*Reshaping Mathematics for Understanding*

# Motion
# Geometry

b. Image

B

of C

Figure C

Hannah Slovin
Linda Venenciano
Melanie Ishihara
Cynthia Beppu

Figure D

**Curriculum Research & Development Group**
University of Hawai'i

# Reshaping Mathematics for Understanding

# Motion Geometry

Hannah Slovin

Linda Venenciano

Melanie Ishihara

Cynthia Beppu

**Curriculum Research & Development Group**

*Curriculum Research & Development Group Staff*
Arthur R. King, Jr., Director
Hannah Slovin, Project Director
Edith K. Kleinjans, Managing Editor
Kathleen Boswell, Editor
M. Jayne Higa, Editor

Book Design by Darrell T. Asato
Cover design by Erin T. Sakihara
Illustrations by Erin T. Sakihara and Gilles Martinez

# Contents

# Acknowledgments

This undertaking would not have been possible without the tireless support of our student helpers Erin Sakihara, Gilles Martinez, Tricia Saito, and Lindsay Miura. Their patience and enthusiasm helped us hold on to our sanity.

Thanks to the generous support from the Curriculum Research & Development Group's Math Section, whose members have included Gary Martin, Barbara Dougherty, Annette Matsumoto, Sid Rachlin, Claire Okazaki, Fay Zenigami, Mary Pat Sjostrom, Jenny Simmons, Gail Hunger, and David Lani.

We want to express our sincere appreciation to the following teachers for testing some of our material in their classrooms and providing us with feedback: Jean Hamai, Phaedra Link, Elizabeth Dudley-Damon, Dexsilyn Navarro, Emma Smith, and Lynele Nayal. Their comments and suggestions have contributed significantly to the project.

We thank the classroom teachers and others who have attended our professional development institutes for encouraging us to publish these materials (formerly the Reshaping Mathematics Program) and make them available for teacher development workshops and classroom use.

We also thank the generations of University Laboratory School sixth- and seventh-graders who, through interviews, class discussions, and journal writings, taught us so much about how they learned mathematics.

Lastly, mahalo nui loa (thank you) to Dr. Anthony Picard of the University of Hawai'i, whose simple question, "Why not start with geometry?" planted the seed for creating a mathematics course for middle-grades students using a geometric perspective.

# About the series . . .

## Reshaping Mathematics for Understanding

*Reshaping Mathematics for Understanding* is a series of fourteen units suitable for sixth- through eighth-grade students that addresses important topics in middle-grades mathematics, including geometry, measurement, proportional reasoning, rational numbers, probability and statistics, and algebra. The entire series is designed to help students learn to think mathematically. It focuses on developing students' understanding of mathematical concepts and on their ability to draw connections among these concepts. It may serve either as the primary resource in the mathematics curriculum or as a complement to other material. The titles for the full series are listed below.

- Getting Started
- Motion Geometry
- Measurement
- Polygons
- Dilations
- Fractions
- Decimals
- Ratio and Proportion
- Area of Polygons
- Solids
- Probability and Statistics
- Integers
- Algebra Patterns and Relationships
- Number Theory

## Access for Every Student

The problems and tasks in the *Reshaping Mathematics for Understanding* series are designed to enable every student to approach mathematics through familiar contexts. Building problem situations on students' past experience makes the study of mathematics more accessible and allows them to expand their thinking. Similarly, to promote genuine engagement, many problems have more than one solution path, giving students opportunities to choose the strategies they prefer using to solve problems. Additionally, to give students opportunities to interact with each concept at varying levels of abstractness and generality, the lessons present new ideas over several days. This design feature also allows students to learn at different rates.

## Learning with Understanding

Students can learn only what they truly understand. To learn, they must understand the concepts that underlie operations; they must make connections among processes and concepts; and they must know when and how to apply concepts and operations to problems. Understanding mathematics means doing more than learning how to perform calculations to get correct answers. The topics in this series, developed through problems and lab explorations, encourage students to deepen their conceptual understanding through the practice of reasoning and problem solving.

## Role of Visual/Spatial Thinking and Reasoning

Mathematics can and should be a lively course of study for students. It should engage them in active inquiry and give them many opportunities to explore problems whose solutions add to their understanding of the world. For many students, however, mathematics involves merely manipulating numbers. With this limited view, they do not learn how to use multiple contexts, tools, and strategies to solve problems or how to integrate mathematical concepts into broader contexts. Transformational (motion) geometry, which plays an important role in the sequence of the concepts in this series, emphasizes the use of visual contexts and spatial thinking. Lesson discussions further encourage students to use their understanding of spatial relations to make connections among concepts. The Motion Geometry and Dilations units, in particular, give students valuable experience with transformations and enhance the study of many related topics.

## Unit Design

Each unit in the *Reshaping Mathematics for Understanding* series can be used individually, or units can be used in clusters. Throughout the series, references among units direct teachers and students to tasks that will help them connect their understanding of new concepts to related experiences.

The lessons in each unit are uniquely designed to enable students to progress through a sequence of tasks that maximize learning with understanding. Rather than present a topic in its entirety in one day, the lessons develop concepts over time. Most concepts begin with an open-ended problem that draws on students' previous experience and intuition and allows for multiple responses. The variety of solutions

students propose to such a problem helps the teacher assess what background knowledge students bring to the topic. The problems and tasks that follow develop the concept through a sequence of approaches that provide several direct examples of the concept, raise questions for clarification, offer alternative viewpoints, and prompt students to summarize ideas.

## The Lessons

There are two forms of lessons in each unit, problem sets and in-class labs. Problem sets consisting of three to five problems are designed to cover several concepts related to the unit topic. Problem sets should be assigned for homework and discussed in class the next day. When students find a problem too difficult to solve on their own, they should be instructed to write questions to help them solve it, and to ask the questions in the discussion. Students work on the labs in small groups in class and debrief afterward, giving them an opportunity to work collaboratively and to concentrate on one strand.

The class discussions are essential in helping students build conceptual understanding. Sharing their solutions and questions allows students to reflect on their thinking and to consider input from others. Both the teacher and students share the responsibility for making discussions productive. As they solve the problems and complete the lab tasks, students explain their thinking, offer alternative responses, and ask questions. Teachers facilitate, asking strategic questions to focus students' thinking on critical ideas. In guiding the discussion, teachers should ensure engagement in the learning process by encouraging students to monitor their learning and by providing a safe, open learning environment in which to share, discuss, and address misconceptions.

## Assessment

Except for the Getting Started unit, designed to orient students to the series, all units include suggested assessment items. To check periodically that students understand and that they are participating in the class discussions, teachers can also create "instant" quizzes to give the day after a problem set discussion or lab debriefing. These quizzes should be unannounced and should take between five and ten minutes at the start of class. They have three purposes:

to emphasize the importance of student discussions;

to emphasize the value of information shared by students;

to highlight the importance of asking questions to clarify understanding.

In creating an instant quiz, teachers should focus on students' ideas from the discussion and write two or three short-answer questions as follows:

- a question that is content-based or refers to an understanding that has been established.

  **Example:** What did we decide it meant to measure the perimeter of a polygon?

- a question that refers to someone's method for solving a problem or alternative point of view.

  **Example:** What method did Leslie use to find the area of the trapezoid?

- a question that highlights an undefined assumption or a statement that needed clarification.

  **Example:** What did Jose mean by *proportional*?

## Materials

A list of the materials needed for each lesson appears at the front of every unit. Although students need no special materials or equipment to complete the homework problems, it would be helpful to have them available during discussions. Encouraging students to use a broad range of tools to explore and express their thinking promotes greater understanding.

# About the unit . . .

The Motion Geometry unit introduces students to concepts of geometry they will use throughout middle-grade and higher-level mathematics courses. These concepts, presented through the study of transformations, provide a framework for other important topics such as number, measurement, proportional reasoning, and graphing on the coordinate plane. As students learn about reflections, translations, and rotations (flips, slides, and turns), they are also learning about the properties of the motions and how they affect objects. The problems in this unit require students to manipulate drawings physically, to be accurate in their work, and to use precise language in analyzing the results of the motions.

## Geometric Thinking

Motion geometry in the middle grades can provide a powerful context for students to develop their reasoning processes. Problems are carefully designed to help them progress from using the informal visual and descriptive bases for thinking about geometric objects which are suitable for elementary grades, to using more formal reasoning based on the logical arguments and justifications that are required in advanced courses.

Motion is a part of everyone's daily life — riding a Ferris wheel, sliding into home plate, or flipping a coin. The study of motions — reflections, translations, and rotations — allows students to draw on their experience and their intuition as they strive to understand the physical world. For example, they can compare two triangles by flipping one over to match the angles, sliding one on top of the other to check the areas, or turning one around to compare its shape to another's. These "common sense" methods, which are grounded in mathematical principles, help students to reason and to justify the relationships between the triangles.

## The Language of Motion Geometry

Although many of the ideas from motion geometry relate to familiar experiences, we use special terminology to name and describe transformations. Some of the terminology can be confusing to students. For example, the term translation means something different in a foreign language class than it does in a mathematics class. The problems in this unit help students build on their previous conceptions to learn new mathematics terminology, and the suggested discussion questions encourage them to clarify their understanding and to re-define terms in a mathematics context.

## Recording Motions on Paper

Solving problems in motion geometry requires students to manipulate drawings physically using pencil, paper, and other tools. They must trace given figures with a reasonable degree of accuracy, use lines and arrows in particular ways, and label drawings effectively to communicate to others how

they solved a problem. Some middle-grades students have difficulty achieving this level of precision. The teacher and students together should determine appropriate expectations for neatness. However, it is important that all students cultivate habits of mind that promote care and mathematical correctness. At the same time, class discussions should include opportunities for students to share and practice techniques for making drawings accurate, and should highlight how the techniques and procedures reflect the mathematical properties of the motions. In this way, the study of the physical and the conceptual components of transformations complement each other. (See page 107 for suggested drawing methods.)

## The Strands in Motion Geometry

**Introduction to Motion:** Students begin by using real objects to explore motion. They experiment with different types of motions, informally describing the properties of the motions they perform. They compare an object before and after the motion; for example, they may focus on its orientation. As they observe that the size, shape, and other physical properties of the objects do not change with rigid motions, students develop a sense of congruence.

**Representation:** Because an actual motion cannot be preserved on paper, we must use representation to convey how an object should be or has been moved. The early tasks in this unit give students opportunities to invent ways to represent the motions they perform and to explain their drawings. These experiences help students understand the importance of using a common scheme to represent motions and to make sense of some conventional schemes.

The problems in this unit use a set of "tools" to represent how one should move an object or to convey how an object has been moved. These tools are the line of reflection, the translation arrow, and the rotation arrow and center of rotation. Each is discussed in its related strand section.

**Reflection:** Most students have had some experience with reflections, which is the easiest motion to perform. The sequence of problems in the reflection strand requires students to reflect figures, find lines of reflection, and locate the position of objects before they were reflected. Reflections are performed over a line of reflection. Students may devise different techniques for doing the reflection, but they must follow all conditions as presented in the problem. For example, they cannot move a given line of reflection to a more convenient location on their paper. Students should be able to draw with accuracy both figures and the line used to create the reflection.
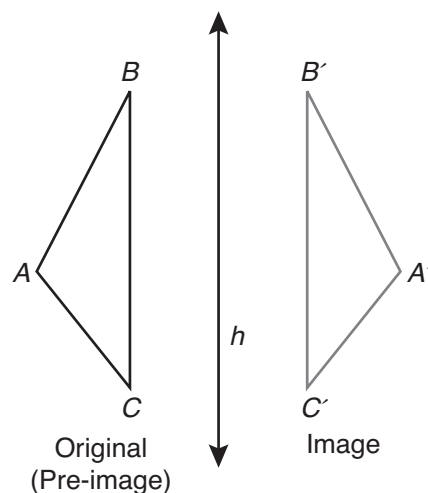


The original triangle (in black) is reflected *over* line *h* to get the image (in gray).

Students may see the connection between their experience of reflections and solving problems involving line symmetry, graphing parabolas, or classifying quadrilaterals.

**Translation:** Translations move a figure a fixed distance in a given direction. Translation arrows (vectors) represent the distance and direction for moving the figure. The problems in this strand give students experience in translating figures, locating figures before they were translated, and suggesting arrows that might have been used in a particular translation. Students need to practice using the translation arrow "tool," which helps them make their drawings accurate. They are expected to draw translation images and arrows accurately and to label the figures correctly.

Students who understand translations can apply this experience to measurement topics—length, area, and volume—to parallel lines, and to graphing on the coordinate plane.
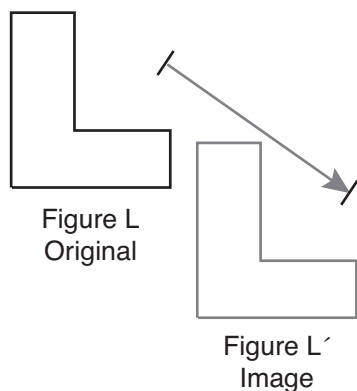


Figure L and its translated image.

**Rotation:** A rotation is a transformation that turns a figure around one point (or, to use the mathematical term, "*rotates* a figure *about* a point") called the center of rotation. Students learn about the tools used to rotate objects and practice doing rotations. They use degrees and fractions to describe rotations.

Rotations are the most challenging motion for students to do. They may not have had enough experience with circles or be sufficiently adept at using a compass to make precise drawings for problems, that, for example, ask them to locate a center of rotation. They should, however, be able to approximate where a center of rotation is and to sketch possible rotation arrows.
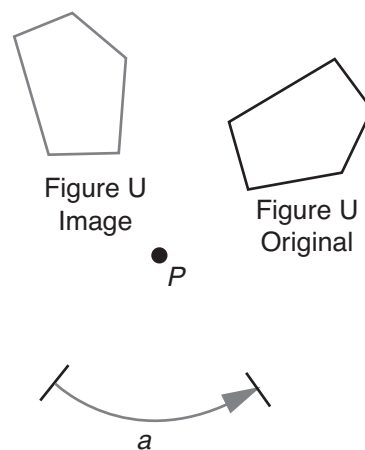


Figure U and its rotation image using arrow *a*.

Rotations help students understand angle measurement, symmetry, and slope.

# Concept Development

| STRAND and FOCUS | Lab A | PS 1 | Lab B | PS 2 | Lab C | PS 3 | PS 4 | Lab D | PS 5 | PS 6 | PS 7 | PS 8 | PS 9 | PS 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Introduction to Motion** | | | | | | | | | | | | | | |
| Moving Objects | 1–5 | 2 | | | | | | | | | | | | |
| Orientation | 6 | 2, 3, 4 | | | | | | | | | | | | |
| Congruence | 1–4, 6 | | | | | | | | | | | | | |
| **Representation** | | | | | | | | | | | | | | |
| Representing Motion | 5 | | | | | | | | | | | | | |
| **Reflection** | | | | | | | | | | | | | | |
| Properties of a Reflection | | 1 | 1 | | | | | | | | | | | |
| Line of Reflection | | | 1, 2 | 1, 2, 3 | 4 | 1, 4 | 1 | 4 | 1 | | 2 | | 2 | |
| Reflection Image | | | | | | | | | | 1 | | | | |
| Comparing Motions | | | | | | | | | | | | 1 | | |
| Finding the Line of Reflection | | | | | | | | | | | | | | 1 |
| **Translation** | | | | | | | | | | | | | | |
| Properties of a Translation | | | | 4 | 1, 2 | 2 | | | | | | | | |
| Direction of Motion | | | | | 3 | | | | | | | | | |
| Distance of a Translation | | | | | | | 4 | | | | | | | |
| Translation Arrow | | | | | | | 4 | 2 | 3 | 2, 3 | 1 | | | |
| Translation Image | | | | | | | | | | | | 4 | 1 | |
| Comparing Motions | | | | | | | | | | | | 1 | | |
| Multiple Translations | | | | | | | | | | | | | 4 | |
| Translation of a Translation | | | | | | | | | | | | | | 2 |
| **Rotation** | | | | | | | | | | | | | | |
| Properties of a Rotation | | | | | | 3 | 2, 3 | 1 | | 4 | 4 | 2 | | |
| Center of Rotation | | | | | | | | 1 | | | | | | |
| Rotation Arrow | | | | | | | | | 2, 4 | 4 | | 3 | | |
| Magnitude of a Rotation | | | | | | | | 3 | | | 3 | | 3 | |
| Comparing Motions | | | | | | | | | | | | | | |
| Arc Measure of a Rotation Arrow | | | | | | | | | | | | | | 3 |

# *Materials*

## Materials used throughout Motion Geometry

Geometric compass, 2 per group

MIRA®, 1 per student

Straight-edge*, 1 per student

Tracing paper**, 2–3 sheets per student per lesson

## Special materials for Motion Geometry

**Lab A**    A variety of flat, traceable objects, 1 object per student

Blank half sheets of paper, 3 per person

*Straight-edges without unit markings are preferable to rulers. Blank index cards could also be used.

**Patty paper cut into 5-inch squares is an economical alternative to the art paper used for tracing.

# Recording and Describing Motions

## Lab A, Nos. 1–5

Strand: Introduction to Motion and Representation

Focus:  Moving objects, congruence, and representation

Task:    Moving objects and representing motion

## PROBLEM

1. Choose an object from the collection you are given.

2. a. Trace the object on your paper.

   b. Move it to a different place on your paper and trace it again.

   c. Make a drawing to show how you moved the object.

3. a. Trace the object again on a second sheet of paper.

   b. Move it to a different place on your paper using a different motion from the one you used in 2.a. Trace it.

   c. Make a drawing to show how you moved the object.

4. a. Trace the object again on a third sheet of paper.

   b. Without lifting it, move your object to a different place on your paper. Trace where it ends up.

   c. Make a drawing to show how you moved the object.

5. What information did you include in your three drawings?

## Teacher's Insight

This lab gives students an initial experience with motions and with representing them on paper. The first task leaves the motion entirely up to students. The second task asks them to use a different motion. The third task puts some constraint on what motion they can use. In the discussion students should begin to talk informally about the properties of the motions and the differences among the motions they used. They may begin to use some terms associated with motions, such as rotate, flip (reflection), slide (translation). Have them explain in their own words what these terms mean. They will come to understand them throughout this unit.

The directions intentionally do not specify what to draw to represent the motions, so students will use their own inventions. During the discussion they can explain what they were trying to convey in their representations and assess the effectiveness of different methods. Having such a discussion at this early point in the unit helps students see the rationale behind establishing conventions to represent motions.

## ANSWER

1–5. Arrows are often used to indicate the direction objects were moved. For example, if an object was slid downward, students may draw an arrow from the initial tracing and point it to the final tracing. If they raised the object off the paper, students may draw an arrow with a loop. Highlight the use of arrows that indicate the precise distance they moved an object. Such arrows resemble the arrows used in motion geometry. Also acknowledge any labels used to indicate the initial and final positions of the objects.

Student drawings may include

Start    End

Start    End

End

Start
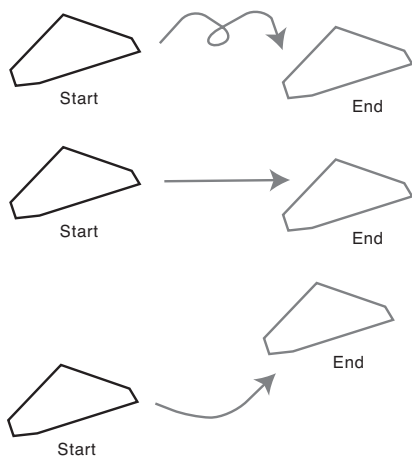
Students should also discuss what about the object changed and did not change when they moved it. Nothing about an object's features changes under a motion, but its orientation may change.

In the last problem students use their experience to interpret someone else's representation. This problem also invites them to use new terms that describe the orientation of a figure.

# Discussion

Start the discussion by asking a few students to show their papers and describe the motions they used. Then have some students show their papers while others try to describe the motions used. Ask what kinds of information helped them to know what motion someone used and why it was helpful.

Have students look at someone's paper and try to recreate the motion used to move the object. Ask if some motions were easier than others to recreate. Why?

How did you know where the object started? How did you know where it ended up?

What makes one motion different from another motion?

What about the object changed? What did not change?
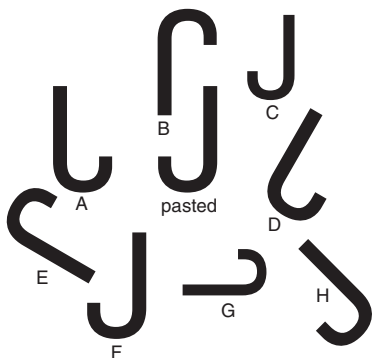
# Lab A, No. 6

Strand: Introduction to Motion

Focus: Orientation and congruence

Task: Comparing orientation with a given figure

## PROBLEM

Now try the following problem:

6. Carrie was pasting some letter J's on a poster. She had pasted only one when a gust of wind scattered the others. Describe clearly the position of each letter J in relation to the one pasted on the poster.



## Teacher's Insight

Students use a variety of ways to describe the orientation of each figure. They may describe the position of each figure in fractions or degrees of a turn. Some may use compass directions or describe the position in clock terms. This problem demonstrates that there can be different ways of describing something and highlights the importance of communicating clearly and checking to understand another person's viewpoint. Have students conjecture what motion resulted in the various orientations of the figures.

This problem also encourages students to consider what makes one figure the same as or different from another. All the *J*'s, except C and G, are congruent. Some students say that figures C and G moved backward and therefore appear smaller. Some students say these two figures were always smaller than the others. Students should distinguish between congruence and orientation.

## ANSWER

6. The positions may be described as turned, flipped, or slid. Students may give specific amounts of turns in terms of degrees or fractions. For example, B was rotated a half-turn or 180˚ clockwise (or counter-clockwise). "Flipped to the left," may be used to describe the position of A. "Slid down to the left in a straight line," may describe F. Combinations of motions may also be used. For example, G was rotated 90˚ and flipped either left or right.

## ★ HOMEWORK

Problem Set 1

## Discussion

Have students describe the position of each *J* compared to the one pasted on the poster.

How do you think the *J* moved in order to end up in that position?

Can you demonstrate how you think the *J* moved?

If students talk about some of the *J*'s (B, E, and H) being turned a certain fractional amount, ask how they decided how much.

If students talk about figure B in terms of 180˚, ask how they knew it turned 180˚.

Are all the *J*'s the same? What makes them the same? What makes them different?

How did the gust of wind affect the figures?

# Identifying Reflections

Motion Geometry      Name _____

**Problem Set 1**

1. Kelly sees a bird soaring over a still, smooth lake. She stands at the edge of the lake and is confused because she also sees what looks like the same bird in the lake.

   a. Draw what Kelly sees in the water.

   b. Compare the bird with its reflection in the water. What is the same about the bird and its reflection? What is different about them?

2. Casey's auntie sent him some alligator chocolates from her candy factory. The pieces got shaken up in the mail, and when Casey opened the box, this is what he found. Describe what happened to each piece of candy.

## Problem Set 1, No. 1

**Strand:** Reflection

**Focus:** Properties of a reflection

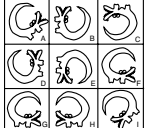**Task:** Comparing an original figure with its reflection image

---

## PROBLEM

1. Kelly sees a bird soaring over a still, smooth lake. She stands at the edge of the lake and is confused because she also sees what looks like the same bird in the lake.

   a. Draw what Kelly sees in the water.

   b. Compare the bird with its reflection in the water. What is the same about the bird and its reflection? What is different about them?

## Teacher's Insight

In this introductory problem students use their previous experience with reflections to think about the properties of a reflection. In reflections, the size and shape of a figure remain congruent, but the orientation changes. Everything on one side of the line of reflection is mirrored on the other, including the distance the objects are from the line of reflection. A common misconception is for students to think of shadows and not reflections. You can address this by asking them what the difference is between a shadow and a reflection or if they can tell what they are looking at. Then focus the discussion on the properties of a reflection.

Some students try to create the reflection of the bird with a freehand sketch. Some try to get a more accurate image by tracing the original bird. Then they fold their paper on the water line and retrace the image. Some rotate the tracing and get a 180° rotation instead of a reflection. One way to help students assess the accuracy of their method is to ask them to predict what the image should look like. Should it be the same size and shape? How should it face? Where should it be in the water? When students clarify what they are trying to accomplish, it is easier for them to assess their methods. When students share their solutions, it is always helpful to have them demonstrate how they did the reflection.

## ANSWER

1. a.
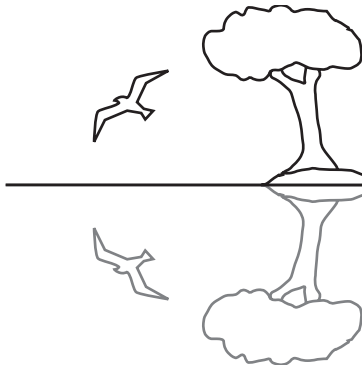


b. The bird and its reflection are the same shape and size. The bird is real but the reflection is just an image; it looks real but it isn't. The reflection looks like an upside-down bird. The bird itself is flying with its right wing higher than its left, but the reflection makes the right wing look lower than the left.

## Discussion

What would you expect the bird in the lake to look like?

How did you draw the reflected image?

What is different about the bird and its reflection?

What is the same about the bird and its reflection?

How did you decide where to put the image of the bird?

If students mention that they made the reflection drawing the same distance from the water line as the bird in the sky, ask why they think that should be so. How could you check the distance without using a ruler?

If students used the water line in their method, ask how the line helped them to create their reflected image.

If students used a folding method, ask where they folded the paper.

Have students agree on names for the actual figure (pre-image or original) and their reflection (image).

What are some ways to make the drawings more accurate?

## Problem Set 1, No. 2

Strand:  Introduction to Motion

Focus:   Moving objects and orientation

Task:    Describing motion

### PROBLEM

2.  Casey's auntie sent him some alligator chocolates from her candy factory. The pieces got shaken up in the mail, and when Casey opened the box, this is what he found. Describe what happened to each piece of candy.
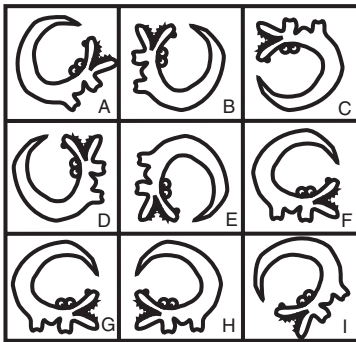


### Teacher's Insight

This problem highlights the need to make and agree on an assumption about the conditions of a problem before discussing its solution. In this problem no particular piece of candy is facing the "correct" way, but students must make an assumption before they can solve the problem. (Some students say that they did not know how the candy started out, so they could not solve the problem.) Students identify the assumptions they made to solve the problem. They listen to other viewpoints and they use a variety of ways to describe what happened to each piece of candy. This problem extends the experience of the *J* problem in the previous lab.

## ANSWER

2. Multiple answers are possible. Most students begin by assuming alligator G is facing the original way and then describe the positions of the others by comparing each to alligator G. Descriptions include such words as turned, flipped over, upside-down, did not move, rotated 90˚, rotated 180˚.

## Discussion

Have students explain their system for describing what happened to each piece of candy (e.g., in clock terms, fractions, or degrees) and check to see if others understand their descriptions of how the pieces are oriented.

How did you decide how the alligators were facing when the candy was mailed?

Did any candy arrive still facing the "right" way? If yes, what made you decide it was the "right" way?

How did you decide what happened to each piece of candy?

If students use fractions and degrees to tell how much the alligators rotated, be sure to probe understanding by asking what they meant by a $\frac{1}{4}$ turn. How do you know it rotated 90°?

What kind of motion(s) could have caused each piece to end up the way it did?

What is the same about all of the candies? What is different?

Reshaping Mathematics for Understanding

Problem Set 1

# Problem Set 1, Nos. 3–4

**Strand:** Introduction to Motion

**Focus:** Orientation

**Task:** Comparing their bodies with reflection images of their bodies

Motion Geometry  Name _____

**Problem Set 1** page 2

3. Look straight into the mirror.
   a. Draw your face.
   b. Close one eye. Draw your face again.
   c. Hold up your outspread hand with the palm facing the mirror. Draw your hand.

4. Write about your drawings.
   a. How are your drawings the same as your actual body?
   b. How are the drawings different from your actual body?
   c. What is special about a reflection?

## PROBLEM

3. Look straight into the mirror.

   a. Draw your face.

   b. Close one eye. Draw your face again.

   c. Hold up your outspread hand with the palm facing the mirror. Draw your hand.

4. Write about your drawings.

   a. How are your drawings the same as your actual body?

   b. How are the drawings different from your actual body?

   c. What is special about a reflection?

## ANSWER

3–4. Students should comment on how it seemed like the opposite side of their body was doing the action when they looked in the mirror. For example, when a student closes her left eye she will draw a picture of herself with her right eye closed. Students should also write about the properties of their reflections. For example, they may comment on how the reflection looked exactly like themselves, it was the size of their actual body, and their skin color and clothes were the same.

## ★ HOMEWORK

None

## Teacher's Insight

This drawing task gives students experience representing a three-dimensional object in a two-dimensional format. It highlights the properties of a reflection. Help them distinguish between the mathematical properties (orientation, size, etc.) and non-mathematical properties (how much the drawing resembles the person) and make comparisons between the real object and the drawing.

## Discussion

Have students share their drawings and explain what they were trying to accomplish in making them.

What did you notice about the reflection?

How could you describe what happened in the mirror when you closed your eye? What happened when you held up your hand?

How does this problem remind you of the bird problem?