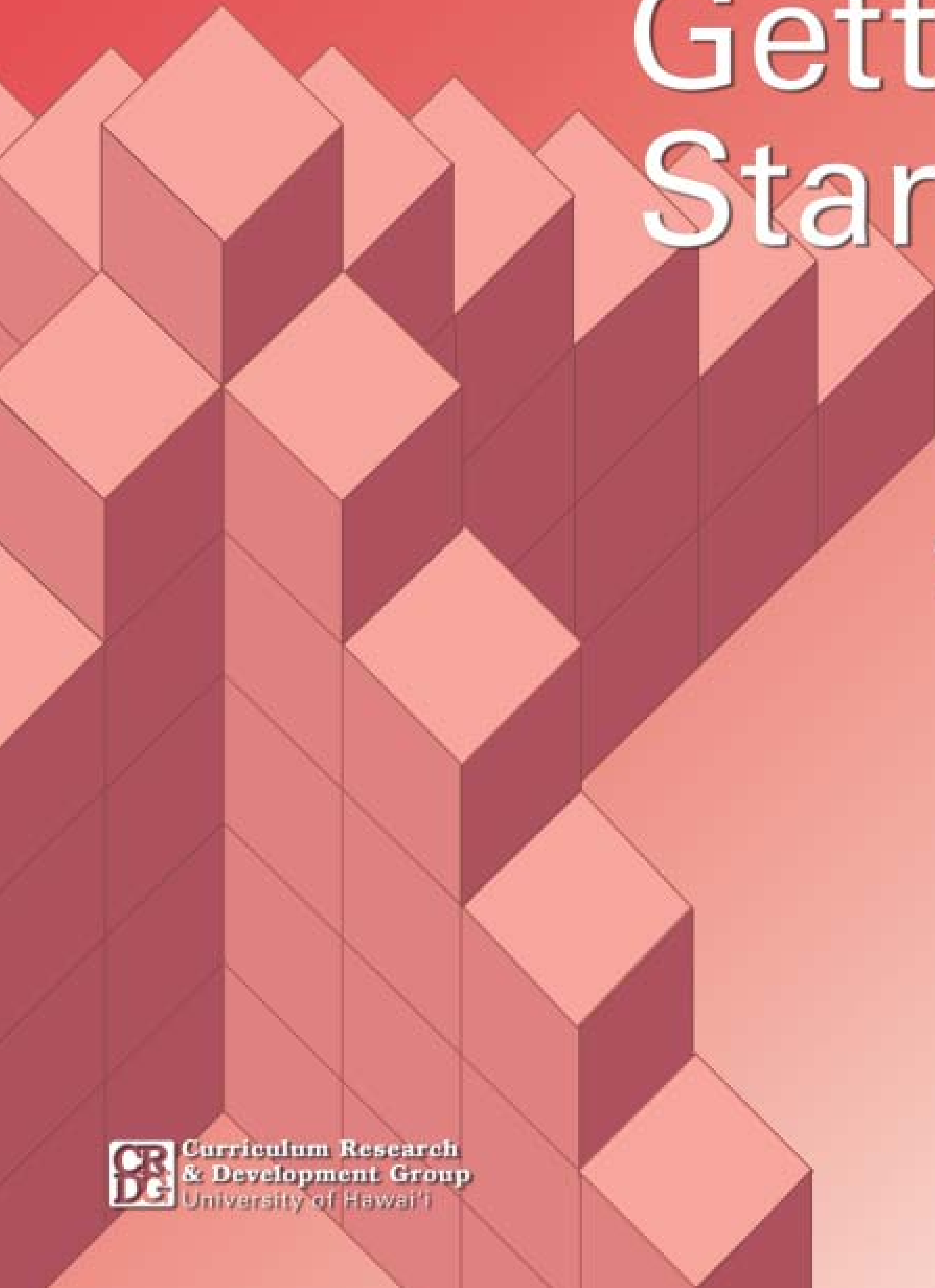
Reshaping Mathematics for Understanding

Getting Started



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Contents

About the unit . . .

The Getting Started unit introduces students to the types of problems and processes used throughout the *Reshaping Mathematics for Understanding* series. The unit may be introduced at the beginning of the school year or whenever it is appropriate to teach strategies that use problem solving to develop mathematics concepts. As students solve the problems in this unit and discuss their solutions, they practice behaviors that deepen their understanding and contribute to a positive learning environment.

Deepening Understanding

The problems in the Getting Started unit deepen students' understanding of mathematics by encouraging them to clarify concepts and challenge their own assumptions. Additionally, by providing opportunities to give and follow directions, create representations, and explain their thinking, the problems in this unit encourage students to recognize and accept multiple strategies and to be flexible in their thinking.

Creating a Positive Learning Environment

The Getting Started unit encourages students to ask questions, test ideas, and offer alternative points of view. These behaviors allow students to practice the skills they need for communicating effectively in a mathematics classroom and encourage them to share the responsibility for making the class successful. At the same time, the problems ask students to explain their thinking, ask productive questions, and challenge answers without being negative. Engaging in mathematics tasks that are specifically designed for students to practice desired behaviors and to establish social norms is the most effective means for building a positive classroom environment.

The Strands in Getting Started

Communication: To communicate effectively, students must explain, describe, clarify, interpret, and question. The problems in this unit are designed to generate active inquiry. Problems that are open to interpretation motivate students to explain their points of view. Encountering problems that they may solve in more than one way or that have more than one correct answer encourages them to exchange ideas and build understanding. Having to describe and explain their processes makes them more aware of their own thinking and gives them more control over when to apply problem-solving strategies. Communication also helps students learn other ways to solve problems. They often ask each other, "How did you know to use that method?" The thinking that underlies the strategy is often a mystery to them. A focused inquiry helps them to be more conscious of the thought processes they use to solve problems.

Problem Solving: While the entire unit takes a problem-solving approach, this strand introduces key processes essential in helping students develop the strategies they need to become effective problem solvers. First, they must understand that solving problems requires them to make certain assumptions about the problem conditions. The problems in this strand lead them to recognize that their assumptions stem from their previous knowledge and experience. Becoming aware of their assumptions leads them to recognize a wider range of possible contexts for problems. Second, to solve these problems, students must use organized strategies. Problems that require them to identify, describe, and extend patterns give them valuable strategic experience in recognizing mathematical relationships. Lastly, to teach them that they must learn to test their solutions independently, many problems in this strand ask them to explain and justify their solutions.

Representation: The ability to represent and interpret representations of problem situations enhances students' problem-solving abilities. Representation helps students mathematically communicate how they understood the problem. In the Getting Started unit, students use diagrams, tables, and symbols to create and interpret mathematical situations.

Visual/Spatial Thinking: Visual/spatial thinking helps all students understand and solve problems. Students who have not attained fluency with numbers may see patterns and relationships more easily when they are represented geometrically. Exercising visual/spatial skills also builds students' conceptual understanding and prepares them for more abstract problem-solving tasks. The entire *RMU* series offers many opportunities for students to use visual/spatial thinking.

Introduction to Motion: Transformational-geometry activities are important in middle-grades mathematics. They provide a context for students to observe figures, reason about relationships among figures, and make connections to important middle-grades topics such as proportion. The problems in the Introduction to Motion strand of this unit ask students to build informal proofs about figures and their properties, a practice that develops the understanding they will need in high school mathematics courses.

Concept Development

The numbers in this matrix refer to the problem numbers in each lesson.

STRAND and Focus	Lab A	Lab B	PS 1	PS 2	PS 3	PS 4	PS 5	PS 6
Communication								
Giving/Following Directions	1–4	Tie a Bow						3
Description			2					
Problem Solving								
Making Assumptions			1		1			
Numerical Patterns					3		1	3
Problem Conditions				1				
Testing Solutions							1	
Representation								
Interpreting Diagrams				3		2		
Representing Motion								3
Visual/Spatial Thinking								
Networks				4		1		
Reconfiguration					2		2, 3	1, 2
Visual Patterns			2	1	4	2		2
Testing Perceptions			3	2, 3		3		
Introduction To Motion								
Using Motion in Problem Solving				2	4	3		
Congruence								2

Materials

Special materials for Getting Started

Lab A	Drawing Activity Cards (see blackline masters at the end of the unit), 1 per student
Lab B	Shoe with ties or laces, 1 per student
Problem Set 1	Geometric compass, 1 per group Straight-edge*, 1 per student Square tiles, 20 per group
Problem Set 2	Straight-edge*, 1 per student Tracing paper
Problem Set 3	Counters, 20 per group Toothpicks, 20 per group Tracing paper
Problem Set 4	Geometric compass, 1 per group Cubes such as centimeter cubes or connecting cubes, 70 per group Straight-edge*, 1 per group Tracing paper
Problem Set 5	Toothpicks, 20 per group
Problem Set 6	Dimes and pennies, 3 of each per group Square tiles, 70 per group Toothpicks, 20 per group

*Straight-edges without unit markings are preferable to rulers. Blank index cards could also be used.

Name ______________________

Lab B

How to Tie a Bow Activity

Exchange your directions for tying a bow with someone in your group. Each of you must *follow the directions exactly as written*. Do not let what you already know interfere with following your partner's directions. Give each other feedback about your directions, using questions like these:

Could you understand and follow the directions?

What was clear? What was not clear?

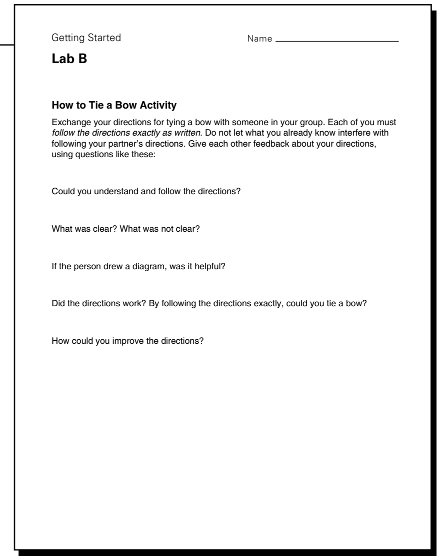
If the person drew a diagram, was it helpful?

Did the directions work? By following the directions exactly, could you tie a bow?

How could you improve the directions?

Lab B

Following Directions: How to Tie a Bow


Getting Started Name

Lab B

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If the person drew a diagram, was it helpful?

Did the directions work? By following the directions exactly, could you tie a bow?

How could you improve the directions?

Lab B

Strand: Communication

Focus: Giving/Following directions

Task: Identifying and describing the steps in doing a routine task

PROBLEM

How to Tie a Bow Activity

Exchange your directions for tying a bow with someone in your group. Each of you must *follow the directions exactly as written*. Do not let what you already know interfere with following your partner's directions.

Give each other feedback about your directions, using questions like these:

Could you understand and follow the directions?

What was clear? What was not clear?

If the person drew a diagram, was it helpful?

Did the directions work? By following the directions exactly, could you tie a bow?

How could the directions be improved?

Teacher's Insight

Many mathematics problems call for writing about the process used to solve a problem. Students must communicate their understanding meaningfully and accurately. They must analyze a strategy for completing a task and convey that method to someone else. This activity gives students the chance to practice becoming conscious of how they perform a familiar action and to write about it. Using diagrams and written explanations helps them to communicate effectively.

ANSWER

Students' directions should be detailed and specific. Students may include step-by-step diagrams with their directions.

★ HOMEWORK

Problem Set 1

Discussion

Before starting the discussion, have students give written feedback to their partners using these prompts:

"I wasn't sure about . . ."

"Something you put in your directions that helped was . . ."

What were some ways that this person made the directions easier to follow?

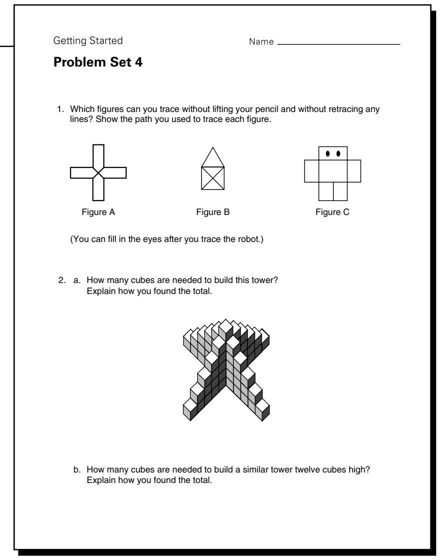
What do you think this activity has to do with math class?

What are your opinions about this homework assignment? Was it easy or difficult? Why?

What made this activity easy? What made this activity difficult? Explain.

Problem Set 4

Justifying Your Answer


Getting Started Name

Problem Set 4

1. Which figures can you trace without lifting your pencil and without retracing any lines? Show the path you used to trace each figure.

Figure A Figure B Figure C

(You can fill in the eyes after you trace the robot.)

2. a. How many cubes are needed to build this tower? Explain how you found the total.

b. How many cubes are needed to build a similar tower twelve cubes high? Explain how you found the total.

Problem Set 4, No. 1

Strand: Visual/Spatial Thinking

Focus: Networks

Task: Using flexibility to find multiple solutions in visual problem solving and to decide that no solution exists

PROBLEM

1. Which figures can you trace without lifting your pencil and without retracing any lines? Show the path you used to trace each figure.

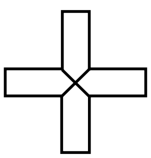


Figure A

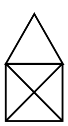


Figure B

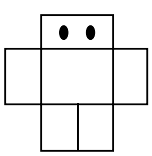


Figure C

(You can fill in the eyes after you trace the robot.)

Teacher's Insight

Because they have had experience with a similar problem, students are more confident approaching this problem. You can focus their attention on explaining how they found their path for tracing the figures and on recording alternate methods. You may be able to highlight some observations about why certain starting points are possible and others are not. However, a lengthy discussion about networks is not appropriate at this time.

Include some discussion, as needed, about how and when students decide that it is not possible to solve a problem.

ANSWER

1. Figures B and C are possible. Figure A is not possible.

Discussion

Could you predict before you started tracing that one figure would be impossible to trace?

How did you make your prediction?

How did you represent the path you took to trace the figures?

Is it possible to use more than one route on any figure?

Why is figure A impossible?

Why should figure A be included in this problem?

How many different starting points are possible for figure B? For figure C?

Do the different starting points have anything in common?

Problem Set 4, No. 2

Strand: Representation and Visual/Spatial Thinking

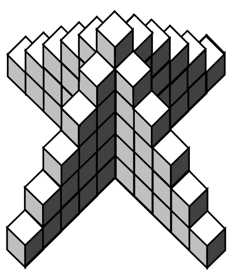
Focus: Interpreting diagrams and visual patterns

Task: Interpreting a diagram and using it to problem solve

PROBLEM

2. a. How many cubes are needed to build this tower?

   Explain how you found the total.



   b. How many cubes are needed to build a similar tower 12 cubes high? Explain how you found the total.

ANSWER

Ask the class to share a variety of explanations of how they decided on their answer.

2. a. 66 cubes

   b. 276 cubes

Teacher's Insight

In this problem, students have to make an assumption about the diagram. Their solution depends on whether they assume that the column of cubes in the middle of the figure is filled or hollow. Students propose and evaluate arguments for why their assumption makes sense.

Students also need to clarify how they interpret the direction "similar tower." Their solutions depend on that interpretation.

Discussion

Did you have any questions about the problem before you tried to solve it?

How did you resolve your questions?

Have students describe and explain solutions, making sure they understand each other's interpretation of the diagram and the directions.

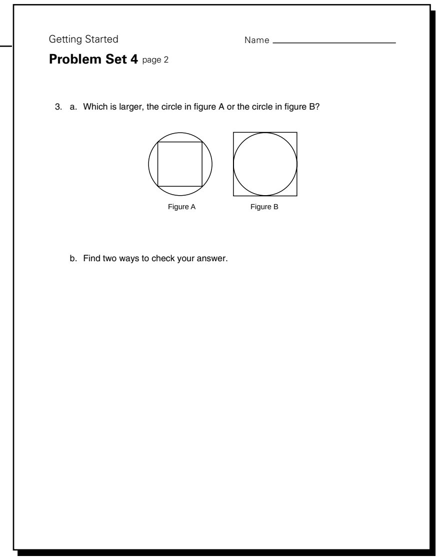
Have them explain why certain interpretations are more productive than others.

If students were not sure how many cubes are in the middle column, ask why that would matter.

How did you interpret "similar tower"?

How did you use your solution for 2.a. to solve 2.b.?

How would you explain to someone who thinks a "similar tower twelve cubes high" takes 132 cubes (66 x 2) that the answer is not correct?


Getting Started Name

Problem Set 4 page 2

3. a. Which is larger, the circle in figure A or the circle in figure B?

Figure A Figure B

b. Find two ways to check your answer.

Problem Set 4, No. 3

Strand: Visual/Spatial Thinking and Introduction to Motion

Focus: Testing perceptions and using motions in problem solving

Task: Applying motions to test the congruence of circles

PROBLEM

3. a. Which is larger, the circle in figure A or the circle in figure B?

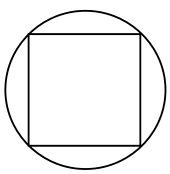


Figure A

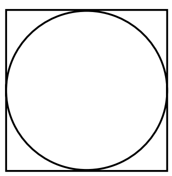


Figure B

b. Find two ways of checking your answer.

Teacher's Insight

This problem uses an optical illusion to help students realize that they can't always trust what they see. Some will try certain measurements to compare the circles. Students often believe that because measuring yields a number, it is a more valid strategy than one that does not yield a number. The problem gives the class an opportunity to begin to see the value of using motions.

Motions (reflection, translation) are intuitive but powerful strategies for comparing geometric figures. Motions are used in many units in this series. For students who have not studied transformations, motions seem "too easy" to be considered a valid mathematical strategy.

Tracing paper will be useful for demonstrating and practicing the motions informally.

Answer

3. a. The circles are congruent.

   b. Trace the circle in figure A, then slide the tracing to the right to cover the circle in figure B. Another way is to fold the paper between the two figures so that the figures overlap each other. The circles will match up perfectly.

★Homework

Problem Set 5

Discussion

How did you determine which circle was bigger?

How did you check your decision?

If students describe measuring or used a compass, ask how they decided where to measure. How did you find the center of the circle? How can you be sure you used the diameter? How can you be sure of your accuracy?

Did one of your checks make you feel more certain of your answer than the other check(s) did? Why?

What is mathematical about tracing and matching the circles to compare their size?

Name ____________________

Problem Set 4

1. Which figures can you trace without lifting your pencil and without retracing any lines? Show the path you used to trace each figure.

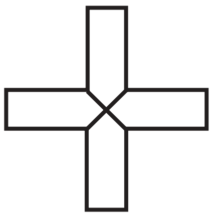


Figure A

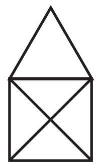


Figure B

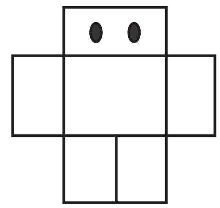
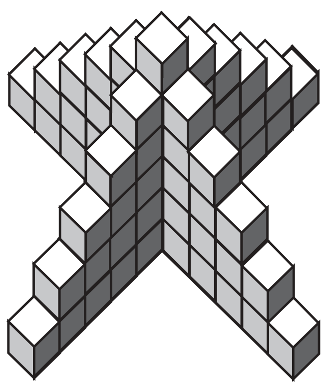


Figure C

(You can fill in the eyes after you trace the robot.)

2. a. How many cubes are needed to build this tower? Explain how you found the total.



b. How many cubes are needed to build a similar tower twelve cubes high? Explain how you found the total.

Name ______________________

Problem Set 4 page 2

3. a. Which is larger, the circle in figure A or the circle in figure B?

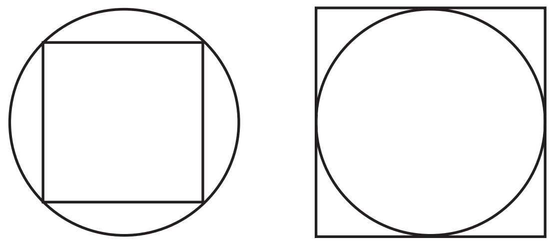


Figure A Figure B

b. Find two ways to check your answer.