

UNIT 1

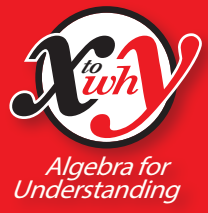
M^{Power}

A Path to Understanding
ALGEBRA



SAMPLE

ENGAGING WITH NUMBER



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GS.1 Setting the Stage

We are beginning a journey that will lead us to some exciting explorations of mathematical topics. Meet the X-Team that will be traveling with us on this journey.



Matt



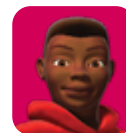
Amanda



Shauna



Marty



RJ

In each unit, the X-Team will be working through the mathematics with us. They will discuss problems and we will try to solve them *before* reading on. The X-Team knows that mathematics is not a spectator sport—to learn mathematics, we must *do* mathematics.



RJ gave Marty and me a problem to solve. The first thing I need to do is understand the problem. It says, ‘Lisa has some pencils. Kim has the same number of pencils. If Lisa gives Kim six pencils, how many more pencils than Lisa will Kim have?’ I think I have a way to solve it. I’m going to use a table with guess-and-check.

| Beginning number of Lisa’s pencils | Beginning number of Kim’s pencils | Ending number of Lisa’s pencils | Ending number of Kim’s pencils | How many more pencils does Kim have? |
|------------------------------------|-----------------------------------|---------------------------------|--------------------------------|--------------------------------------|
| 24 | 24 | 18 | 30 | 12 |
| 20 | 20 | 14 | 26 | 12 |
| 100 | 100 | 94 | 106 | 12 |



The table helps me notice something! No matter how many pencils each girl started with, after Lisa gave Kim six pencils, she had 12 fewer pencils than Kim.

I solved it a different way, Shauna. I used a diagram. I noticed that when Lisa gave Kim six pencils, Lisa was not only *increasing* Kim's pile by six pencils but was also *decreasing* her own pile by six pencils.



REFLECT ON THIS How are Marty's method and Shauna's method alike? How are they different? Which method do you prefer?

Characteristics of good problem solvers include

- understanding the problem,
- devising a plan,
- carrying out the plan, and
- looking back.

Sometimes we have to go back and read the problem again to understand it. Or we try a method and it doesn't work, so we have to devise a new plan.

REFLECT ON THIS What strategies do you use to understand the problem?

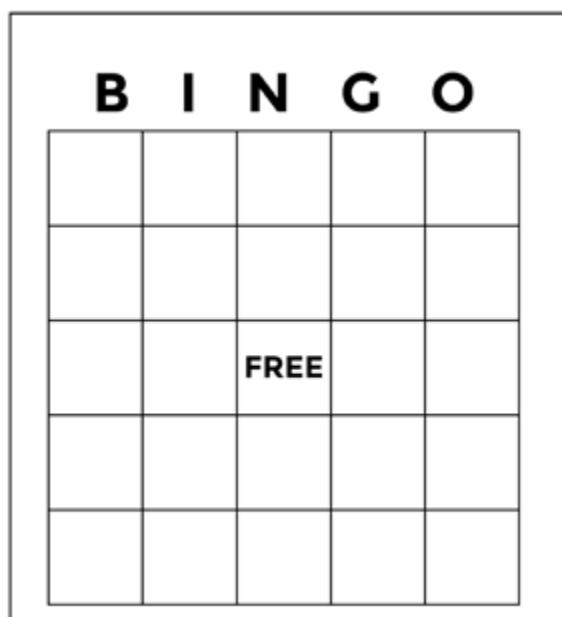
EXPLORATION

BINGO CARD PROBLEM

Materials: Calculator (optional)

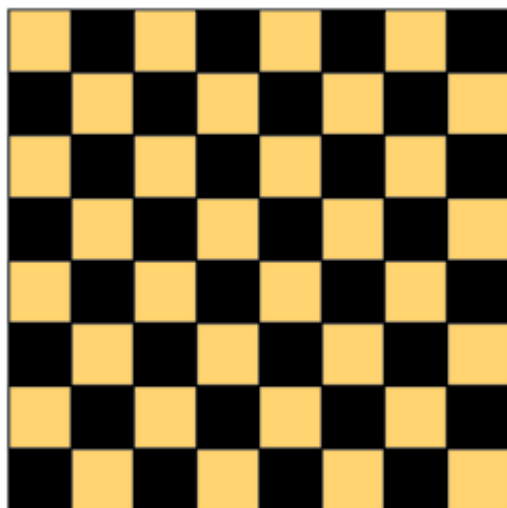
Work with a partner. Write your answers on your own paper. Be sure to write legibly and to explain your reasoning.

1. How many squares can you find on this Bingo card?



2. Describe the method you and your partner used to solve this problem. Be sure to include any patterns that you noticed.

3. How many squares are there on an ordinary checkerboard?



4. How did solving the Bingo card problem help you solve the checkerboard problem?

EXPLORATION

FURNITURE PROBLEM

Work with a partner. Write your answers on your own paper. Be sure to write legibly and to explain your reasoning.

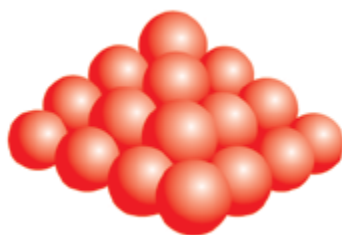
1. The U-Bought-It-U-Own-It Furniture Outlet is having a sale. As a promotion, it is advertising some chairs and stools at ridiculously low prices. Although all sale merchandise is sold in “as is” condition, the outlet promises that each chair will have four legs and each stool will have three legs. The outlet also guarantees that there will be at least two more chairs than stools.

If there are 53 legs altogether, how many chairs and stools are being advertised?

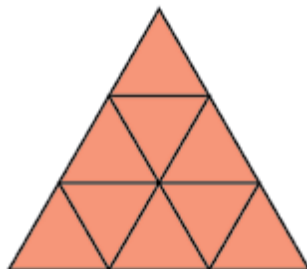
2. If there were 72 legs and *at least twice* as many chairs as there were stools, how many chairs and stools would the outlet have?
3. Describe how solving problem 1 helped or did not help to solve problem 2.

PROBLEM SET GS.1

1. Masha's team, the Homerun Hitters, has won 18 games. Unfortunately, the team has lost 4 more games than it has won. How many games have the Hitters played?
2. Oranges are piled in a pyramid with 1 orange in the top layer, 4 in the next layer, 9 in the third layer, and 16 in the fourth layer. How many oranges will you need to make a pile with 10 layers?



3. Kona, on the island of Hawai'i, is 95 miles from Hilo and 41 miles from Waimea. If Waimea is on the road between Hilo and Kona, how far is Hilo from Waimea?
4. How many equilateral triangles are there?



1.7 Multiplication/Division Fact Teams

Marty wondered if she could use patterns to multiply two negative numbers. She decided to use what she learned about multiplying a positive number with a negative number and recorded a list of products where -3 was always the second factor. Marty shared her pattern with Shauna.

$$\begin{aligned} 5 \cdot (-3) &= -15 \\ 4 \cdot (-3) &= -12 \\ 3 \cdot (-3) &= -9 \\ 2 \cdot (-3) &= -6 \\ 1 \cdot (-3) &= -3 \\ 0 \cdot (-3) &= 0 \\ -1 \cdot (-3) &= ? \end{aligned}$$

REFLECT ON THIS What do you notice about Marty's products each time she decreases the first factor by 1 and multiplies it by the second factor of -3 ? How can the pattern help her find the product of $-1 \cdot (-3)$? How will it help to find $-2 \cdot (-3)$?

Shauna was excited and inspired by Marty's pattern of products.

Shauna's Fact Team

$$\begin{aligned} 12 &= -4 \cdot (-3) \\ 12 &= -3 \cdot (-4) \\ -4 &= 12 \div (-3) \\ -3 &= 12 \div (-4) \end{aligned}$$

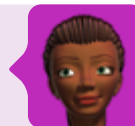
Marty, I used your pattern to find the product of -4 and -3 , and I wrote its fact team. What do you think?





I agree with your equations, Shauna. Fact teams are really helpful because once you know how to multiply integers, you also know the related division facts!

That's what I thought too, but then I wrote a couple of fact teams that have me wondering. I tried the fact team for $0 \cdot (-3) = 0$, but something just did not seem right with one of my four equations. So then I tried another fact team with a similar equation, $10 \cdot 0 = 0$, and now I'm really perplexed. Are these really fact teams?



REFLECT ON THIS Write the fact team for $0 \cdot (-3) = 0$ and the fact team for $10 \cdot 0 = 0$. What do you think has Shauna perplexed? Do you think these are fact teams?

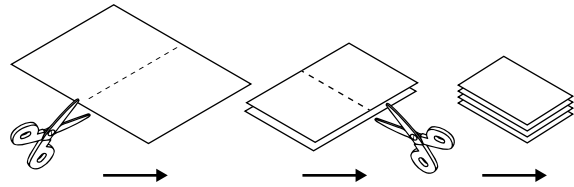
EXPLORATION

Pile High

Materials: Calculator (optional), paper (optional)

Work with a partner. Write your answers on your own paper. Be sure to write legibly and to explain your reasoning.

- Suppose you were to cut a sheet of paper in half, stack one piece on top of the other, tear those in half, stack all the pieces together, and repeat this process 20 times. A sheet of notebook paper is about 0.003 of an inch thick. How high do you predict the stack of paper would be?



- Use a sheet of paper and test your prediction. Record your findings in a table. What height did you find? Describe your method to determine what the height would be.
- How close to your prediction is the actual height you found? What might explain the difference between your prediction and the actual height you found?
- What relationships do you notice between the number of cuts and the number of pieces?
 - What relationships do you notice between the number of pieces and the height?
 - What relationships do you notice between the number of cuts and the height?

| No. of cuts | No. of pieces | Height (inches) |
|-------------|---------------|-----------------|
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PROBLEM SET 1.7

1. Write fact teams to match the conditions given in each part below.
 - a. Write an addition/subtraction fact team in which two of the numbers are negative and one is positive.
 - b. Write a multiplication/division fact team in which two of the numbers are negative and one is positive.
 - c. Write an addition/subtraction fact team in which all three numbers are negative.
 - d. Write a multiplication/division fact team in which all three numbers are negative.
2. What number divided by 6 equals -4 ?
3. Find the following sums.
 - a. $9 + (-12)$
 - b. $-\frac{4}{5} + (-\frac{1}{5})$
 - c. $4\frac{1}{2} + 2\frac{1}{2}$
 - d. $-15 + (-37)$
4. Find the following differences.
 - a. $-15 - 37$
 - b. $4\frac{1}{2} - (2\frac{1}{2})$
 - c. $9 - 12$
 - d. $-\frac{4}{5} - \frac{1}{5}$
5. Gina opened a checking account on September 1 with \$200. Here are her deposits and debits for the first two weeks of September. How much was in the account as of September 16?

| Sept. | Note | Debit | Deposit |
|-------|-----------------|---------|----------|
| 1 | Cash | | \$200.00 |
| 5 | Game Shop | \$30.97 | |
| 7 | Cash withdrawal | \$40.00 | |
| 12 | Shoe Mart | \$57.20 | |
| 15 | Paycheck | | \$115.00 |



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