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The "Write" Way Mathematics Journal Prompts and More

FOR GEOMETRY

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Curriculum Research & Development Group

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Introduction

he Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) includes a separate process standard for Communication for grades 3 through 8 and 9 through 12. This standard advocates using multiple activities that give students opportunities to demonstrate their understandings. These tasks should allow students to express themselves in a variety of ways such as with words, concrete models, pictures or drawings, or abstract symbols. Each of these forms lets students communicate with others about their thinking. At the same time, students are developing a deeper understanding of the mathematics because they have to be clear and explicit when they are writing to an audience that cannot see their facial expressions and gestures, or ask clarifying questions. Thus students understand more and retain their understanding longer.

Communication can include at least five types of activities: (1) speaking, (2) listening critically, (3) reading, (4) modeling or constructing, and (5) writing. Speaking or talking about mathematics should be encouraged and used daily in mathematics classes. Students can talk in pairs or small groups as well as present their ideas to the whole class. If students are talking about mathematics, others should be critically listening so that any student should be able to comment on the ideas presented by another. Students should also be reading in a mathematics class. The reading could be centered on the textbook but can be expanded to include students' writing or trade books. Modeling or constructing means that students can also communicate to others by using concrete models (such as Algebra Tiles[™]), drawing pictures or diagrams, or using technology such as graphing calculators. Finally, communication also includes writing such as journal writing and extended or expanded problem-solving tasks.

The following sections describe different types of journal writing, extended problem-solving tasks, and the use of these two forms of writing in assessment.

Journal Writing

Journal writing can be structured to give teachers cohesive and comparable information about students and their thinking while challenging them through contextual situations. A structure for journal writing includes prompts that focus on (1) mathematical content, (2) mathematical processes, and (3) student attitude or affect. Journal prompts give situations or questions to which students respond. Responses may include words, pictures or drawings, or symbols. Students are encouraged to support their ideas and to clearly explain what they mean. They can give specific examples as part of their explanations or use counterexamples.

Content prompts relate or connect topics within and outside of mathematics, targeting important or meaningful concepts and skills. They can also provide situations that focus on areas where students often have misunderstandings or misconceptions. The responses to the prompts give teachers (and students) insight into how a student has interpreted a mathematical idea.

Process prompts promote the awareness of how students solve or approach problems or algorithms. The responses to these prompts can give insight into students' preferences for problem-solving strategies or algorithms and into how they learn or remember. As students become aware of how they learn and solve problems, they grow more confident in approaching new or novel problems.

Attitudinal or affective prompts focus on students' feelings about themselves as mathematicians and students of mathematics. Students' responses allow teachers to assess how positive attitudes about mathematics and mathematicians are developing in the classroom environment.

Extended Problem-solving Tasks

Extended or expanded problem-solving tasks provide opportunities for students to explore and solve problems that require novel solution approaches. For this purpose, problem solving is defined as confronting a problem that does not have an obvious solution or solution path. In most cases, a non-routine solution method (or combination of methods) is required such as making a list, drawing a diagram, working backwards, guessing-and-testing, or creating a table.

Extended problem-solving tasks require more time and thought to solve than routine problems. Students draw on their previous knowledge and experiences to reason through the problem. Because their thought processes will be more complex, writing an expanded solution is an important part of communicating their methods or processes to others. Writing a response to an extended problem-solving task also helps students create a solution process as they clarify what the problem is asking, what information is given in the problem, and what solution methods would be appropriate.

Many students believe problem solving to be a linear process. That is, they read a problem, think of a solution method, solve the problem, and check their answer. Problem solving is more complex. It often requires re-reading a problem or abandoning one solution method for another.

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Journal Prompts and More for Geometry

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Assessment Tasks Requiring Writing

Assessing student understanding can be done in a variety of ways including journal writing, homework problems, problem-solving write-ups, quizzes, and tests. Any assessment should encompass at least three types of tasks:

(1) problem solving, (2) conceptual understanding, and (3) skill acquisition.

Of the three types of assessment tasks, skill acquisition is most often assessed. These tasks would include solving equations and inequalities or using formulas by primarily symbol manipulation. Students often apply an algorithm that may or may not convey their understanding.

Items that are designed to assess students' conceptual understanding or ability to problem solve can provide a rich means by which students demonstrate their thinking and interpretations of concepts through expanded responses. The inclusion of these types of items link assessment with classroom practice. If students are required in mathematics classes to explain their thinking in class discussions or on their homework papers, it is important that assessments also include similar tasks. Likewise, if state assessments include selfconstructed response items, students will develop skill in responding to such items when these types of tasks are regularly included on a daily basis as well as on assessments.

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Description of Materials and Teaching Suggestions

here are over 150 journal prompts here for teachers to choose from. Some of them may be used more than once to ascertain how students are growing in their understandings of and beliefs about mathematics. The prompts may also be used as assessment tasks for expanded response or self-constructed response items. As they use these prompts teachers may generate other ideas that can be used to create their own journal prompts.

Journal Prompts

- Each mathematical content prompt includes a short description of what to expect or consider in students' responses. The mathematical content response should reflect a student's progress in understanding the concept or skill.
- The content prompts were selected to match recommendations from the National Council of Teachers of Mathematics (*Principles* and Standards for School Mathematics, 2000) rather than to fit a specific text. Locate the topic students are studying and then select the prompt that best fits the class. The wording may be changed if students will not understand the terminology or vocabulary used.
- Process prompts give students opportunities to show how they do a mathematical process or algorithm (step-by-step procedure). When teachers use these prompts, they gain insight into how students have interpreted or altered procedures. In some cases, they may find student-invented algorithms that are appropriate to share with the class.
- Affective or attitudinal prompt responses show students' beliefs. If used more than once, teachers should be able to detect changes in beliefs or feelings about mathematics as a discipline as the year or semester progresses. Similarly, students' views of themselves as mathematicians or as students of mathematics can be assessed.

Teachers can use writing prompts daily or intermittently, depending on the class, and prompts can be repeated from time to time. In prompts that use students' names, they may change the names to represent their students. If teachers use journal prompts daily, it is recommended that students respond to at least three content prompts, no more than one process prompt, and no more than one attitudinal prompt in a week. The more frequently writing is used as a regular part of mathematics class, the better students become at responding. More specific directions for implementing journal writing are included at the end of this section.

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- There are 12 extended problem-solving tasks. Each task requires more time to solve than one class period. Students often provide the best solutions if they are given 10 days in which to solve it. The teacher may decide to use one of these every 3 weeks or so.
- In some cases, teachers may assign the tasks for the entire class to work individually. These tasks also give teachers and students the opportunity to use pair or group problem solving. Regardless, it is important that students write their responses in a way that a reader can see the flow of their thinking and understand the solution method or path that they used.
- It is recommended that students do extended problem solving on a regular basis. This practice supports their development of problemsolving strategies and boosts their confidence to solve complex problems.
- For each problem-solving task a solution has been given. However, there are multiple methods to solve each problem. Teachers should be open to creative ways that students may approach these problems.
- There are 10 assessment items included here. These items represent a conceptual approach to a particular mathematical topic. There should be no more than one of these items on a chapter test. If used independent of the chapter quizzes or tests, however, it is possible to use more than one. Additionally, any of the content or process journal prompts can be used on formative or summative assessments in a similar manner.
- It is important to recognize that students should have experience with talking and writing mathematics daily in order for the use of these assessment items to be representative of student understanding. If a teacher uses a lecture-based approach, some of these items may need alteration to allow for optimal responses from students.

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Evaluating Students' Responses

Students tend to give the writing more thought if it is to be scored. One method used was to score the responses on content rather than on grammar and spelling. An essay grading method was used. By reading two or three papers to get a feel for students' responses these first papers formed the baseline for scoring other papers. This method, however, did not provide students a guideline for writing in advance.

When teachers use rubrics and metrics, however, scoring guidelines can be provided in advance. A rubric establishes qualitative levels that define what characteristics a response has or what criteria it meets. The qualitative levels are not used as points. On the other hand, a metric is a point system. The levels of a metric have specific criteria associated with each point value. The same criteria may be used for either a rubric or a metric system. The difference is in whether or not points or qualitative levels are established for the response.

There are many ways to create a rubric or metric. Teachers can develop one alone, or students can work with the teacher to develop a rubric or metric specific to their class or to the task. Sample rubrics and metrics, developed with students' participation, follow on the next page.

Although grammar and punctuation are not usually scored, students should communicate their ideas in ways that are understandable. Some students may use drawings, tables, charts, or other means to convey their ideas. They should be encouraged to use whatever ways they need to make their ideas clear.

As writing tasks are used, excerpts can be taken from students' papers to illustrate qualities that you consider important. Both high- and low-quality responses can be used to show students the comparison with rubric or metric criteria. Of course, authorship of whatever responses are selected should be kept anonymous.

Rubrics or metrics to score the problem-solving tasks as extended types or as assessment items can be developed for each individual task or created as a general guide for student performance. The rubrics or metrics that follow can also be adapted to serve as a generalized set of criteria to guide students' solution approaches. If rubrics or metrics are being used by the entire mathematics department, it may be appropriate to have departmentwide discussions to agree upon criteria. This will provide a means by which to motivate consistent and cohesive student work across grades, courses, and teachers.

With any form of writing and any type of rubric or metric, students can self-evaluate their responses or conduct peer evaluations. This allows you to see if students truly understand the criteria outlined in the rubric or metric. This activity also requires students to think at a much higher level as they analyze critically others' work .

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General Metric

4 points The student's work includes—

- completed prompt or an answer to the question posed
- support for statements made by using either examples or counterexamples
- ideas clearly communicated to the reader
- legible writing, drawings, pictures, charts or tables, and diagrams
- accurate mathematics or information

3 points Omission of one criterion from level 4

- **2 points** Omission of two criteria from level 4
- **I point** Omission of three criteria from level 4
- **0 points** Omission of more than three criteria from level 4

Three-level Rubric (or Metric)

The student's work shows a response that—

Exceeds standard

- addresses the question raised in the prompt
- has correct or accurate mathematics
- 🔊 is legible
- has support or justification for any statements made
- makes sense to the reader

Meets standard

- addresses the question raised in the prompt
- has some correct or accurate mathematics
- 🔊 is legible
- does not support or justify some of the statements made
- makes sense to the reader

Below standard

- does not address the question raised in the prompt
- has incorrect or inaccurate mathematics
- 🔊 is not legible
- does not support or justify statements made

Five-level Rubric (or Metric)

- 4 The student's work shows response that
 - addresses the question raised in the prompt
 - Shas correct or accurate mathematics
 - 🔊 is legible
 - has support or justification for any statements made
 - $\$ makes sense to the reader.
- 3 The student's work shows response that
 - addresses the question raised in the prompt
 - * has correct or accurate mathematics
 - 🔊 is legible
 - does not have fully justified or supported statements

2 The student's work shows a response that—

- addresses the question raised in the prompt
- has some incorrect or inaccurate mathematics
- 🔊 is legible

L

- does not have justified or supported statements
- \otimes is somewhat clear to the reader

The student's work shows a response that—

- addresses the question raised in the prompt
- \otimes has incorrect or inaccurate mathematics
- S is partially legible
- does not have justified or supported statements
- \odot does not make sense to the reader

0 The student's work shows a response that—

- does not address the question raised in the prompt
- $^{\label{eq:stable}}$ has incorrect or inaccurate mathematics
- 🛸 is not legible
- * does not make sense to the reader

Journal Prompts and More for Geometry

Implementing Journal Writing in Your Classroom

- To begin using writing in your classroom, you will need to make sure your students understand your expectations for writing. The following offers one method for helping students learn what is meant by *writing in mathematics*.
- Share with students the rubric or metric you will be using. You may opt to create a rubric or metric with your students rather than creating one yourself. Make copies of the rubric or metric for students to keep in their notebooks. Post one copy in the classroom for easy reference.
- Give students a practice prompt to write. If it is used as a warm-up, allow about 6 minutes for them to respond. The practice prompt can be any type, but the rubric or metric may work better with a content prompt. Select one that you feel all students in your class can attempt.
- Have students compare what they wrote with their partner or table mates. They should check the rubric or metric. Have students focus on 3 things they could do to improve their writing to the next higher level. If their writing already includes all the indicators for the top level, ask them to write another question that this prompt made them think about.
- Allow 3 minutes for students to correct or revise their work. They should strive to reach the top two levels of the rubric or metric.
- Collect the work. Score it with your rubric or metric. However, to allow students time to learn to meet your expectations, you may not want to record the score yet.
- For the next several days, whether you assign prompts as a warm-up or for homework, allow students time to revise or correct their work. You should stress that they should strive to reach the top two levels. Repeat this phase as often as needed to help students understand your expectations for their writing.
- If possible, show students samples of other students' writing. Use this sample to illustrate what you mean by your criteria in the rubric or metric. A sample of student writing in the middle grades follows.

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Sample of Student Work

Prompt: Corey said, "A rhombus and a rectangle are the same thing." Do you agree with Corey? Why or why not?

\bigcirc	No, I don't agree with Corey. The only time that a
	rhombus and a rectangle are the same thing is when the
	rectangle and mombus are a square. A square can be
	a rhombus and a square can be a rectangle. That's the
	only time when they would be the same thing.
	I think some people might agree with Corey if they think
	that there are some things about a rhombus and a
	rectangle that are the same. Like they both have two pairs
	of parallel sides and opposite sides are the same length.
	He should be more careful and remember to think about
	everything.

Journal Prompts

n this section 168 prompts are provided in categories described in the introduction and description of materials. As a teacher you can give prompts as a homework assignment in conjunction with a textbook assignment. Or, you can use them as a warm-up for the first 5–7 minutes of the class period. Student responses can be collected and graded or students can share their ideas within a whole class discussion. Their ideas can be recorded on chart paper, on transparencies, or on the board. The chart paper provides a permanent record or archive of student responses so that they can be revisited later in the unit of study or in an associated unit. An archive for responses allows the progress of students' understandings, or thinking about mathematical concepts or skills, to be monitored.

You can also use the prompts in a problem-solving context where students create a solution in a pair, group, or individually. If pairs or groups work on the problems, they can share their ideas before the whole class. Individual student responses can be shared by having other students read them and analyze the response.

Prompts are organized as follows:

Content PromptsII
Axioms, Postulates, and Reasoning
Points, Lines and Planes 12
Angles
Polygons
Polyhedra
Measurement 21
Similarity and Congruency
Transformational Geometry 27
Circles
Coordinate Geometry
Process Prompts
Affective Prompts

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Content Prompts

Axioms, Postulates, and Reasoning

1. I use inductive reasoning when ...

Students discuss the use of particular instances to find a generalization in inductive reasoning. Watch for students who confuse inductive reasoning with deductive reasoning, finding specific examples using a generalization.

2. Tarika asked her teacher, "Why do we need so many definitions in geometry?" What do you think her teacher told her? Be specific.

Having a common agreement on the meaning of concepts or terms is important in communicating with each other. Students may offer specific ideas related to the class discussion about definitions and their use.

3. "I think a postulate is like a theorem," said Travis. "I don't agree with that," said Desiree. Who do you agree with? Why? Explain.

A postulate is accepted as true, but a theorem can be proven. Students should clearly distinguish the two and cite supporting examples.