Alignment of

Curriculum Research & Development Group Mathematics programs

Algebra I: A Process Approach
Geometry: A Moving Experience
Reshaping Mathematics **(in development)**

with the

Content Standards - HCPS II

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Curriculum Research & Development Group
Mathematics Programs

The Mathematics Section of the Curriculum Research & Development Group, University of Hawai‘i, has developed three programs: *Algebra I: A Process Approach, Geometry: A Moving Experience,* and *Reshaping Mathematics.*

Our programs have four consistent characteristics. They
1. are based on research about students' thinking and learning.
2. develop concepts and skill over extended time.
3. provide opportunities for students to make sense of the mathematics.
4. support teacher change and the implementation of the curriculum in the classroom.

Algebra I: A Process Approach

Algebra I: A Process Approach is a first-year algebra program designed to help students of all ability levels develop problem-solving processes. A basic premise of the program is that students must do more than memorize formulas and get right answers. They must learn to think mathematically and communicate their thinking through speaking and writing.

Algebra I: A Process Approach alters content and instructional methods to foster solid mathematical understanding. It is based on research into how students think and learn. Problem tasks promote open-ended inquiry appropriate for individual differences in any classroom. Sequencing of tasks allows time for students to grasp concepts, arrive at generalizations, and refine their skills.

Algebra I: A Process Approach has been identified as a promising program in mathematics education by the Laboratory Network Program of the U. S. Department of Education's Office of Educational Research and Improvement. The accompanying required professional development component has been selected by the National Staff Development Council as one of 26 programs that has evidence linking teacher learning with student learning.

Our program recognizes that teachers need support as they implement programs that require a new look at teaching and learning. Teachers using *Algebra I: A Process Approach* must successfully complete a professional development institute. Varying levels and types of follow-up support are available to new users.

Geometry: A Moving Experience

Geometry: A Moving Experience is a full-year course for high school geometry. The materials have been successfully used with a diverse range of students.

Transformational geometry is the basis for developing geometric ideas. Students begin early in the school year validating their reasoning and solutions, moving to a more formal notion of proof as their understandings and experiences increase. Topics are developed over time so that all students have access to the geometric content. *Geometry: A Moving Experience* provides opportunities for multiple ways of solving a problem, often requiring students to bring together a variety of understandings.

As with other mathematics programs, teachers using *Geometry: A Moving Experience* must complete a two-part professional development institute. The first part, 45 hours, introduces teachers to new approaches to teaching high-school geometry and includes geometry content based on transformations, research on student learning, and new instructional approaches developed as part of the program. The second part provides additional practical information for teachers who will be implementing the program during the following school year. This teacher-user phase of the course consists of up to three half-days. Teachers who use *Geometry: A Moving Experience* participate in various follow-up activities throughout the school year.

Reshaping Mathematics (In Development)

The *Reshaping Mathematics* staff is designing a two-year middle-grades program. Transformational geometry provides the basis for developing mathematics through visual perception and spatial tasks.

Professional development assists teachers in making the transition from a traditional classroom model to one that focuses on student intuition and discourse.

Curriculum Research & Development Group/University Laboratory School

The Curriculum Research & Development Group (CRDG), including the University Laboratory School, conducts systematic research, design, development, publication, staff development, and related services for elementary and secondary schools. The CRDG has curriculum development projects in science, mathematics, English, Asian studies, marine studies, environmental studies, Hawaiian and Polynesian studies, Japanese language and culture, music, nutrition, art, technology, health, and computer software. Research and school service projects focus on educational evaluation, teacher development, reduction of in-school segregation of students, and programs for students educationally at risk.

The CRDG is the senior member of a cooperative program of thirteen universities in the United States to improve schooling in science, health, and technology in elementary and secondary schools. It is a founding member of the Pacific Circle Consortium of universities, major school systems, and educational ministries in Australia, Canada, Japan, New Zealand, and the United States.

CRDG-developed programs are being used throughout the United States and in other countries. The CRDG provides professional development institutes and support services for all its projects. CRDG publishes and distributes its materials nationally and internationally.

Mathematics Materials and Projects
Alignment with *Mathematics Content Standards: Moving from the Blue Book to HCPS II*

The *Mathematics Content Standards: Moving from the Blue Book to HCPS II* (1999) is based on six principles:

1. All students can learn and want to learn important mathematics.

Our programs include interesting problems that encourage and motivate students to engage in the mathematics. Our materials are designed to meet the needs of a broad range of students.

2. All students should have equal access to important and meaningful mathematics through curricula that are coherent with the standards and comprehensive.

Our materials provide full-year courses. These materials align with national and state standards for content, instruction and assessment.

3. All students should have access to competent, mathematically confident, and caring teachers who will help them understand and use mathematics.

We understand that teachers need support as they use new materials and change their teaching methods. Each program has created teacher support materials such as a teacher's guide and ancillaries. Our programs also include a minimum 45-hour teacher institute with additional follow-up support at the school level. The National Staff Development Council (1999) cited our algebra program's staff development institute as a program that results in changes in teaching and learning.

4. All students are entitled to the range of learning experiences that will enable them to understand and use mathematics and be successful in their learning.

Combining strong mathematics materials with a collaborative classroom environment empowers students to build understandings and be confident of their mathematical abilities.

5. All students should have access to and use technology appropriately to help them understand mathematics and help prepare them to use mathematics in an increasingly technological world.

Our programs incorporate technology in both student and teacher components. Students use calculators to assist in developing conceptual understandings.

6. All students should have multiple opportunities and multiple ways to demonstrate what they know and can do in mathematics.

Our programs include multiple types of assessments and suggestions for aligning grading and evaluation techniques with standards-based teaching and learning. The National Laboratory Network (1994, 1998) cited our algebra program's assessment strategies as suggestions for multiple techniques.

The *Mathematics Content Standards* include five content strands: Number and Operation; Measurement; Geometry and Spatial Sense; Patterns, Functions, and Algebra; and Data Analysis, Statistics, and Probability. The process standards, which include communication, connections, problem solving, logic and reasoning, and representation, are not explicitly mentioned in the content standards but are embedded throughout.

The tables on subsequent pages show which content standards are addressed by our programs. The following gives a snapshot of how our projects address the process standards.

Communication. Communication is central in each of our programs. Students communicate in at least five ways: (1) reading, (2) writing, (3) listening critically, (4) speaking, and (5) modeling. Within each daily lesson, students have opportunities to share their understandings through multiple ways in settings such as large and small groups and individually.

Connections. Problems and tasks in our programs are structured so that students naturally connect topics. As they begin a new topic, introductory problems carefully link more mature understandings with new concepts. As students gain sophistication with concepts, associated skill development follows naturally.

Problem solving. Our programs are enmeshed with problem solving in two ways. First, students engage in problem-solving tasks daily. These tasks introduce new concepts, build the use of non-routine strategies, and provide situations for applications of skills and other developed concepts.

Second, teachers using our programs consistently use questions that promote higher-order thinking. Students are often asked to solve problems in multiple ways, generalize their findings, make conjectures about patterns they notice, and extend the problem.

Logic and reasoning. Students are expected to explain how and why they selected particular processes or strategies to solve a problem. They justify their answers not only to a degree of accuracy but also to its reasonableness.

Representation. Problem sets encourage students to represent mathematics in multiple ways with tables, symbols, diagrams, and manipulatives. Each representation is linked carefully with appropriate mathematical development of the concept or skill.

Mathematics Content Standards: Moving from the Blue Book to HCPS II Alignment for Grades 6–8

Content Standard	Grade Cluster Benchmarks Grades 6–8	Benchmarks Addressed by	
		Reshaping Mathematics (In Development)	***Algebra I: A Process Approach***
Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.	Recognize and use signed numbers (e.g., order and compare integers and rationals), proportions, and percents (e.g., use 15.1% or 4 2/3%)	✓	✓
	Explain and apply number theory concepts, (e.g., triangular and square numbers, prime and composite numbers, factors and multiples).	✓	✓
	Represent and use exponents, absolute values, scientific notation, and square and cube roots.	✓ (except for cube roots)	✓ (except for cube roots)
Number and Operations: Understand the meaning of operations and how they relate to each other.	Describe situations when addition, subtraction, multiplication, and division of integers, rationals, and numbers involving whole number non-negative exponents are appropriate.	✓	✓
	Recognize and use properties (closure, associative, commutative, distributive, identity, and inverse) and order of operations on rational numbers.	✓	✓
	Understand and use inverse relationships among and within operations of addition and subtraction, multiplication and division, and squares and square roots.	✓	✓
Number and Operations: Use computational tools and strategies fluently and when appropriate, use estimation.	Select and use appropriate strategies for computing with rationals, ratios, exponents, square roots.	✓	✓
	Use estimation as a means to check the reasonableness of results; and form upper and lower bounds.	✓	✓

Content Standard	Grade Cluster Benchmarks Grades 6–8	Benchmarks Addressed by	
		Reshaping Mathematics (In Development)	***Algebra I: A Process Approach***
Measurement: Understand attributes, units, and systems of units in measurement; and develop and use techniques, tools, and formulas for measuring.	Select appropriate units to estimate and measure angles, circumferences and areas of a circle, and surface area and volume of regular solids.	✓	
	Estimate and measure angles in plane figures.	✓	
	Develop and use formula for circumference and area of a circle.	✓	
	Develop and use formulas to find surface area and volumes of regular solids.	✓	
	Select appropriately precise tools to achieve a desired accuracy in measurement.	✓	
	Use ratios and proportions to solve problems related to measurement	✓	✓
	Determine an appropriate scale and make scale drawings; use scale drawings or models in applications.		
	Solve simple problems involving rates and other derived measures (e.g., miles per hour, gms per cc or cm^3).		✓
Geometry and spatial sense: Analyze properties of objects and relationships among the properties.	Analyze and describe geometric relationships among two- and three-dimensional figures (e.g., relationship between rectangles and squares, and between equilateral and isosceles triangles).	✓	
	Critique logical arguments concerning geometric ideas and relationships.	✓	
	Describe and apply geometric ideas and relationships to solve problems (e.g., Pythagorean theorem, similar triangles).	✓	✓

Content Standard	Grade Cluster Benchmarks Grades 6–8	Benchmarks Addressed by	
		Reshaping Mathematics (In Development)	***Algebra I: A Process Approach***
Geometry and spatial sense: Use transformations and symmetry to analyze mathematical situations.	Describe changes in size, position, and orientation of plane figures under dilation, reflection, rotation, and translation.	✓	
	Use reflectional and rotational symmetry to describe polygons and polyhedra.	✓ (not polyhedra)	
	Describe changes in size, position, or orientation under compositions of transformations.		
Geometry and spatial sense: Use visualization and spatial reasoning to solve problems both within and outside of mathematics.	Draw two-dimensional representations of three-dimensional objects.	✓ (in graphing unit negative dilations)	
	Compose and decompose two- and three-dimensional figures in order to solve problems and develop formulas (e.g., $(a + b)^2 = a^2 + 2ab + b^2$).	✓	✓ (two-dimensional figures)
Geometry and spatial sense: Select and use different representational systems, including coordinate geometry.	Use coordinate geometry to describe geometric relationships and properties of geometric figures.		
	Use relationships found in right triangles to solve problems (e.g., Pythagorean and sine, cosine, and tangent ratios).		✓ (chapter 11)
Patterns, functions, and algebra: Understand various types of patterns and functional relationships.	Analyze, create, and generalize numeric and visual patterns including patterns that have a recursive nature (e.g., Pascal's triangle, Fibonacci numbers, powers of rational numbers).	✓	✓ (except recursive)
	Use patterns to solve problems.	✓	✓
	Analyze and use linear relationships among two variables.	✓	✓

Content Standard	Grade Cluster Benchmarks Grades 6–8	Benchmarks Addressed by	
		Reshaping Mathematics (In Development)	***Algebra I: A Process Approach***
Patterns, functions, and algebra: Use symbolic forms to represent, model, and analyze mathematical situations.	Represent and translate among a variety of patterns, relations and functions with tables, graphs, verbal rules, and symbolic rules.	✓	✓
	Use symbolic algebra to represent and solve situations involving change, rates of change, linear equations, and inequalities.		✓
	Relate slopes of line to constant rates of change.	✓	✓
Data analysis, statistics and probability: Pose questions and collect, organize, and represent data to answer those questions.	Design experiments and surveys with consideration for issues of sampling (e.g., size, bias).		
	Describe different types of data and organize collections of data (e.g., nominal, ordinal, interval, ratio).	✓	
	Choose, create, and use various representations of data (e.g., histograms, stem and leaf plots, box and whisker plots).	✓	
Data analysis, statistics and probability: Interpret data using methods of exploratory data analysis.	Describe and interpret measures of the center of a data set and know which measure to use for particular situations.	✓	
	Describe and interpret the spread of a set of data (e.g., range, quartile).	✓	
	Analyze and interpret relationships between variables (e.g., scatterplots).		
	Analyze different representations of the same data to determine effects of the representation.	✓	
Data analysis, statistics and probability: Develop and evaluate inferences, predictions, and arguments that are based on data.	Develop conclusions about a characteristic in the population.	✓	

Content Standard	Grade Cluster Benchmarks Grades 6–8	Benchmarks Addressed by	
		Reshaping Mathematics (In Development)	***Algebra I: A Process Approach***
	Explain that differences in data may indicate an actual difference in the populations from which the data were collected or that the differences may result from random variation in the samples.	✓	
	Use data to answer the questions that were posed, describe the limitations of those answers, and pose new questions that arise from the data.	✓	
Data analysis, statistics and probability: Understand and apply basic notions of chance and probability.	Judge the likelihood of uncertain events and connect these judgments to percents or proportions.	✓	
	Understand what it means for events to be equally likely and a game or process to be fair.	✓	
	Calculate theoretical probabilities based on assumptions about sample space, and compare with experimental results.	✓	

Mathematics Content Standards: Alignment for Grades 9–12

Content Standard	Grade Cluster Benchmarks Grades 9–12	Benchmarks Addressed by	
		Algebra I: A Process Approach	***Geometry: A Moving Experience***
Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.	Recognize and use real and complex numbers and infinity.	✓ (For real numbers)	
	Represent real and complex numbers variously (e.g., number line, coordinate plane, rational exponents, and logarithms).	✓ (all except logarithms)	
	Model situations appropriately with vectors or matrices.	✓ (in 2001 edition labs)	
Number and Operations: Understand the meaning of operations and how they relate to each other.	Add, subtract and scalar multiply vectors.		
	Represent and operate with matrices no larger than 3 X 3.	✓ (in 2001 edition labs)	
Number and Operations: Use computational tools and strategies fluently and when appropriate, use estimation.	Recognize conditions governing use of formulas (e.g., discriminant in the quadratic formula, $(a^b)^c = a^{bc}$ can be false when a is negative).		
	Identify computational limitations of calculators and computers (e.g., dividing by small numbers).		
	Understand the effects of measurement error on computed values.		
	Analyze effects of rounding in various situations.		
	Use vectors or matrices to solve problems.	✓ (in 2001 edition labs)	
Measurement: Understand attributes, units, and systems of units in measurement; and develop and use techniques, tools, and formulas for measuring.	Analyze how changes in the measurement of one or more attributes of an object relate to other measurements (e.g., "In doubling the volume of a cube, what happens to the lengths of the sides?").		✓

Content Standard	Grade Cluster Benchmarks Grades 9–12	Benchmarks Addressed by	
		Algebra I: A Process Approach	*Geometry: A Moving Experience*
	Explain rate of change as a quotient of two different measures (e.g., velocity = change in displacement / change in time).		
	Use degree measures in problem situations.		✓
	Determine precision, accuracy and measurement errors; identify sources and magnitudes of possible errors in a measurement setting; describe how errors can propagate within computations; and determine how much imprecision is reasonable in various measurements.		
	Experimentally determine and use formulas for the volume of a sphere, cylinder, and cone.		✓
	Apply limit concepts to develop concepts of area under a curve and instantaneous rate of change.		
	Combine measurements using multiplication or ratios to produce measures such as force, work, velocity, acceleration, density, pressure, or trigonometric ratios.		
Geometry and spatial sense: Analyze properties of objects and relationships among the properties.	Make and evaluate conjectures about, and solve problems involving classes of two- and three-dimensional geometric objects (e.g., "Are all squares rectangles?").		✓
	Use logical reasoning to create and defend valid geometric conjectures.		✓
Geometry and spatial sense: Use transformations and symmetry to analyze mathematical situations.	Represent transformations of objects in the plane with coordinates, vectors, or matrices; describe the effects of a given transformation.		
	Apply transformations to three-dimensional objects.		✓
Geometry and spatial sense: Use visualization and spatial reasoning to solve problems both within and outside of mathematics.	Sketch three-dimensional objects and spaces from different perspectives and interpret two- and three-dimensional drawings of objects.		✓

Content Standard	Grade Cluster Benchmarks Grades 9–12	Benchmarks Addressed by	
		Algebra I: A Process Approach	***Geometry: A Moving Experience***
	Analyze and describe cross-sections, truncations, and compositions / decompositions of three-dimensional objects.		✓
Geometry and spatial sense: Select and use different representational systems, including coordinate geometry.	Solve problems involving two- and three-dimensional figures.		✓
	Analyze and apply coordinate systems on a sphere (distance and place on the earth's surface, positions of stars in heavens.)		
Patterns, functions, and algebra: Understand various types of patterns and functional relationships.	Describe and use advanced functions (e.g., absolute value, piece-wise defined, step, trigonometric, logarithmic, exponential, polynomial).		
	Analyze and use linear relations among three variables.		
	Analyze and use quadratic relations between two variables.	✓	
Patterns, functions, and algebra: Use symbolic forms to represent, model, and analyze mathematical situations.	Represent advanced functions with graphs, tables, and symbolic rules and translate among these representations.	✓ (with linear functions)	
	Model phenomena with a variety of functions and explain how and why a particular function can model many different situations.		
	Approximate and interpret accumulation and rates of change for functions representing a variety of situations (e.g., compound interest).		
Data analysis, statistics and probability: Pose questions and collect, organize, and represent data to answer those questions.	Design and carry out investigations or experiments with two variables.		
	Select appropriate methods for collecting, recording, organizing, and representing data; and describe how a change in representation affects the likely interpretation of the information.		

Content Standard	Grade Cluster Benchmarks Grades 9–12	Benchmarks Addressed by	
		Algebra I: A Process Approach	***Geometry: A Moving Experience***
Data analysis, statistics and probability: Interpret data using methods of exploratory data analysis.	Compute, identify and interpret measures of center and spread (including standard deviation).		
	Look for patterns in data and understand their use in interpretation of the data.		
	Explain how sample size or transformations of data affect shape, center, and spread.		
	Explain trends and use technology to determine how well different models fit data (e.g., line of best fit).		
Data analysis, statistics and probability: Develop and evaluate inferences, predictions, and arguments that are based on data.	Identify good models for phenomena (e.g., exponential model for population growth).		
	Apply models to predict unobserved outcomes.		
	Evaluate conclusions based on data and support inferences with valid arguments.		
Data analysis, statistics and probability: Understand and apply basic notions of chance and probability.	Identify relationships among events (e.g., inclusion, disjoint, complementary, independent, and dependent).		
	Compute probabilities of two events under different relationships, unions, intersections.		
	Use fundamental counting principle, permutations, and combinations as counting techniques to solve problems.		
	Compute the theoretical probabilities of repeated experiments with replacement and repeated experiments without replacement.		
	Recognize random variables in real situations (e.g., insurance, life expectancy) and estimate and compute expectations.		