 0.

Not all 2×2 matrices have an inverse. If the determinant of a matrix is 0, then it does not have an inverse.

20. Keila asked, "How is e like pi?" "I know," said Jake. What do you think Jake told Keila? Describe Jake's ideas and include examples. Be specific.

Jake may share the definition of e by showing the numerical computation for deriving it. It could be compared to pi in that it is irrational, and it is represented by a letter, like π.

21. Mr. Thomas told his class that finding a square root and squaring a number are opposite operations. What did he mean by that? Be specific in your explanation.

Students can discuss the processes by which each operation is done so that the 'undoing' of one or the other is described.

22. Find four pairs of numbers whose product is 1. Write the multiplication sentences for your pairs. What do you notice?

The pairs are multiplicative inverses. If one is one of the numbers, then the pair is made up of two ones. The same holds true for –1. To extend this prompt, you can ask students if 0 has a multiplicative inverse.

23. Stormi dropped spaghetti sauce on her math paper. What do you think is under the sauce she dropped? Why?

$\frac{2}{0} =$ 

Division by 0 is undefined and students should give a rationale as to why it is undefined.

24. Megan challenged Amanda to find the larger quantity without using a calculator or computing it with paper and pencil.

Which is larger: 6^9 or 9^6? How do you know?

6^9 is the larger amount. Students may have various ways of determining which one is larger.

25. Courtney said she could use the distributive property of multiplication over addition to help her multiply mixed numbers. Here is what she wrote:

$$\begin{aligned} 3\tfrac{2}{3} \times 4\tfrac{1}{2} &= (3 \times 4) + (\tfrac{2}{3} \times \tfrac{1}{2}) \\ &= 12 + \tfrac{1}{3} \\ &= 12\tfrac{1}{3} \end{aligned}$$

Is her method correct? Why or why not?

Her method is not correct. She did not apply the distributive property appropriately. Thus her product is too small.

26. Myrna says all numbers have reciprocals. Nicky disagrees. Who is right? Why do you think so?

Not all numbers have reciprocals. Nicky is correct. Zero does not have a reciprocal.

27. Jensen said, "The square root of a number is always smaller than the radicand." What do you think? Why?

This is not true. As one counterexample, the square root of one-fourth is one-half. One-half is greater than one-fourth.

28. Kevala noticed that $\frac{7}{\sqrt{7}}$ is equal to $\sqrt{7}$. How can you explain this?

The original fraction was multiplied by $\frac{\sqrt{7}}{\sqrt{7}}$ or 1 to obtain the radical.

29. Chester wrote $\sqrt{-25} = -5$. Lester wrote that $\sqrt{-25}$ is impossible. Who is right? Why?

Chester is incorrect because –5 raised to the second power would be 25. Lester is correct if imaginary numbers cannot be used. Depending on the level of your class, students may indicate they are both incorrect because $\sqrt{-25}$ = 5.