UNIT 1

MPower

A Path to Understanding

ALGEBRA

ENGAGING WITH NUMBER

TEACHER NOTES

Algebra for Understanding

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To Teachers . . .

The *x* to Wh*y* project of the Curriculum Research & Development Group at the University of Hawai‘i has as its primary goal to produce curriculum materials that will help students be able to solve mathematical problems and communicate mathematically. To that end, *M^Power A Path to Understanding Algebra* was designed using instructional strategies that support students who struggle to develop concepts and skills that build confidence and competence in mathematics.

A basic premise of *M^Power* is that learning algebra requires more than memorizing formulas and finding answers. Therefore, students engage in open-ended problems and explorations, build conceptual understanding as they acquire skills, and develop generalizations. Your role as a teacher is to model how to engage in mathematical inquiry and discussion, guide the classroom discourse with thought-provoking questions, and facilitate whole class and small group work to optimize student engagement. The bulk of class time is spent cultivating the why and how of algebra. This strategy is driven partly by the problems and tasks and partly by the instructional approach.

Research Base

The research base for *M^Power* integrates research from mathematics education and special education to provide opportunities for students who struggle in mathematics to become confident and competent in their learning of algebra. The following points describe the research base.

1. Distributed practice. The units are structured so that students have multiple opportunities to engage in problems that move from concept to skill. They have a minimum of 5 days on a concept as it moves to more skill-type problems so there is more time to learn. Each problem set has no more than five problems so that students do not feel overwhelmed with long assignments.
2. Open-ended problems. The problems have multiple entry points so that students can engage in a task at their level, but you can extend the task by asking questions that move student learning further. These types of problems also allow all students to participate in the problem discussion rather than only one or a few students answering.

3. Multiple representations. Concrete (physical materials), semiconcrete (sketches, tables, graphs), and abstract (numerical and algebraic symbols) representations are included across the units. The different representations are presented concurrently so that students see connections across them.
4. Systematic instruction. The mathematics progressions connect prior knowledge to the new content so that students build a cohesive understanding of algebra. They do not see each new topic or lesson as a standalone and, thus, retain the content longer.
5. Scaffolds. Scaffolds are suggested to assist students who may have difficulty engaging in a task or understanding the content. The scaffolds include ways of organizing their work so that students can retrieve and explain their thinking.
6. Modified text. Unlike more traditional textbooks, *M^Power* includes dialogue from a fictitious group of students called the *X*-team. The *X*-team members share their thinking, providing students with a model for how to talk about mathematics.

Lesson Structure

Each lesson in *M^Power* has four components. **Get Started** provides a task that is a review of previously learned concepts or skills, a problem-solving task, or an advance organizer for new content. In the **Share** section, an instructional strategy is recommended for the discussion of homework (the detailed explanations of those strategies can be found in the appendix). The **Explore** section is the heart of the lesson where students have an opportunity to engage in a hands-on exploration of a mathematical topic. And finally, as you move into the **Reflect** part of the lesson, the big ideas or new concepts from the lesson are summarized.

1.7 Multiplication/Division Fact Teams

Mathematics Focus

- Multiplication/division fact teams represent relationships among positive and negative numbers.

Content Standards

7.NS.2: Apply and extend previous understandings of operations of multiplication and division and of fractions to multiply and divide rational numbers.

Standards for Mathematical Practice

Make sense of problems and persevere in solving them. (SMP 1)
Reason abstractly and quantitatively. (SMP 2)
Look for and make use of structure. (SMP 7)

Materials Needed

1 Pile High exploration per student
Video: *X-Power: Powers and Exponents* (0:47)—
https://www.youtube.com/watch?v=1NibmxadFjs
1 calculator per student or pair of students
1–2 sheets of paper (scratch paper is fine)
1 ruler per pair of students (optional)

LESSON OVERVIEW

Lesson Component	Teacher Actions	Student Actions	Approximate Time
Get Started	Present the task.	Complete the task.	5 minutes
Share	Facilitate a homework discussion using collaborative groups.	Discuss the homework.	20 minutes
Explore	Distribute the Pile High exploration. Clarify the process. Show the *X-Power* video.	Work individually, then discuss in pairs. Share your reasoning and insights. Watch the video.	25 minutes
Reflect	Summarize the lesson.	Read the lesson. Write fact teams for $0 \cdot (-3) = 0$ and/or $10 \cdot 0 = 0$.	10 minutes

Homework to assign: Problem Set 1.7

LESSON OUTLINE

1. GET STARTED

- Display the two problems.

  Find four pairs of numbers whose product is positive.
  Find four pairs of numbers whose product is 1.

- Allow 1 minute for students to write their pairs of numbers.
- Have students share.

?
- What patterns do they notice in each problem? In the first problem, they should notice that the factors are either both positive or both negative. In the second problem, the factors are reciprocals.
- Extend the second problem if students suggest that reciprocals are flipped numbers. Ask, What is the reciprocal of 2.75?

- Emphasize the definition of reciprocals as two numbers whose product is 1 rather than 'flipping' a number.

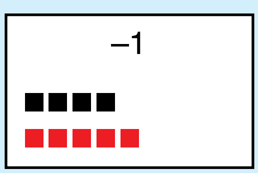
2. SHARE PROBLEM SET 1.6
Collaborative group discussion

- Place students in groups of 4.
- Ask each group to select one of the problems assigned for homework, OR assign a problem to each group. There may be multiple groups assigned to one problem depending on the number of groups in your class.
- Allow about 5–7 minutes for groups to prepare their problem presentation.
- Use any method to randomly select someone from a group to present their problem solution.

Problem Set 1.6

1. Write the fact team for each set of three numbers. Use the diagrams to help model your thinking.

a. 4, –5, –1



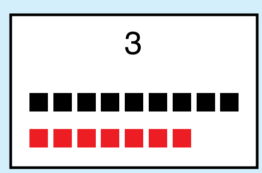
–1 = 4 + (– 5)

–1 = –5 + 4

–5 = –1 – 4

4 = –1 – (–5)

b. –6, 3, 9



–6 + 9 = 3

9 + (–6) = 3

3 – 9 = –6

3 – (–6) = 9

? How did students decide if each of three numbers was a fact team?

2. Write a multiplication/division fact team in which two of the three numbers are zero.

Example: $5 \cdot 0 = 0$

$0 \cdot 5 = 0$

$0 \div 5 = 0$

Students may write fact teams with a fourth equation, e.g., $0 \div 0 = 5$. Use their examples to discuss division by zero. The discussion should lead to the generalization that when two of three numbers in a multiplication/division fact team are zero, there are only three fact team equations. Students should recognize that 0 can be the dividend but not the divisor.

? Is it possible to have a fact team with only one 0? With three 0s? Why or why not?

3. Without using a calculator, find each sum.

a. $1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$ 5,050

?
- What methods did students use? Share the story of mathematician Carl Gauss, who correctly added the numbers $1 + 2 + 3 + \ldots + 100$ in a few seconds when he was a young boy. If students did not already share this method, suggest Gauss's technique of adding the numbers in "101-pairs" using addends from the "ends" inward ($1 + 100$, $2 + 99$, $3 + 98$, and so on).
- How might the solution to problem 3.a help them find solutions to problems 3.b and 3.c?

b. $1 + 3 + 5 + \ldots + 95 + 97 + 99$ 2,500

c. $-10 + (-9) + (-8) + \ldots + 3 + 4 + 5$ –40

? Did any students find addends that form zero pairs as a strategy?

4. Complete the magic square. The numbers in each row, column, and diagonal sum to –6.

1	–6	–1
–4	–2	0
–3	2	–5

? What methods did students use to fill the cells of the magic square?

5. Find four pairs of numbers to match each of the following conditions.

a. Find four pairs of numbers whose product is negative. Answers will vary.

If students give only pairs of integers, solicit pairs of fractions or decimals so students can generalize that the product of a positive factor and a negative factor is negative.

Students should be able to make other generalizations such as the product of two positive factors (or two negative factors) is positive. What if the product is 0? Students should describe that one or both factors must be 0.

b. Find four pairs of numbers whose quotient is negative. Answers will vary.

Students should generalize that the quotient is negative when a positive dividend is divided by a negative divisor, or when a negative dividend is divided by a positive divisor. A similar discussion should be conducted for a positive quotient.

c. Find four pairs of numbers whose product is 1. Answers will vary.

Students should generalize that the factors are reciprocals. If they only show fractions, tell them one factor is 0.62. What is the other factor if the product is 1? $\frac{100}{62}$ or any equivalent fraction or decimal.

3. EXPLORE

- Distribute the Pile High exploration, but do not allow students to begin working. (You may want to demonstrate, as in the next bullet, before you distribute the explorations.)
- Demonstrate folding and cutting a sheet of paper in half, then stacking the pieces. Repeat three times so students understand the process.
- Have students read problem 1 and allow not more than 30 seconds for them to record their prediction. Have students share their estimates and the methods they used to predict them.
- Allow 15 minutes for students to complete the exploration. Students should work individually but may consult with their partner.
- Facilitate a discussion on students' solutions and solution methods.
- Play the video *X-Power: Powers and Exponents* (00:47) to summarize the exploration and introduce exponential notation. https://www.youtube.com/watch?v=1NibmxadFjs
- Another related 4-minute TED video on YouTube, *How Folding Paper Can Get You to the Moon*, may help students understand the magnitude of exponential growth.

4. REFLECT

- Have students read Lesson 1.7.
- Discuss Reflect on This as time allows.

EXPLORATION
PILE HIGH

Name__________________________

Date___________________________

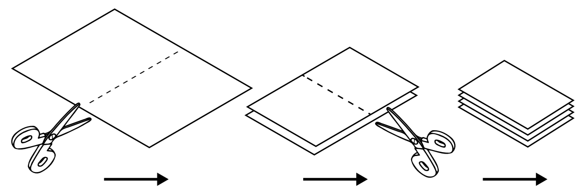
Materials: Calculator (optional), paper, scissors

Work with a partner. Write your answers on your own paper. Be sure to write legibly and explain your reasoning.

1. Suppose you were to cut a sheet of paper in half, stack one piece on top of the other, tear those in half, stack all the pieces together, and repeat this process 20 times.

A sheet of notebook paper is about 0.003 of an inch thick.

How high do you predict the stack of paper would be?



2. Use a sheet of paper and test your prediction. Record your findings in the table. What height did you find? Describe your method to determine what the height would be.

No. of cuts	No. of pieces	Height (inches)

3. How close to your prediction is the actual height you found? What might explain the difference between your prediction and the actual height you found?

4. a. What relationships do you notice between the number of cuts and the number of pieces?

b. What relationships do you notice between the number of pieces and the height?

c. What relationships do you notice between the number of cuts and the height?

PILE HIGH TEACHER NOTES

1. Suppose you were to cut a sheet of paper in half, stack one piece on top of the other, tear those in half, stack all the pieces together, and repeat this process 20 times. A sheet of notebook paper is about 0.003 of an inch thick. How high do you predict the stack of paper would be?

Answers may vary. Do not provide calculators or paper and pencil until this problem is discussed, and allow only a short time (such as 30 seconds) for students to make their predictions. Predictions will vary. Have students share how they estimated their predictions.

2. Use a sheet of paper and test your prediction. Record your findings in the table. What height did you find? Describe your method to determine what the height would be.

Answers may vary. If no one notices a pattern, suggest rewriting the numbers of pieces to show the number of times 2 is a factor (e.g., with 3 cuts there are 2 • 2 • 2, or 8, pieces of paper, or with 10 cuts there are 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2, or 1,024 pieces).

If students calculate using the paper thickness of 0.003 of an inch, then the stack will be approximately 3,145.728 inches thick, or about 262 feet tall. A local reference helps students relate to the magnitude of the number (e.g., in Hawai'i, the Aloha Tower is 184 feet tall).

Introduce exponential notation, where 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 can be written as 2^{10}, 2 is the **base** of the notation and 10 is the **exponent.** The exponent indicates how many times the base is used as a factor. Use other examples, such as 4^3, $(0.1)^5$, and 9^1, as needed.

No. of cuts	No. of pieces	Height (inches)
0	1	0.003
1	2	0.006
2	4	0.012
3	8	0.024
4	16	0.048
5	32	0.096
6	64	0.192
7	128	0.384
8	256	0.768

3. How close to your prediction is the actual height you found? What might explain the difference between your prediction and the actual height you found?

Answers may vary. Students may describe that there are 2^{20} sheets of paper.

4. a. What relationships do you notice between the number of cuts and the number of pieces?

Answers may vary. Students may notice that the number of pieces is 2 to the power of the number of cuts. Students may wonder if $2^0 = 1$; this will come up later when properties of exponents are explored (e.g., $\frac{2^x}{2^x} = 2^{x-x}$). However, it can be modeled with the whole sheet of paper where no cuts were made and thus, the total number of sheets is 1.

b. What relationships do you notice between the number of pieces and the height?

Answers may vary. Students may notice a doubling pattern with consecutive heights or describe that the height is the product of 0.003 (thickness of paper) and the number of pieces.

c. What relationships do you notice between the number of cuts and the height?

Answers may vary. Students may notice that the product of 0.003 (thickness of paper) and 2 to the power of the number of cuts is the height.

EXPLORATION
PILE HIGH

Name____________________________

Date_____________________________

Materials: Calculator (optional), paper, scissors

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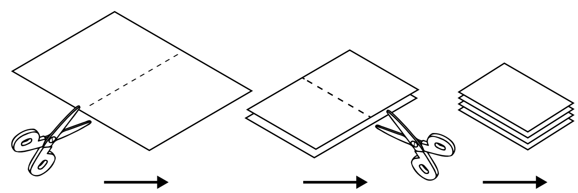

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X to whY

Algebra for Understanding

M Power — A Path to Understanding Algebra

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