About the series . . .

Reshaping Mathematics for Understanding is a series of fourteen units suitable for sixth- through eighth-grade students that addresses important topics in middle-grades mathematics, including geometry, measurement, proportional reasoning, rational numbers, probability and statistics, and algebra. The entire series is designed to help students learn to think mathematically. It focuses on developing students' understanding of mathematical concepts and on their ability to draw connections among these concepts. It may serve either as the primary resource in the mathematics curriculum or as a complement to other material. The titles for the full series are listed below.

- Getting Started
- Motion Geometry
- Measurement
- Polygons
- Dilations
- Fractions
- Decimals
- Ratio and Proportion
- Area of Polygons
- Solids
- Probability and Statistics
- Integers
- Algebra Patterns and Relationships
- Number Theory

Access for Every Student

The problems and tasks in the *Reshaping Mathematics for Understanding* series are designed to enable every student to approach mathematics through familiar contexts. Building problem situations on students' past experience makes the study of mathematics more accessible and allows them to expand their thinking. Similarly, to promote genuine engagement, many problems have more than one solution path, giving students opportunities to choose the strategies they prefer using to solve problems. Additionally, to give students opportunities to interact with each concept at varying levels of abstractness and generality, the lessons present new ideas over several days. This design feature also allows students to learn at different rates.

Learning with Understanding

Students can learn only what they truly understand. To learn, they must understand the concepts that underlie operations; they must make connections among processes and concepts; and they must know when and how to apply concepts and operations to problems. Understanding mathematics means doing more than learning how to perform calculations to get correct answers. The topics in this series, developed through problems and lab explorations, encourage students to deepen their conceptual understanding through the practice of reasoning and problem solving.

Role of Visual/Spatial Thinking and Reasoning

Mathematics can and should be a lively course of study for students. It should engage them in active inquiry and give them many opportunities to explore problems whose solutions add to their understanding of the world. For many students, however, mathematics involves merely manipulating numbers. With this limited view, they do not learn how to use multiple contexts, tools, and strategies to solve problems or how to integrate mathematical concepts into broader contexts. Transformational (motion) geometry, which plays an important role in the sequence of the concepts in this series, emphasizes the use of visual contexts and spatial thinking. Lesson discussions further encourage students to use their understanding of spatial relations to make connections among concepts. The Motion Geometry and Dilations units, in particular, give students valuable experience with transformations and enhance the study of many related topics.

Unit Design

Each unit in the *Reshaping Mathematics for Understanding* series can be used individually, or units can be used in clusters. Throughout the series, references among units direct teachers and students to tasks that will help them connect their understanding of new concepts to related experiences.

The lessons in each unit are uniquely designed to enable students to progress through a sequence of tasks that maximize learning with understanding. Rather than present a topic in its entirety in one day, the lessons develop concepts over time. Most concepts begin with an open-ended problem that draws on students' previous experience and intuition and allows for multiple responses. The variety of solutions students propose to such a problem helps the teacher assess what background knowledge students bring to the topic. The problems and tasks that follow develop the concept through a sequence of approaches that provide several direct examples of the concept, raise questions for clarification, offer alternative viewpoints, and prompt students to summarize ideas**.**

The Lessons

There are two forms of lessons in each unit, problem sets and in-class labs. Problem sets consisting of three to five problems are designed to cover several concepts related to the unit topic. Problem sets should be assigned for homework and discussed in class the next day. When students find a problem too difficult to solve on their own, they should be instructed to write questions to help them solve it, and to ask the questions in the discussion. Students work on the labs in small groups in class and debrief afterward, giving them an opportunity to work collaboratively and to concentrate on one strand.

The class discussions are essential in helping students build conceptual understanding. Sharing their solutions and questions allows students to reflect on their thinking and to consider input from others. Both the teacher and students share the responsibility for making discussions productive. As they solve the problems and complete the lab tasks, students explain their thinking, offer alternative responses, and ask questions. Teachers facilitate, asking strategic questions to focus students' thinking on critical ideas. In guiding the discussion, teachers should ensure engagement in the learning process by encouraging students to monitor their learning and by providing a safe, open learning environment in which to share, discuss, and address misconceptions.

Assessment

Except for the Getting Started unit, designed to orient students to the series, all units include suggested assessment items. To check periodically that students understand and that they are participating in the class discussions, teachers can also create "instant" quizzes to give the day after a problem set discussion or lab debriefing. These quizzes should be unannounced and should take between five and ten minutes at the start of class. They have three purposes:

to emphasize the importance of student discussions;

to emphasize the value of information shared by students;

to highlight the importance of asking questions to clarify understanding.

In creating an instant quiz, teachers should focus on students' ideas from the discussion and write two or three short-answer questions as follows:

- a question that is content-based or refers to an understanding that has been established.

  Example: What did we decide it meant to measure the perimeter of a polygon?

- a question that refers to someone's method for solving a problem or an alternative point of view.

  Example: What method did Leslie use to find the area of the trapezoid?

- a question that highlights an undefined assumption or a statement that needed clarification.

  Example: What did Jose mean by *proportional*?

Materials

A list of the materials needed for each lesson appears at the front of every unit. Although students need no special materials or equipment to complete the homework problems, it would be helpful to have them available during discussions. Encouraging students to use a broad range of tools to explore and express their thinking promotes greater understanding.