Learning Place Value through a Measurement Context

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How does a measurement context support student understanding of place value?

- By measurement context we include the conceptual development of unit, part-whole relationship, and iteration using continuous quantities

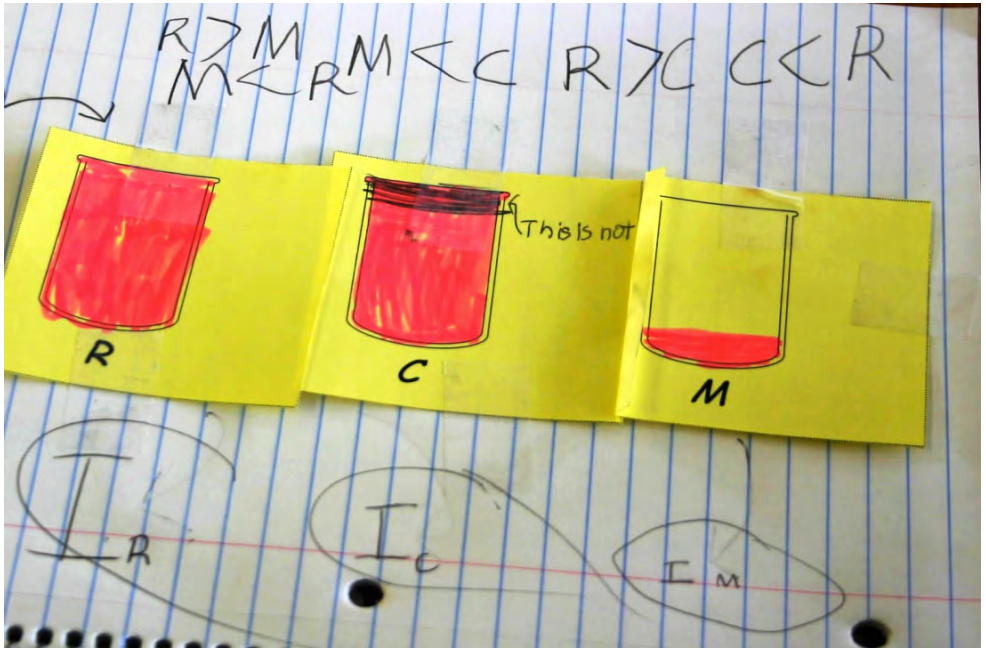
- These quantities are length, area, volume, and mass




Objectives for this presentation

- To describe a measurement approach to number structure and place value
  - The Measure Up (MU) curriculum project is being developed at the University of Hawai‘i
- To share evidence about the effectiveness of this approach

Measure Up background, Davydov origins



Contrasting approaches to school mathematics

Discrete counting approach

- Number as the result of counting objects
- Begins with a focus on specific cases
- Quantities viewed as collections
- Builds toward generalized cases
- Focuses on *how many*

MU quantitative approach

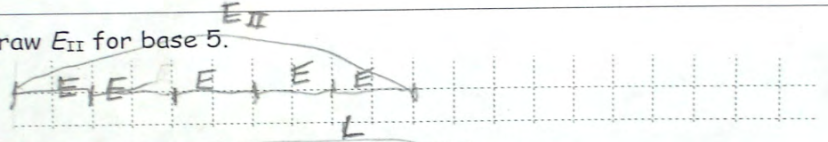
- Number represents the relationship between a unit and a larger quantity
- Counting as a means to document measurement
- Students must define the unit
- Starts from generalized cases
- **Counting implies action: using a unit and iterating a number of times**

Place value as a structure

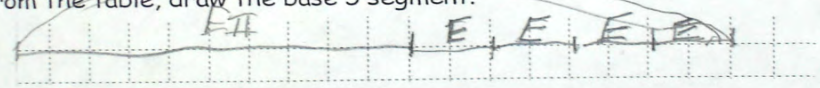
4. a. E is the main measure.

II	I	
1	4	(5)
2	1	(3)

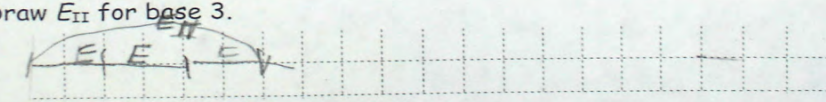
Draw E_{II} for base 5.



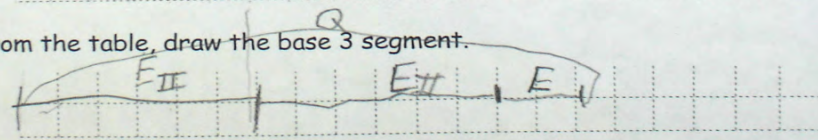
From the table, draw the base 5 segment.

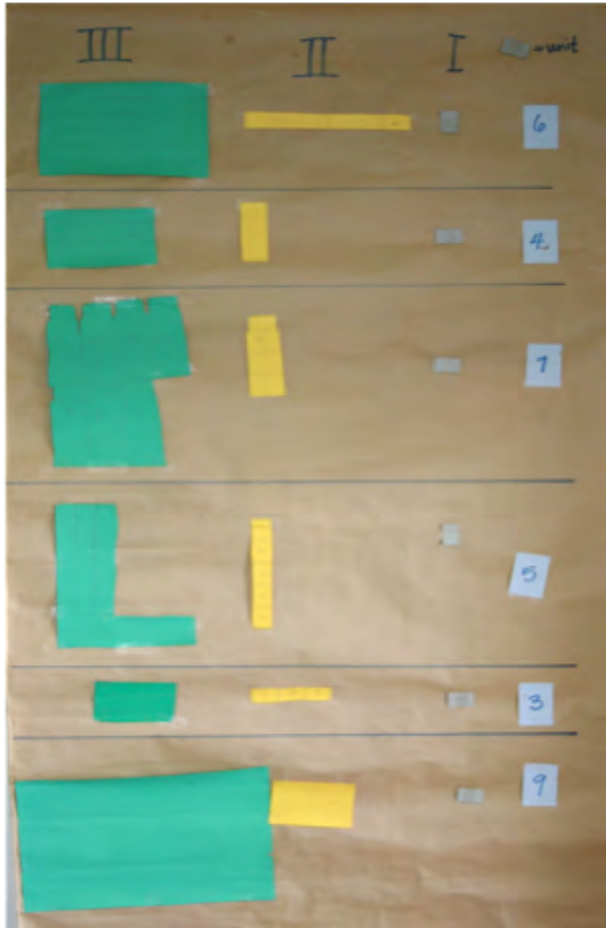


Draw E_{II} for base 3.



From the table, draw the base 3 segment.

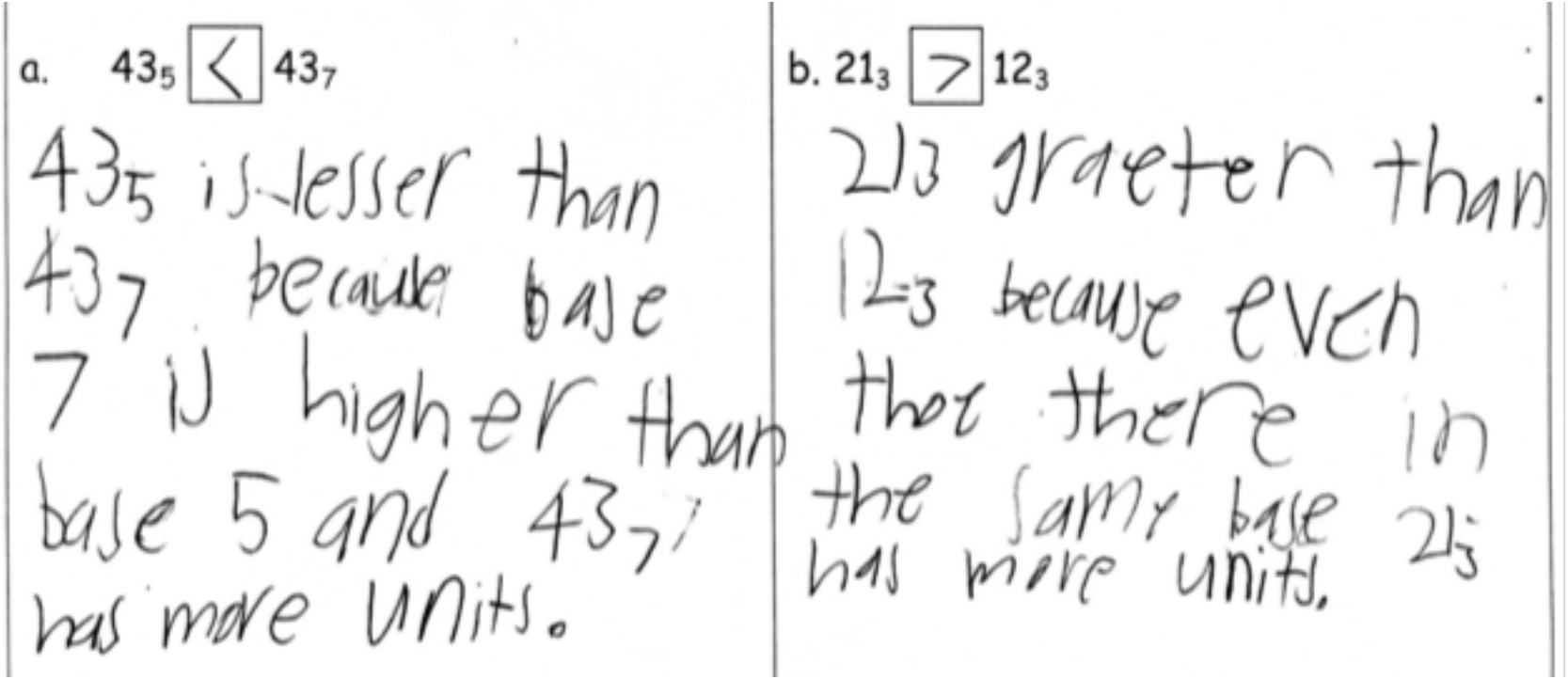




Our study

Research question: How does a measurement context support student understanding of place value?
Subjects: Thirty students, 7–8 years old, in their second year of the MU mathematics curriculum
Treatment: MU mathematics, daily instruction in a class of 10 (laboratory school setting), 40–45 minutes per session, teacher-researcher roles
Instrument: MU written assessment on the number system
Data analysis: Responses analyzed for evidence of mathematical structural understanding

MU mathematics foundational to place value



Assessment problems

Problem 1. Use your units and the information from the table to draw area J.

III	II	I	
1	3	2	(four)

Problem 2. Complete the statements below and explain your answers.

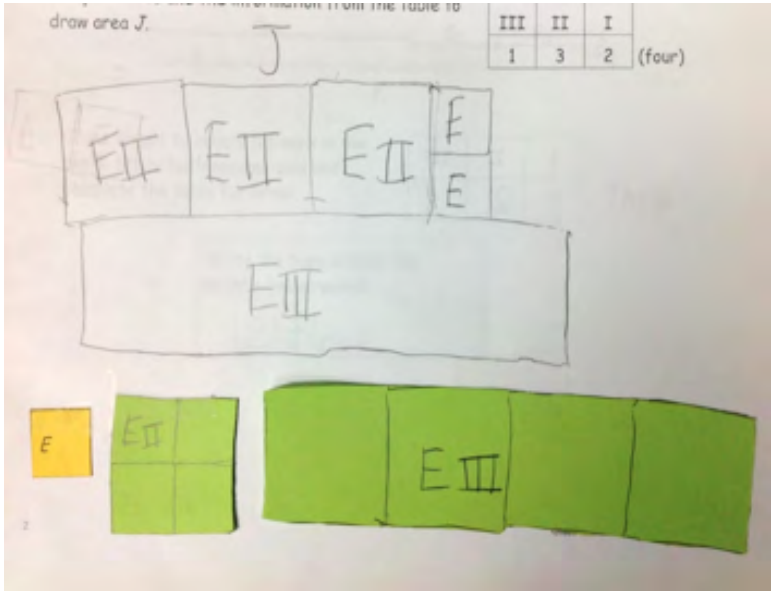
$$4_5 \square 4_7$$

$$21_6 > 21_{\square}$$

$$3\square_9 > 38_9$$

Sample response and analysis

Koa



- Koa's work is an example of generalized understanding of place value
  - Koa uses the unit to create a place II measure, then uses the place II as a unit to iterate and create a place III
  - Individual unit markings in the larger measures are irrelevant, focus is on the relationship among the place measures

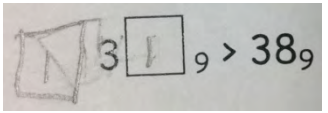
Sample response and analysis, continued

Logan

In Logan's work we see individual units marked on the place III area, but not on the traced area. This is an indication that the student is at a developmental stage.

Responses to the assessment, $3\square_9 > 38_9$

- I am stuck It says > but they took the big anser [*sic*] I can't think.
- It dosent [*sic*] make sense because the other statement is 38_9 and so I cant write anything
- It's eight so I can't use digit 9
- I had to put another box so the statement would be true
- Because if I put in aswer [*sic*] it will be lesser and it spote [*sic*] to be greater

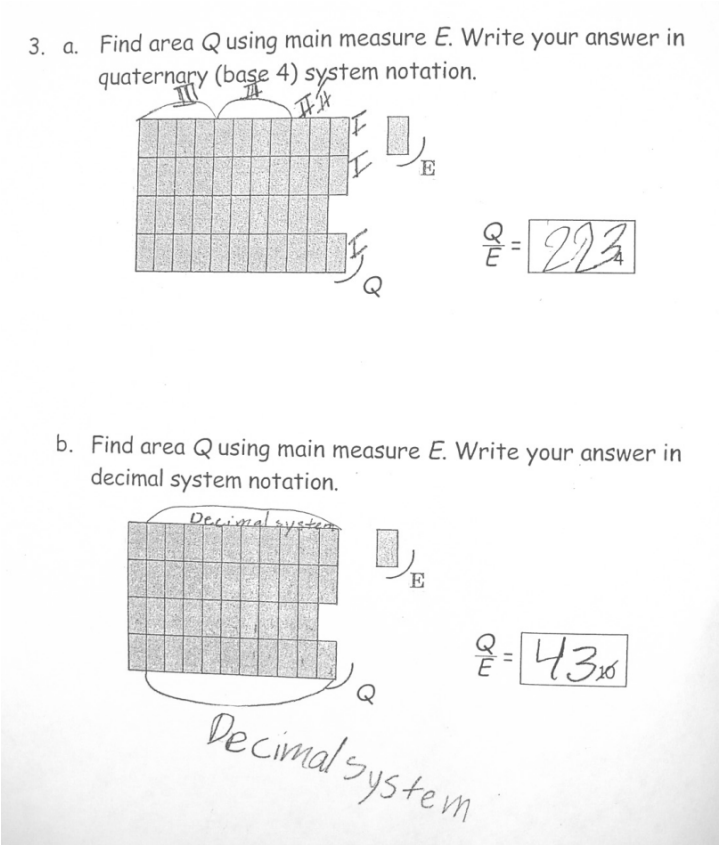


Analysis of responses to $3\square_9 > 38_9$

- Responses to the second task indicate that students developed a relational view of multi-digit numbers in base 9
  - These responses address the structure of place value and the logic students expect of the mathematics
  - One student changes the context of the problem to be able respond

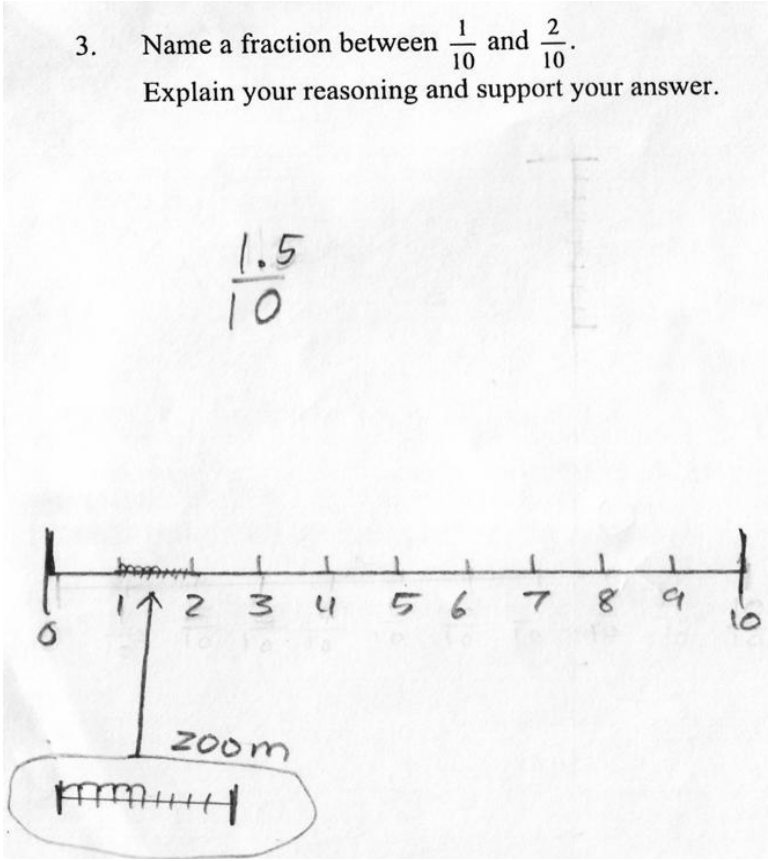
Connections to the decimal system

- Tasks involving measurement in different bases
- Students identify place II and place III measures
- Considerations for representations of quantities
- Discussion about $223_4 = 43_{10}$ reveal robustness of students' thinking



This approach is maintained with the introduction of later topics

- Opportunity to explore the density of numbers
- Student “zoom” indicates preservation of base ten
- Concept of partial units maintains the base number and extends place value



Discussion and concluding thoughts

- MU students develop the capacity to focus on the supplementary measures, (e.g., E_{II} and E_{III}) as units themselves rather than as counted collections of discrete pieces.
- Measurement context is a powerful means for young children to develop mathematics understanding
- Learning place value and the number system are not trivial tasks
- The representation of magnitude in the measurement context enables students to focus on the multiplicative relationship of the numbers, as opposed to numbers as mere counts