# First Graders' Representation and Relational Thinking: A Qualitative Study

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# Relational Thinking

- A key construct for Early Algebraic Thinking
- Comparing quantities and determining if a quantity is equal to, less than, or greater than another
- Focus is on relations as opposed to computing or operating with numbers

# Davydov (1975a)

- Early stages of mathematics learning should rely on concepts of comparing and relating quantities
  - Transitivity property
  - Symmetric property of equality
  - Reflexive property of equality
- A quantitative approach allows students to think about and understand mathematical ideas at a generalized level

# Research Question

What role do representations play in supporting first grade students' relational thinking of quantitative comparisons?

# Our Work

- Curriculum research and development project in a public charter school
- Ten Grade 1 students (6–7 years old)

# Representation in the Measure Up Curriculum

- Use of multiple concurrent representations
  - Students represent the mathematics in different ways, e.g., as continuous quantities, with paper strips, pictorially, and symbolically
  - These representations are done concurrently rather than sequentially
- Links concept and physical action
  - The physical actions allow students to think abstractly and make links to mathematical concepts being introduced without numbers
- Generalized statements (e.g., $V < M$) gives meaning to number examples

# Methods

- Two assessments, given 6 weeks apart
- Semi-structured interviews in one-to-one sessions, conducted by an MU teacher-researcher
- Took place during the implementation of curriculum
- Data from interviews were analyzed for indications of relational thinking as observed via student's actions, words, and symbols
    - Codes were developed through a series of memo-writing (Corbin & Strauss, 2008)

# Sample

Ava

Ben

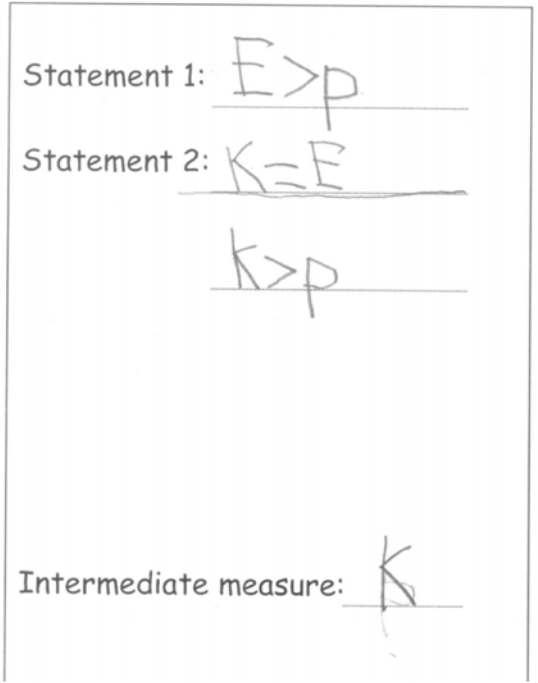
Noah

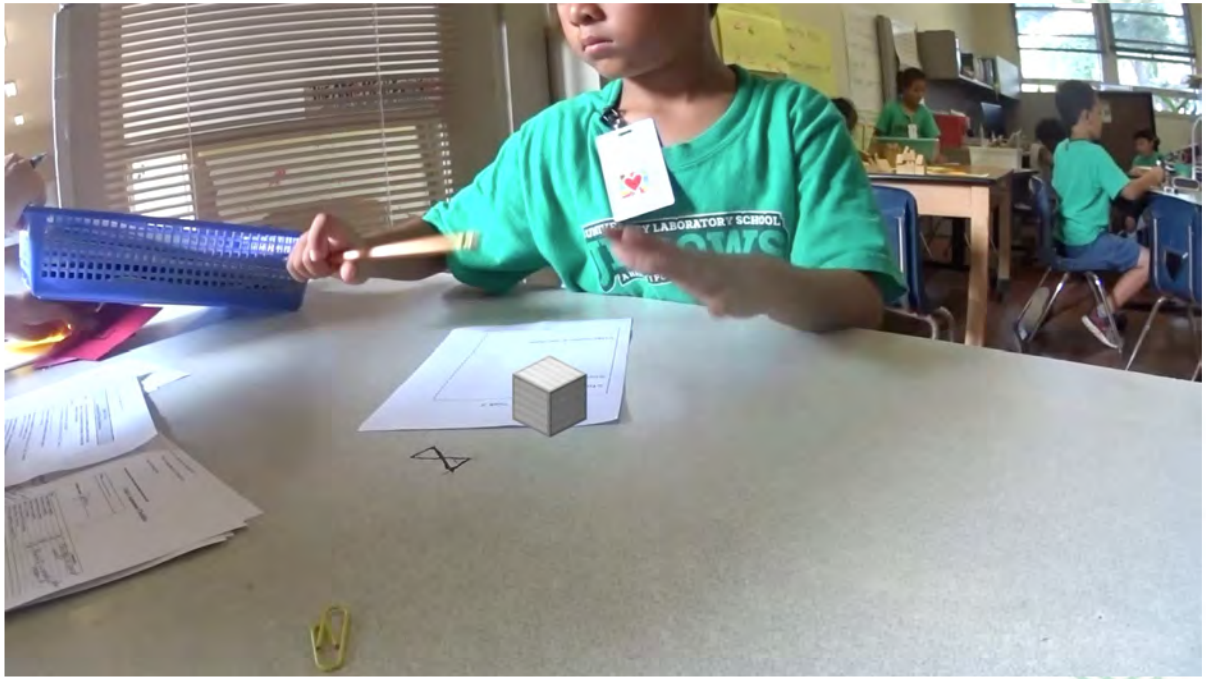
# Assessments Interview: Noah

# Data: Relational thinking between two quantities

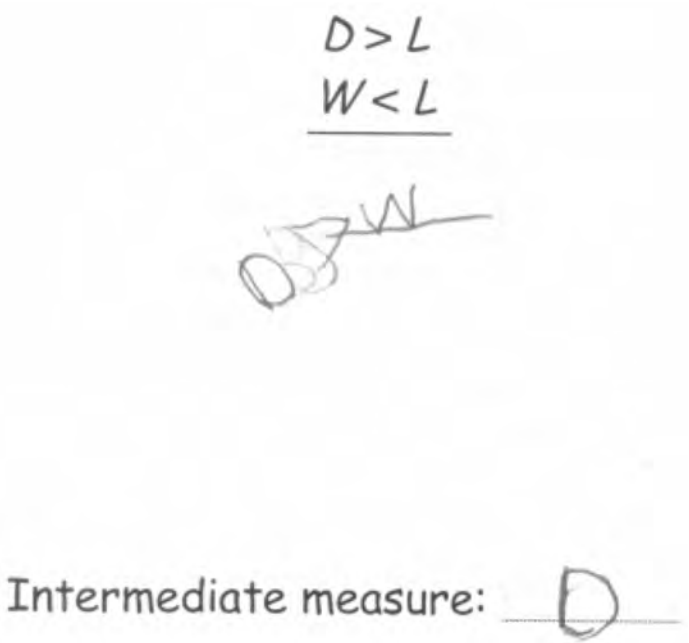
| Ava | Ben | Noah |
| --- | --- | --- |
| Coordinates verbal explanation, written representations, reading statements, and physical comparison, though not according to conventions | Indicates relationship between two quantities, mostly accurate with describing and representing relationships | Indicates relationship between two quantities, accurate with describing and representing relationships. Some irregularity with method of comparison |

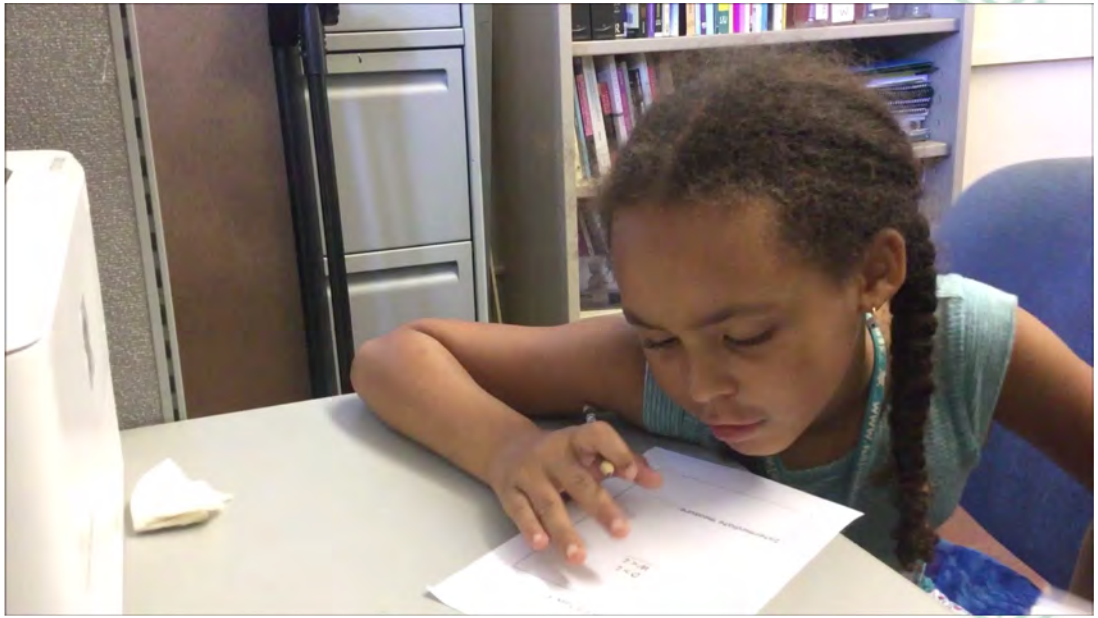
# Assessments Interview: Ben



# Assessments Interview: Ava



# Data: Transitive reasoning with an intermediate measure

| Ava | Ben | Noah |
| --- | --- | --- |
| Used two direct comparisons to deduce an indirect comparison. Did not attend to the role of the intermediate measure | Used two direct comparisons to deduce an indirect comparison, developing understanding about the role of the intermediate measure. Explained that the intermediate measure is not in the final statement | Used two direct comparisons to deduce an indirect comparison, strong understanding about the role of the intermediate measure. Described the intermediate measure as “the medium” |

# Findings

Codes referring to representation and student thinking about quantitative relationships

1. Spoke with mathematics vocabulary
2. Used hand gestures
3. Interpreted symbolic representations of comparisons
4. Recognized comparison relationships (equal, strict non-equal) using physical quantities
5. Represented with relational statements (letters and symbols)
6. Represented relationships with diagrams (line segments)
7. Exhibited deductive reasoning

# Discussion

- Representations were used concurrently and allowed for flexibility in going between representations
- The representations were used to convey relationships between and among quantities
- Through the representations, students explained how they made sense of relationships

# Implications

For Early Algebraic Thinking

- “Thus, while children in the U.S. are having pre-algebraic experiences that are numerical, Russian children using Davydov’s curriculum are having pre-numerical experiences that are algebraic” (Schmittau & Morris, 2004, p. 61)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